GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

OCTOBER / NOVEMBER 2005
OKTOBER / NOVEMBER 2005

ADDITIONAL MATHEMATICS HG

TIME: 3 hours

MARKS: 400

INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections should be answered from Sections B, C, D and E.
- Each section should be answered **in a separate answer book and the relevant section should be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing all the answer books in together.
- A diagram sheet is provided on page 16 for answering Question 2.2. Detach it and put it in your answer book.
- Unless otherwise indicated, non-programmable calculators may be used.
- This examination paper consists of 19 pages. Statistical tables and relevant formula sheets can be found on pages 17 to 19.
- All essential calculations should be clearly shown.
- All angles are measured in radians and answers should be given in radians.
- Writing should be legible.

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

1.1 Define the following concepts fully:

1.1.1 The function f is continuous at the point where $x = a$. (6)

1.1.2 The function f is differentiable at $x = a$. (4)

1.2 Given the function

$$f(x) = \begin{cases} -x + \frac{p}{2} & \text{if} \quad x < 0 \\ \arccos x & \text{if} \quad 0 \le x < 1 \\ 2 & \text{if} \quad x = 1 \\ (x-1)^2 & \text{if} \quad x > 1 \end{cases}$$

1.2.1 Discuss the continuity of $f(x)$ for the following values of x. Classify any discontinuities.

(a) $x = 0$ (8)

(b) $x = 1$ (8)

1.2.2 Discuss the differentiability of $f(x)$ at $x = 0$ (8)

[34]

QUESTION 2

2.1 Without using a calculator, determine:

2.1.1 $\arcsin\left(\frac{-\sqrt{3}}{2}\right)$ (4)

2.1.2 $\cos(\arccos\sqrt{2})$ (2)

2.1.3 $\arctan\left(\cot\left(\frac{4\pi}{3}\right)\right)$ (6)

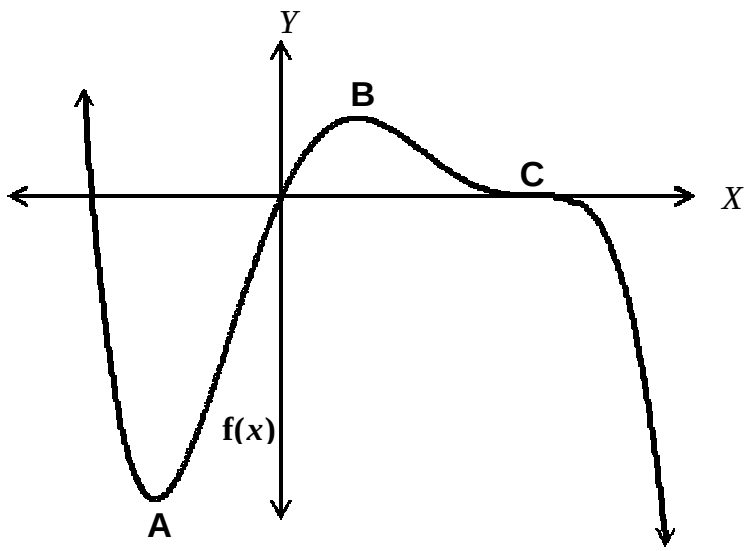
2.2 A sketch graph of $f(x)$ is provided with the following properties:

– $f(x)$ is continuous

– Three stationary points occur at **A** (local minimum), **B** (local maximum) and **C** (point of inflection).

– Three zeroes (x – intercepts) occur at $x = -1{,}5$; 0 and 2.

– $f(x) \to -\infty$ if $x \to \infty$, and $f(x) \to \infty$ if $x \to -\infty$.



Use the diagram sheet provided on page 16 and draw on the same set of axes a rough sketch of $f'(x)$. Show clearly where $f'(x)$ has a maximum, minimum or zero. Detach it and place it in your exam book. (10)

2.3 Determine the value of the following limits, if they exist:

2.3.1 $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1}$ (8)

2.3.2 $\lim\limits_{?\to 0} \dfrac{\sin ?}{|?|}$ (8)

[38]

QUESTION 3

3.1 If $f(x) = \arcsin x$ and $g(x) = 2x - 3$

3.1.1 Determine $(f \circ g)(x)$. (2)

3.1.2 Show that $D_x(f \circ g)(x) = \dfrac{1}{\sqrt{3x - x^2 - 2}}$ (8)

3.1.3 Hence calculate the value of $\int_{3/2}^{7/4} \dfrac{dx}{\sqrt{3x - x^2 - 2}}$ (8)

3.2 Find $\dfrac{d}{dx}\left(\sqrt{1-2x}.\sec x^2\right)$ (10)

3.3 If $f(x) = (1-2x)^n$, determine $f^{(n)}(x)$, the n^{th} derivative of the function. (12)

[40]

QUESTION 4

4.1 Use the Newton-Rhapson method to find a positive solution for the equation $1 + x = 2\tan x$ in the interval $\left[0; \dfrac{\pi}{2}\right]$. Use 0,8 as a starting value and give the answer correct to 3 decimal places. (12)

4.2 A force, F, drags an object with weight W along a horizontal plane. The force acts along a rope attached to the object. If the rope makes an angle of θ with the horizontal, then the magnitude of the force is given by the equation:

$F(\theta) = \dfrac{aW}{a\sin\theta + \cos\theta}$, where a and W are positive constants.

Show that F will be a minimum when $\tan\theta = a$. (12)

[24]

QUESTION 5

Find the area under the curve $f(x) = -x^2 + 2x + 4$ between $x = 0$ and $x = 3$ by using n strips of equal width, the Riemann Sum, and then letting $n \to \infty$. **[20]**

QUESTION 6

Determine the following integrals:

6.1 $\int \cos 5x \sin 2x\,dx$ (10)

6.2 $\int \dfrac{x}{\sqrt{1-4x^4}}\,dx$ (8)

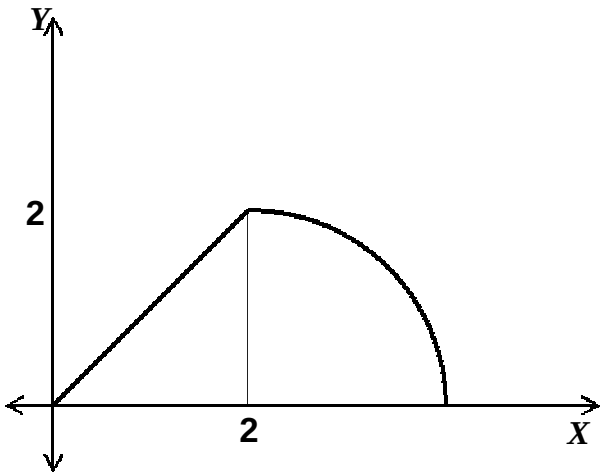
6.3 $\int_0^{\frac{1}{2}} \dfrac{x^3}{\sqrt{1-4x^4}}\,dx$ (Leave the answer in surd form.) (10)

[28]

QUESTION 7

The piecewise function $f(x)$, sketched below, is defined by

$$f(x) = \begin{cases} x & \text{if} \quad 0 \le x < 2 \\ \sqrt{4x - x^2} & \text{if} \quad 2 \le x \le 4 \end{cases}$$



If $f(x)$ is rotated about the x-axis, determine the volume of the resulting solid of revolution. Leave your answer in terms of p. **[16]**

TOTAL FOR SECTION A: [200]

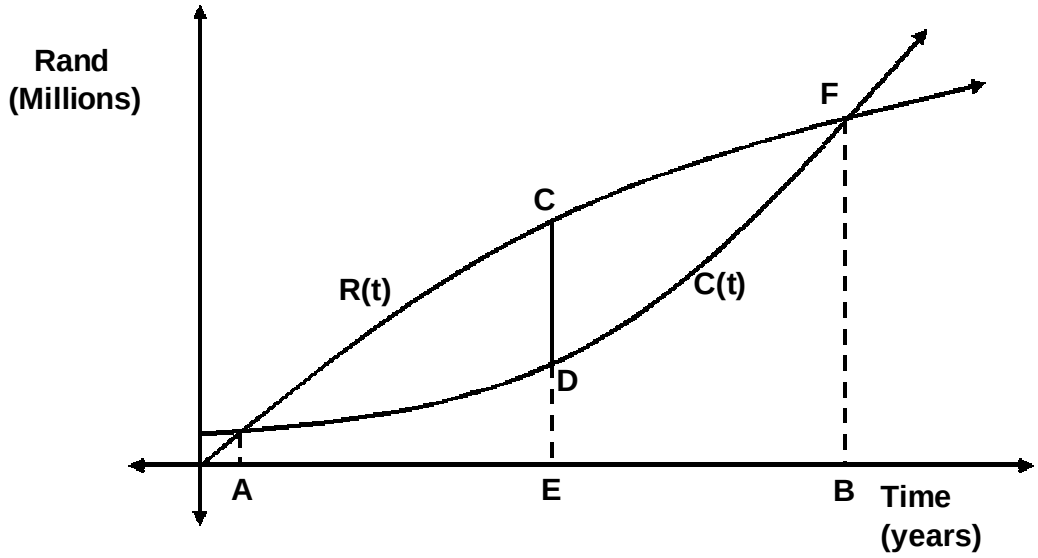
Answer any TWO of the following FOUR sections.

SECTION B
FINANCIAL MATHEMATICS

QUESTION 8

The cost and revenue functions for a platinum mine are given respectively by

$C(t) = 0{,}05t^2 + 40$
$R(t) = -0{,}02t^2 + 7t$ where t is the time (in years) that the mine operates for and $C(t)$ and $R(t)$ are in millions of Rands. The graphs of $C(t)$ and $R(t)$ are given below.



8.1 What does point **A** represent? (2)

8.2 What does point **B** represent? (2)

8.3 If **CD** is a vertical line somewhere between **A** and **F**, what does **CD** represent? (2)

8.4 Determine expressions for the marginal cost C'(t) and marginal revenue R'(t) and hence find the maximum profit. (10)
[16]

QUESTION 9

9.1 Calculate each of the following and write down which is a better investment.

A: An amount **x** is invested at a simple interest rate of 12% per annum for 8 years.
B: An amount **x** is invested at a compound interest rate of 10% per annum for 8 years. (10)

9.2 What compound interest rate would yield the same amount as investment **A**? Give your answer correct to 4 decimal places. (4)

[14]

QUESTION 10

The Minister of Transport pays a deposit of R y on a new car costing R764 346,00. He takes out a bank loan for the balance and pays this off in 3 instalments;

R y in 2 years' time,
R $3y$ in 5 years' time, and
R $5y$ in 7 years' time.

Interest is paid at 17% compounded quarterly for the first 3 years and 19% compounded semi-annually for the remaining 4 years. How much was his deposit? **[18]**

QUESTION 11

Vivaldi wants to buy a new sound system costing R4 700,00. He is offered a hire purchase agreement where the simple interest rate is 10% per annum and payments are due every month for 3 years.

11.1 Calculate Vivaldi's monthly payments to the nearest cent. (8)

11.2 After one year (i.e. 12 payments), Vivaldi finds he is bankrupt. He still owes R4 073,33. A bank agrees to give him a loan for this amount on condition that he pays back x rands per month over 4 years to amortise the debt. He starts paying 5 months after this loan is granted and must make 44 payments. The compound interest on the loan is 12% per annum compounded monthly. Calculate x. (16)

[24]

QUESTION 12

A medical centre has just purchased an x-ray machine for R3,5 million.

12.1 The centre began saving for this machine a few years ago. R60 000 was paid at the end of every 3 months into an account earning 9% interest per annum, compounded quarterly. Their last payment was with the purchase of the machine. Calculate for how many months they saved in order to have enough money to buy the machine. (12)

12.2 In 8 years from now the medical centre will have to replace the x-ray machine. They want to avoid taking out another loan for this. Taking inflation and depreciation into account, they work out they will need R3 705 104 immediately after the last payment to replace the machine. If the centre sets up a sinking fund, starting in one month's time, to pay for the new machine, what will the monthly payments be (to the nearest cent)? The interest is now 9% p.a., compounded monthly. (8)

12.3 After the 80^{th} payment of R26 492,25 into the account, the old machine breaks and has to be replaced. At this time a new machine costs R5 million. They do not have enough money in the sinking fund to buy this and have to take out a loan for the balance. Calculate the value of this loan. (8)

[28]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY

QUESTION 13

13.1 Two lines l_1 and l_2 are defined by the equations:

$l_1: \quad x - 2y + 3 = 0$

$l_2: \quad 2x + 3y + 1 = 0$

13.1.1 Find the acute angle between l_1 and l_2. Give your answer correct to 2 decimal places. (10)

13.1.2 Find the point **P** which is symmetrical to **(-2; 3)** in the line l_1. (14)

13.2 Find the equation of the line which passes through the intersection of $3x - y = 4$ and $x - 2y = 18$ and is perpendicular to $2x - 3y - 7 = 0$. (12)

[36]

QUESTION 14

14.1 The equation of a parabola is $y - x^2 - 5x = 0$. Find the equation of the tangent and the normal at the point on the parabola where $x = -2$. (14)

14.2 An ellipse is defined by the equation $\frac{(x-3)^2}{81} + \frac{(y+4)^2}{121} = 1$.

Determine the . . .

14.2.1 eccentricity (6)

14.2.2 foci (4)

14.2.3 directrices and (6)

14.2.4 area enclosed by the ellipse (4)

[34]

QUESTION 15

Two circles have the following equations:

$$(x+4)^2+(y-4)^2=100$$
$$(x+7)^2+(y-8)^2=225$$

15.1 Show that the circles touch internally. (8)

15.2 Find the equation of the common tangent. (8)

[16]

QUESTION 16

16.1 Write down the equation of the plane V, parallel to the plane $2x+3y-z-2=0$ if V intersects the y axis at **-4**. (6)

16.2 Determine the perpendicular distance between the two planes. (8)

[14]

TOTAL FOR SECTION C: [100]

SECTION D
ALGEBRA

QUESTION 17

17.1 Prove the following assertion with the help of Mathematical induction:

$a+ar+ar^2+.....+ar^{n-1}=\frac{a(1-r^n)}{1-r}$ for all a and r in R with $r\neq 1$ for all $n\in \mathrm{N}$. (14)

17.2 Rationalize the denominator of $\frac{1}{a^2-a+2}$ if $a=\sqrt[3]{-4}$. (24)

[38]

QUESTION 18

Decompose the following into partial fractions:

$$\frac{x^3-6x^2+x}{x^4-1}$$

[18]

QUESTION 19

19.1 State Eisenstein's Criterion. (6)

19.2 Decompose the following polynomial completely into factors in **Z[*x*]** if $3+2\sqrt{2}$ is a zero: $5x^5-32x^4+17x^3+8x^2-60x+10$ (18)

[24]

QUESTION 20

$$f(x)=\frac{x^2+2x-3}{x-4}$$

f(x) has a local minimum at (8,6 ; 19,1) and a local maximum at (-0,6 ; 0,8).

20.1 Determine the intercepts with the ***x***-axis and ***y***-axis. (6)

20.2 Determine all the vertical, horizontal and oblique asymptotes of this function. (6)

20.3 Make a neat sketch of the graph of ***f(x)***, and show all the intercepts, asymptotes and turning points. (8)

[20]

TOTAL FOR SECTION D: [100]

SECTION E
STATISTICS

QUESTION 21

21.1 In how many different ways can three green, five yellow and eight red books be arranged on a shelf? (6)

21.2 A box of "Smarties" contains a total of 28 "Smarties" of which six are blue. If I take out six (chosen randomly), find the probability that three will be blue. (6)

21.3 How many times do I have to throw an ordinary die in order to be 95% sure of obtaining at least one six? (10)

21.4 Mamre and Dlamini play table tennis against each other. The first person to win two times, wins the match. Mamre has a chance of 0,75 to win each time. Use a tree diagram to determine the probability that Dlamini will win the game. (12)

[34]

QUESTION 22

The total monthly rainfall (in mm) for two towns **A** and **B** for the first half of 2005 is given below:

	Jan	Feb	Mar	Apr	May	Jun
A	112	98	75	23	12	10
B	39	47	60	98	163	186

22.1 What is Town **A**'s average monthly rainfall for the first half of 2005? (4)

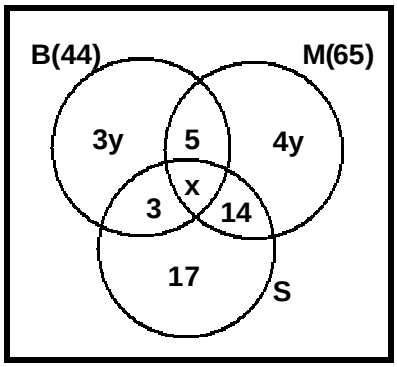
22.2 What is the standard deviation of the readings in Question 22.1? (2)

22.3 Town **B**'s average monthly rainfall is 99 mm and the standard deviation for this data is 57 mm. Compare this with your answers obtained in Questions 22.1 and 22.2 and write down a conclusion which can be drawn regarding the averages and a conclusion regarding the standard deviations. (4)

[10]

QUESTION 23

At Hlanganiphile High School there are 120 Grade 12 learners. A survey is conducted to see how many take the subjects Biology, Mathematics and Science. The Venn Diagram below shows three sets representing the number of learners taking Biology (B) (a total of 44); Mathematics (M) (a total of 65) and Science (S).



23.1 Using the numbers given in the Venn diagram, find x and y. (8)

23.2 Hence write down . . .

23.2.1 $n(B \cap M' \cap S')$ (2)

23.2.2 $n(M \cap (B \cup S)')$ (2)

23.2.3 the probability that a pupil chosen at random will take none of the three subjects. (4)

[16]

QUESTION 24

The probability density function for the lifespan of a certain insect species is given by

$$f(x) = \begin{cases} -\frac{3}{16}x^2 + \frac{3}{4} & 0 \leq x \leq m \\ 0 & elsewhere \end{cases}$$ where x is the age of the insect in years.

Find m, the maximum lifespan of these insects. **[10]**

QUESTION 25

A very prestigious cross-country race takes place annually in the town of Baleka. In 2004 the times taken by all the competitors to complete the race were normally distributed about a mean of $\mu = 90$ minutes and standard deviation s.

25.1 If 80% of the competitors took less than 2 hours to complete the race, what is the value of s to the nearest minute? (10)

25.2 Only the top 5% of competitors are awarded the famous 'Mvundla' award. Assuming that s is 36 minutes, find the cut-off time for this award (to the nearest minute). (10)

[20]

QUESTION 26

It is believed that 15% of the population in South Africa have blue eyes. If a random sample of South Africans is taken, how large would the sample have to be, to be 95% sure of obtaining an estimate to within 2%? **[10]**

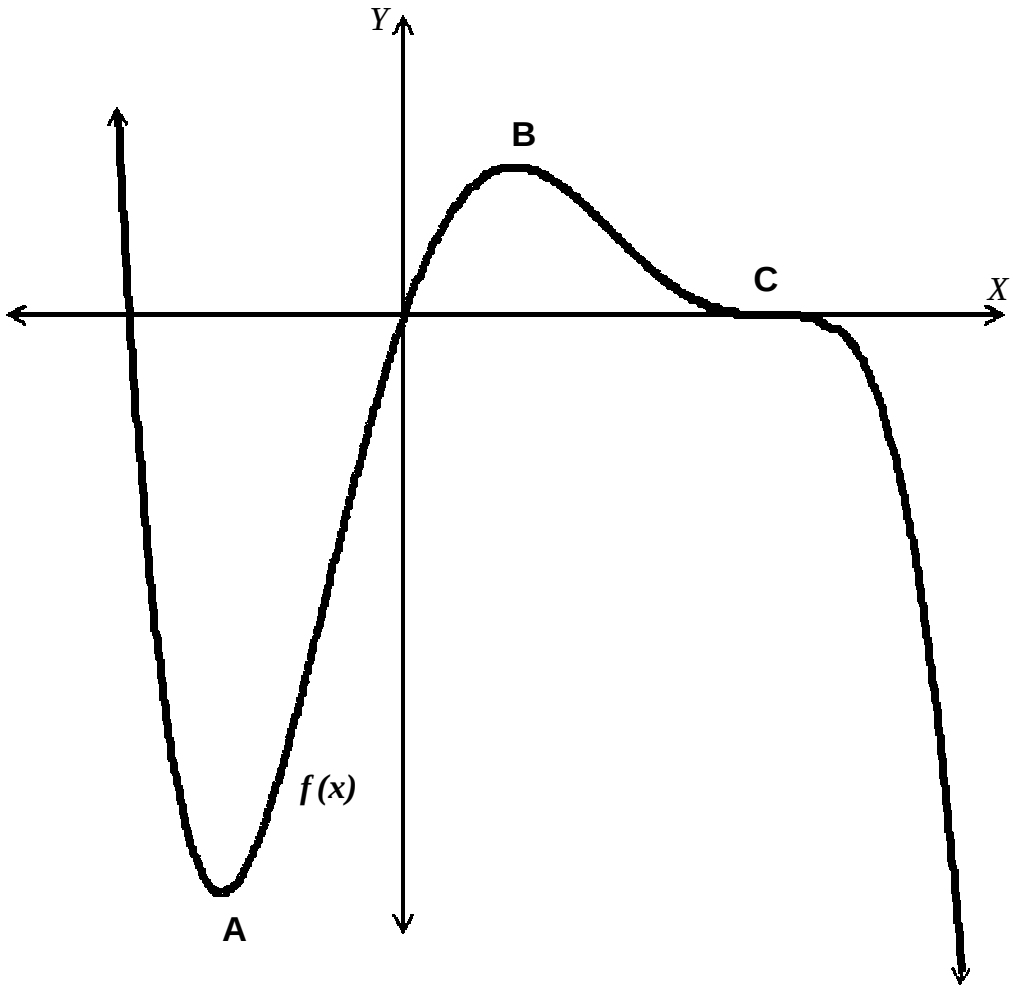
TOTAL FOR SECTION E: [100]

TOTAL: 400

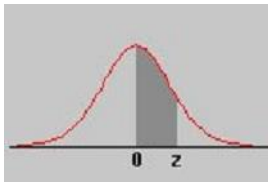
Diagram Sheet / *Diagramblad*

Examination number / *Eksamennommer*

QUESTION 2.2 / *VRAAG 2 .2*



Normal Distribution/ *Normaalverspreiding*



$$P(X \le x) = \frac{1}{\sqrt{2p}} \int_{-\infty}^{x} e^{\frac{-x^2}{2}} dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

FORMULA SHEET/ *FORMULEBLAD*

Differential and Integral Calculus

Differensiaal- en Integraalrekene

$s = r\theta$

$\sin^2 x = \frac{1}{2}(1-\cos 2x) \quad \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin A.\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$\sin A.\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A.\cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$Riemann\ Sum = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$F(x)$	$F'(x)$
$a.x^n$	$na.x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x.\tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
$\arcsin x$ $\operatorname{bgsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$ $\operatorname{bgcos} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$ $\operatorname{bgtan} x$	$\frac{1}{x^2+1}$
$f(x).g(x)$	$f'(x).g(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x)-f(x).g'(x)}{[g(x)]^2}$
$f(g(x))$	$f'(g(x)).g'(x)$

Finance/ *Finansies*

$F=P(1+i)^n \qquad F=P(1-i)^n$

$F=P(1+in) \qquad F=P(1-in)$

$$P = x.\frac{1-(1+i)^{-n}}{i} \qquad F = x.\frac{(1+i)^n-1}{i}$$

Analytical Geometry/ *Analitiese Meetkunde*

$y=4ax^2 \qquad yy_1=2a(x+x_1)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Algebra

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha+\beta+\gamma=-\frac{b}{a}$$

$$a.ß=\frac{c}{a} \qquad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}$$

Statistics / *Statistiek*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!r!}$$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=x) = \frac{\binom{p}{x}\binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z = \frac{X-\mu}{\sigma}$$

$$P\left(X - 1.96\frac{\sigma}{\sqrt{n}} < \mu < X + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(p - 1.96\sqrt{\frac{p(1-p)}{n}} < \pi < p + 1.96\sqrt{\frac{p(1-p)}{n}}\right) = 0,95$$

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1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $T_n = a + (n-1)d$

3. $S_n = \dfrac{n}{2}(a + l)$

4. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

5. $T_n = ar^{n-1}$

6. $S_n = \dfrac{a(1 - r^n)}{1 - r}$

7. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

8. $S_\infty = \dfrac{a}{1 - r}$

9. $A = P(1 + \dfrac{r}{100})^n$

10. $A = P(1 - \dfrac{r}{100})^n$

11. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

13. $y = mx + c$

14. $y - y_1 = m(x - x_1)$

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

16. $m = \tan\theta$

17. $(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2})$

18. $y^2 + x^2 = r^2$

19. $(x - p)^2 + (y - q)^2 = r^2$

20. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

21. $a^2 = b^2 + c^2 - 2bc.\cos A$

22. $\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

23. $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

24. $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

25. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

26. $\cos 2A = \cos^2 A - \sin^2 A$

27. $\sin 2A = 2\sin A \cos A$

END / *EINDE*

END / *EINDE*