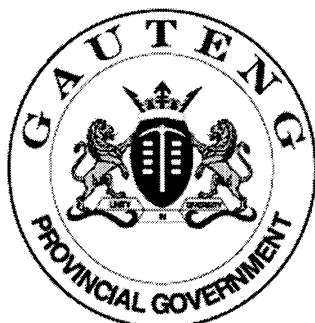


**SENIOR CERTIFICATE
EXAMINATION
SENIORSERTIFIKAAT-EKSAMEN**



**FEBRUARY / FEBRUARIE
MARCH / MAART**

2005

ADDITIONAL MATHEMATICS

***ADDISIONELE
WISKUNDE***



302-1/0

ADDITIONAL MATHEMATICS HG

**17 pages
17 bladsye**



X05



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GAUTENGSE DEPARTEMENT VAN ONDERWYS

SENIORSERTIFIKAAT-EKSAMEN

ADDISIONELE WISKUNDE HG

TYD: 3 uur

PUNTE: 400

INSTRUKSIES:

- Hierdie vraestel bestaan uit VYF afdelings.
 - Afdeling A is VERPLIGTEND.
 - Beantwoord ook nog enige TWEE ander afdelings uit Afdeling B, C, D en E.
 - Elke afdeling moet in 'n **aparte antwoordboek beantwoord word en die betrokke afdeling moet duidelik op die buiteblad aangetoon word**. Plaas alle antwoordboeke in die antwoordboek vir Afdeling A voordat jy dit inlewer.
 - Nie-programmeerbare sakrekenaars mag gebruik word, tensy daar anders aangedui word.
 - Hierdie vraestel bestaan uit 14 bladsye. Statistiese tabelle en formuleblaaie kom onderskeidelik voor op bladsye 15, 16 en 17.
 - Alle noodsaaklike berekeninge moet duidelik getoon word.
 - Alle hoeke is in radiale en antwoorde moet ook in radiale gegee word.
 - Skryfwerk moet leesbaar wees.
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GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

TIME: 3 hours

MARKS: 400

INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections must be answered from Sections B, C, D and E.
- Each section must be answered **in a separate answer book and the relevant section must be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing them in.
- Unless otherwise indicated, non-programmable calculators may be used.
- The examination paper consists of 14 pages. Statistical tables and formula sheets can be found on pages 15, 16 and 17, respectively.
- All essential calculations must be clearly shown.
- All angles are in radians and answers must also be given in radials.
- Please write legibly.

AFDELING A
VERPLIGTEND
CALCULUS

VRAAG 1

- 1.1 Skets op dieselfde assestelsel die grafieke van
 $f(x) = bgtan2x$
 $g(x) = bgtanx$
 Toon op beide grafieke die punt $(x; \frac{\pi}{4})$ aan. (10)

- 1.2 Bereken die waardes van die volgende, sonder die gebruik van 'n sakrekenaar:

1.2.1 $2 bgtan(\frac{-1}{\sqrt{3}})$ (4)

1.2.2 $bgsin(sin \frac{3\pi}{2})$ (6)

1.2.3 $tan(bgcos(-\sqrt{2}))$ (6)
[26]

VRAAG 2

'n Funksie $f(x)$ word gegee:

$$f(x) = \begin{cases} -1 & x < -3 \\ -2x - 5 & -3 \leq x < -1 \\ x^2 - 4 & -1 \leq x < 2 \\ x - 2 & x > 2 \end{cases}$$

- 2.1 Gebruik die definisie van kontinuïteit en bepaal of $f(x)$ kontinu is of nie by die volgende punte. Indien nie kontinu nie, noem ook die tipe diskontinuïteite.

2.1.1 $x = -3$ (8)

2.1.2 $x = -1$ (8)

2.1.3 $x = 2$ (6)

- 2.2 Bepaal algebraïes, met motivering, of $f(x)$ differensieerbaar is by $x = -1$. Dit is nie nodig om vanuit eerste beginsels te werk nie. (8)

[30]

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

- 1.1 Sketch on the same system of axes the graphs of
 $f(x) = \arctan 2x$
 $g(x) = \arctan x$
 Show on both graphs the point $(x; \frac{\pi}{4})$. (10)
- 1.2 Determine the values of the following, without using a calculator:
- 1.2.1 $2 \arctan\left(\frac{-1}{\sqrt{3}}\right)$ (4)
- 1.2.2 $\arcsin\left(\sin \frac{3\pi}{2}\right)$ (6)
- 1.2.3 $\tan(\arccos(-\sqrt{2}))$ (6)
- [26]

QUESTION 2

A function $f(x)$ is given:

$$f(x) = \begin{cases} -1 & x < -3 \\ -2x - 5 & -3 \leq x < -1 \\ x^2 - 4 & -1 \leq x < 2 \\ x - 2 & x > 2 \end{cases}$$

- 2.1 Use the definition of continuity and determine whether or not $f(x)$ is continuous at the following points. If not continuous, state what kind of discontinuity occurs.
- 2.1.1 $x = -3$ (8)
- 2.1.2 $x = -1$ (8)
- 2.1.3 $x = 2$ (6)
- 2.2 Determine algebraically, with motivation, if $f(x)$ is differentiable at $x = -1$. It is not necessary to work from first principles. (8)
- [30]

VRAAG 3

3.1 Bepaal algebraïes $\lim_{x \rightarrow 2} \frac{x^2 - 4}{|x - 2|}$ (10)

3.2 Indien $f(x) = \sqrt{2x + 1}$ en $g(x) = bg \cos x$, bepaal

3.2.1 $(g \circ f)(x)$ (4)

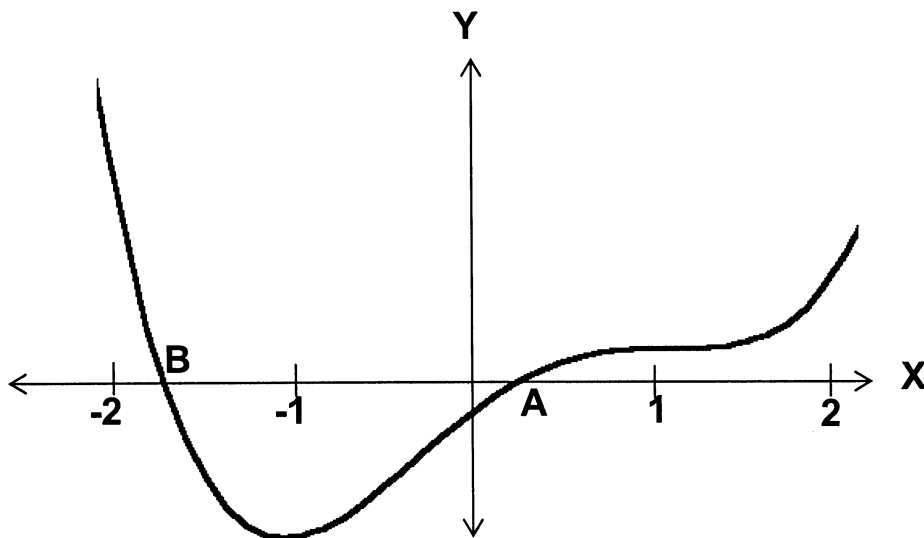
3.2.2 $D_x [(g \circ f)(-\frac{1}{4})]$. (10)

3.3 Bepaal $h'(x)$ as $h(x) = \frac{1 - \cos 3x}{2 \sin x}$ (6)

3.4 Indien $y = \frac{1}{1 + 2x}$, bepaal 'n formule vir die n de afgeleide $\frac{d^n y}{dx^n}$. (12)
[42]

VRAAG 4

'n Sketsgrafiek van die kromme $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x - 2$ word gegee.



Stasionêre punte kom voor by $x = -1$ en $x = 1$. Die x - afsnitte is aangedui met A en B.

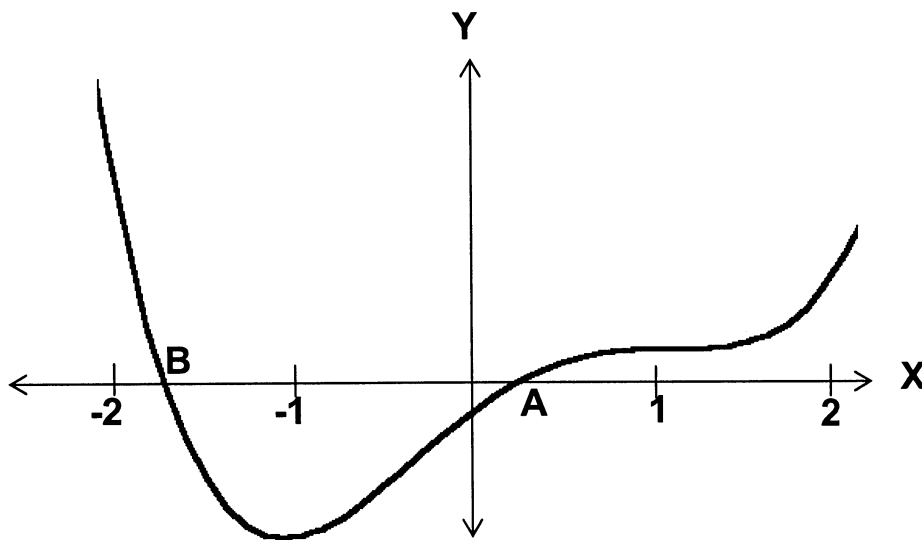
Drie Addisionele Wiskunde-kandidate word gevra om die waarde van die x - afsnit by A te bepaal met behulp van Newton se interpolasie metode (ook bekend as die Newton-Raphson metode). Hulle word gevra om 'n beginwaarde van a_0 iewers tussen 0 en 1 te kies.

QUESTION 3

- 3.1 Determine algebraically $\lim_{x \rightarrow 2} \frac{x^2 - 4}{|x - 2|}$ (10)
- 3.2 If $f(x) = \sqrt{2x + 1}$ and $g(x) = \arccos x$, determine . . .
- 3.2.1 $(g \circ f)(x)$ (4)
- 3.2.2 $D_x [(g \circ f)(-\frac{1}{4})]$. (10)
- 3.3 Determine $h'(x)$ if $h(x) = \frac{1 - \cos 3x}{2 \sin x}$ (6)
- 3.4 If $y = \frac{1}{1 + 2x}$, determine a formula for the n th derivative $\frac{d^n y}{dx^n}$. (12)
[42]

QUESTION 4

A sketch graph of the curve $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x - 2$ is given.



Stationary points occur at $x = -1$ and $x = 1$. The x intercepts are marked A and B.

Three Additional Mathematics candidates are asked to determine the value of the x intercept at A using Newton's interpolation method (also called the Newton Raphson method). They are told to use a starting value of a_0 somewhere between 0 and 1.

- 4.1 Savlon, die eerste kandidaat, kies $a_0 = 0,8$ en, deur gebruik te maak van die volgende formule $a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$ bepaal hy . . .

$$a_1 = -1,4494 \quad (\text{korrek tot 4 desimale plekke})$$

$$a_2 = -1,6527$$

Hy staak die proses want hy is onder die indruk dat daar iets verkeerd is. Verduidelik volledig wat gebeur het.

(8)

- 4.2 Dettol, die tweede kandidaat, begin met $a_0 = 1$ maar kan glad nie 'n waarde vir a_1 bereken nie. Verduidelik wat sy probleem is.

(6)

- 4.3 Methiolate, die derde kandidaat (wat dink dat hy sny punte die beste kan bepaal) begin met $a_0 = 0,5$ en kry

$$a_1 = 0,0138888\dots$$

Onder die veronderstelling dat sy berekening tot dusver korrek is, help hom om die volgende benadering a_2 te bereken (korrek tot 4 desimale syfers).

(6)

[20]

VRAAG 5

Bepaal die volgende integrale.

5.1 $\int 2 \cos^2 3x \cdot \sin 3x \, dx$ (8)

5.2 $\int_0^{\frac{\sqrt{3}}{4}} \frac{dx}{\sqrt{1-4x^2}}$ (10)
[18]

4.1 Savlon, the first candidate, chooses $a_0 = 0,8$ and, using the formula

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)} \text{ he determines . . .}$$

$$a_1 = -1,4494 \text{ (correct to 4 decimal places)}$$

$$a_2 = -1,6527$$

He stops because things seem to be going wrong.

Explain fully what is happening.

(8)

4.2 Dettol, the second candidate, begins with $a_0 = 1$ but can't determine a value for a_1 .
Explain what his problem is.

(6)

4.3 Methiolate, the third candidate (who claims to be the expert at determining intercepts!), starts with $a_0 = 0,5$ and determines

$$a_1 = 0,0138888\dots$$

On the assumption his calculation so far is correct, help him determine the next approximation a_2 (correct to 4 decimal places).

(6)

[20]

QUESTION 5

Determine the following integrals.

5.1 $\int 2 \cos^2 3x \cdot \sin 3x \, dx$

(8)

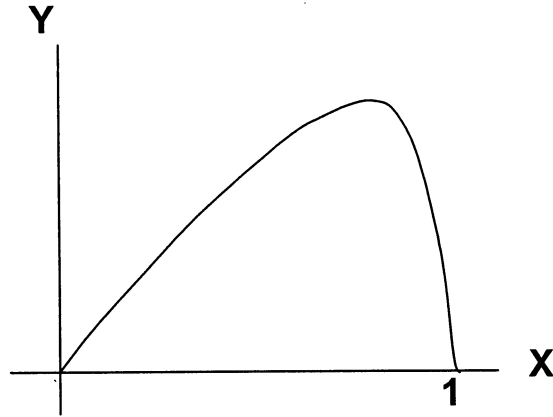
5.2 $\int_0^{\frac{\sqrt{3}}{4}} \frac{dx}{\sqrt{1-4x^2}}$

(10)

[18]

VRAAG 6

Die bygaande skets word gegee vir $f(x) = x\sqrt{1-x^2}$ vir die interval $0 \leq x \leq 1$.



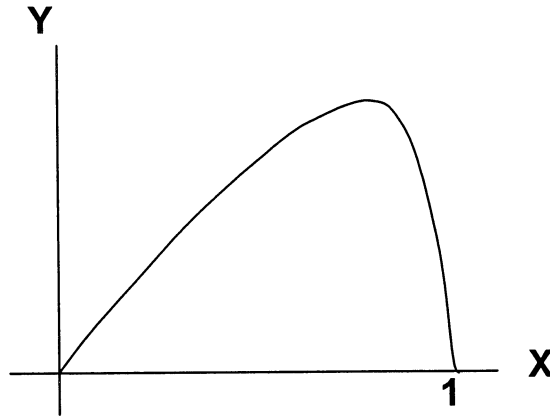
- 6.1 Bereken die presiese oppervlakte tussen die kromme en die x - as. (12)
- 6.2 As $f(x)$ rondom die x as geroteer word, bepaal die volume van die omwentelingsliggaam wat so ontstaan. Laat jou antwoord in terme van π . (10)
- [22]

VRAAG 7

Gegee $f(x) = 2x - x^2$. Gebruik die Riemann-somformule om die oppervlakte tussen $f(x)$ en die x -as tussen $x = 0$ en $x = 2$ te bereken. [18]

QUESTION 6

The accompanying sketch is given for $f(x) = x\sqrt{1-x^2}$ in the interval $0 \leq x \leq 1$.



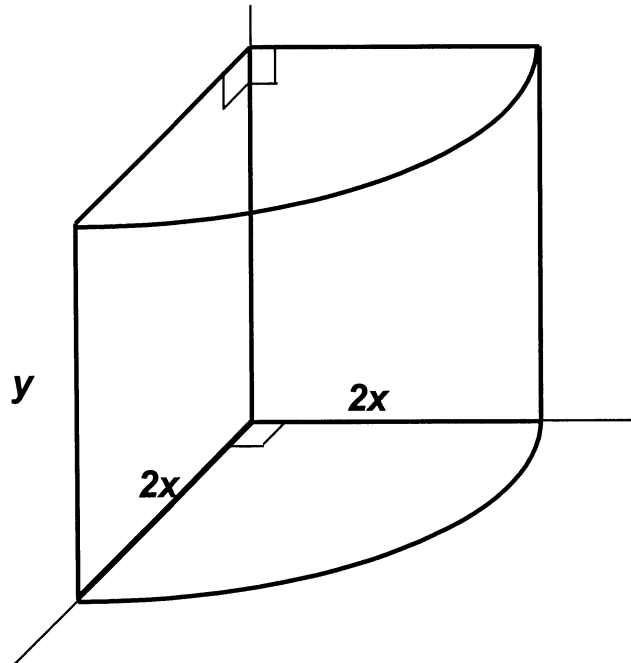
- 6.1 Calculate the exact area between the curve and the x -axis. (12)
- 6.2 If $f(x)$ is rotated about the x axis, determine the volume of the solid of revolution that is generated. Leave your answer in terms of π . (10)
[22]

QUESTION 7

Given $f(x) = 2x - x^2$. Use the Riemann sum formula to determine the area between $f(x)$ and the x axis from $x = 0$ to $x = 2$. [18]

VRAAG 8

'n Losstaande metaalhouer (wat bo oop is) moet gemaak word vir die hoek van jou werkswinkel. Die houer se dwarsdeursnee is in die vorm van 'n kwart sirkel (sien diagram).



Die radius van die basis is $2x$ en die hoogte van die houer is y .

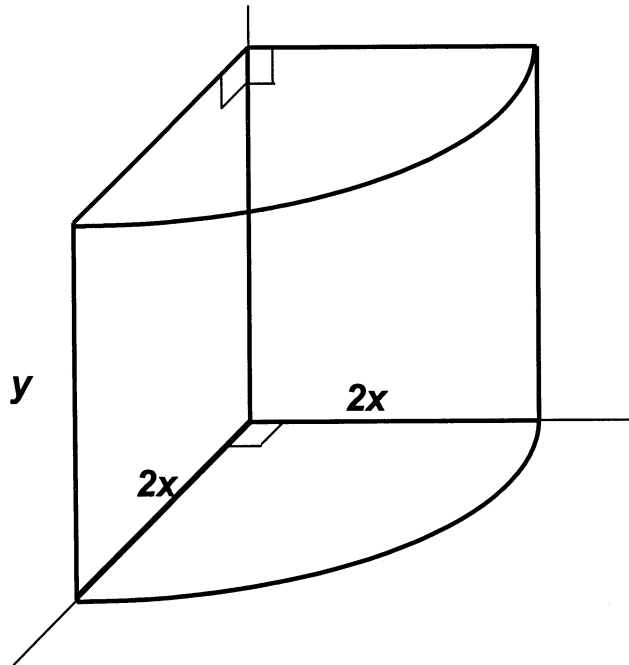
- 8.1 Indien die volume van die houer 2 m^3 (kubieke meter) is, toon aan dat $y = \frac{2}{\pi x^2}$. (4)
- 8.2 Toon aan dat die oppervlakte van die houer gegee word deur die formule $A = \pi x^2 + xy(4 + \pi)$ (6)
- 8.3 Gebruik jou antwoorde van Vrae 8.1 en 8.2 om die waarde van x te bepaal sodat 'n minimum hoeveelheid metaal benodig word. Dit is nie nodig om aan te toon dat die antwoord wel 'n minimum sal lewer nie. Gee die antwoord korrek tot 3 desimale syfers. (14)

[24]

TOTAAL VIR AFDELING A: [200]

QUESTION 8

A free-standing metal box (open at the top) is to be made for the corner of your workshop. The box is to have a cross-section in the shape of a quarter circle (see diagram).



The radius of the base is $2x$ and the height of the box y .

8.1 If the box has a volume of 2 m^3 (cubic metres), show that $y = \frac{2}{\pi x^2}$. (4)

8.2 Show that the area of the box is given by the formula $A = \pi x^2 + xy(4 + \pi)$. (6)

8.3 Use your answers to Questions 8.1 and 8.2 to determine the value of x which will enable the minimum amount of metal to be used. It is not necessary to show that this answer gives a minimum. Give the answer correct to 3 decimal digits. (14)
[24]

TOTAL FOR SECTION A: [200]

Beantwoord enige TWEE van die volgende VIER afdelings.

AFDELING B
WISKUNDE VAN FINANSIES

VRAAG 9

9.1 Dit is die begin van 2003 en Mnr. en Mev. Wittebrood is pas getroud.

Terwyl sy 'n mediese student was, het Helpende Bank aan Mev. Wittebrood verskeie studenteleninge toegeken. Die eerste was 'n bedrag van x rand aan die begin van 1997, die tweede 'n bedrag van $2x$ rand aan die begin van 1999 en die derde 'n bedrag van $3x$ rand aan die begin van die jaar 2000. Die rente op die lenings was 12,5% jaarliks saamgestel vir die eerste drie jaar (vanaf 1997) en 15% jaarliks saamgestel vir die laaste drie jaar. Tans (aan die begin van 2003) skuld sy R60 900,35. Bereken die waarde van x .

(12)

9.2 Aan die begin van 1982, die jaar waarin sy gebore is, het Mev. Wittebrood se oupa 'n bedrag van R3 000 vir haar belê by 'n belegger wat aan hom 'n rentekoers van 14% per jaar halfjaarliks saamgestel belowe het. Sy kan die bedrag nou ontvang (aan die begin van 2003) want sy is pas 21. Sy wil hierdie bedrag gebruik om haar studentelening af te betaal. Sal daar genoeg geld wees? Toon al die nodige berekenings om jou antwoord te motiveer.

(8)

[20]

VRAAG 10

10.1 Mnr. en Mev. Wittebrood trek na 'n leë huis wat hulle huur. Terwyl hulle op soek is na meubels, besluit hulle om 'n sitkamerstel, wat vir R7 500 geadverteer is, te koop. Die verkoopsman bied aan hulle 'n huurkoop-ooreenkoms in terme waarvan hulle R289 aan die einde van elke maand vir 3 jaar sal moet betaal. Bereken die enkelvoudige rentekoers, korrek tot een desimale syfer, van hierdie ooreenkoms.

(6)

10.2 Mnr. Wittebrood se pa waarsku hom om versigtig te wees vir huurkoop-ooreenkomste en stel voor dat hy eerder 'n banklening uitneem. Helpende Bank is bereid om 'n lening vir die bedrag van R7 500 aan hom toe te ken teen 'n rentekoers van 15,5% maandeliks saamgestel. Wat sal Mnr. Wittebrood se maandelikse paaieremente wees as hy die lening wil delg in dieselfde 3 jaar periode? Hy begin deur die eerste paaierement aan die einde van die eerste maand te betaal.

(10)

10.3 Mnr. Wittebrood besluit om die ooreenkoms in Vraag 10.2 te aanvaar en betaal gereeld sy maandelikse paaierement van presies R261,83. Wat is die bedrag uitstaande op die lening direk na sy 12de paaierement?

(8)

Answer any TWO of the following FOUR sections.

SECTION B
FINANCIAL MATHEMATICS

QUESTION 9

- 9.1 It is the beginning of 2003 and Mr and Mrs Honeymoon are newly married.

While she was a medical student, Mrs Honeymoon was given a series of student loans by Helping Bank. The first was an amount of x rands at the beginning of 1997, the second an amount of $2x$ rands at the beginning of 1999 and the third an amount of $3x$ rands at the beginning of the year 2000. The interest on the loans was 12,5% compounded annually for the first 3 years (from 1997) and 15% compounded annually for the last 3 years. At present (beginning of 2003) she owes R60 900,35. Calculate the value of x .

(12)

- 9.2 When Ms Honeymoon was born at the beginning of 1982 her grandfather invested for her an amount of R3 000 with an investor who offered a rate of 14% compounded half yearly. She may receive this money now (beginning of 2003) as she has just turned 21. She wishes to use this to pay off the student loan. Will it be enough money? Show the necessary calculations to support your answer.

(8)

[20]

QUESTION 10

- 10.1 Mr and Mrs Honeymoon move into an empty house which they have rented. In shopping around for furniture, they find a lounge suite advertised for R7 500. The salesman offers them a hire purchase agreement in terms of which they will have to pay only R289 at the end of each month for 3 years. Calculate the simple interest rate, correct to one decimal place, of this agreement.

(6)

- 10.2 Mr Honeymoon's father warns him to be wary of Hire Purchase agreements and suggests he rather take out a bank loan. Helping Bank is prepared to give him a loan for the required amount of R7 500 at an interest rate of 15,5% compounded monthly. What would Mr Honeymoon's monthly payments have to be to amortise (pay back) the loan within the same 3 year period, if he starts the payments at the end of the first month?

(10)

- 10.3 Mr Honeymoon chooses the arrangement in Question 10.2 and faithfully pays the amount of exactly R261,83 every month. What is the balance outstanding on the loan immediately after his 12th payment?

(8)

- 10.4 Aan die begin van die tweede jaar van terugbetaling is die uitstaande bedrag op die lening R5 373,68. Die Wittebrood-egpaar begin finansiële probleme ontwikkel en besluit dat hulle vir die volgende 3 maande geen terugbetalings sal kan bekostig nie. Hierna kan hulle slegs R200 per maand bekostig, aan die einde van elke maand. Hoeveel addisionele paaieimente van R200 sal dit neem om die lening te delg? Sluit die laaste betaling wat minder as R200 sal bedra uit in jou berekeninge. (12)
[36]

VRAAG 11

Mev. Wittebrood sluit aan by 'n mediese praktyk, wat 'n X-straalmasjien vir R270 000 moet aankoop. Nadat die masjien gekoop is, bring die praktyk 'n delgingsfonds op die been om die masjien na vyf jaar te vervang.

- 11.1 As die waarde van die bestaande masjien verminder teen 9,5% per jaar op 'n verminderde saldobasis en die nuwe masjien se verwagte koste R350 000 oor vyf jaar sal wees, bepaal die vervangingswaarde van die ou masjien as dit ingeruil kan word op 'n nuwe. (6)
- 11.2 Bo en behalwe die vervanging van die ou masjien, wat hulle beraam R187 000 sal kos, moet die delgingsfonds ook 'n jaarlikse diens van R5 000, aan die einde van elke jaar, vir die eerste 4 jaar finansier. Paaieimente in die delgingsfonds word kwartaalliks gemaak. Die rente op die kwartaallikse paaieiment sowel as die R5 000 betalings word deur 12% per jaar, kwartaalliks saamgestel, bereken. Bepaal wat die kwartaallikse paaieimente sal wees indien dit aan die einde van elke drie maande gedoen word. Die eerste paaieiment is na drie maande en die laaste paaieiment is met die aankoop van die nuwe masjien. (20)
[26]

VRAAG 12

Die Wittebrood-egpaar besluit om 'n Labrador hondjie te koop wat 'n aansienlike hoeveelheid hondebeskuitjies verorber. Dit word deur 'n maatskappy met die naam van "Happy Pet" vervaardig. Die koste- en inkomstefunksie in Rande van hierdie maatskappy word as volg gegee:

$$C(x) = 10x^3 - 30x^2 + 40x + 1000$$

$$R(x) = -45x^2 + 130x$$

waar x die aantal hondebeskuitjies, in tonne, verteenwoordig wat weekliks geproduseer word.

- 12.1 Bereken die weeklikse wins indien 5 ton hondebeskuitjies weekliks vervaardig word. (6)

- 10.4 At the beginning of the second year of repayment, the outstanding balance on the loan is R5 373,68. The Honeymoon couple begin to experience financial difficulties and determine they cannot pay anything for 3 months. After this they can only afford to pay back R200 per month at the end of each month. How many more payments of R200 will it now take to amortise the loan? Exclude in your answer the final payment which will be less than R200.

(12)
[36]

QUESTION 11

Mrs Honeymoon joins a medical practice which needs to buy an X-ray machine costing R270 000. Having bought the machine, the practice sets up a sinking fund to replace the machine after 5 years.

- 11.1 If the value of the existing machine depreciates at 9,5% per annum on a reducing balance scale and a new machine is expected to cost R350 000 in 5 years' time, determine the replacement cost of the old machine if the old one can be traded in for a new one.

(6)

- 11.2 In addition to replacing the old machine, which they estimate at R187 000, the sinking fund also has to finance an annual service costing R5 000 at the end of each year for the first 4 years. Payments into the fund are made quarterly and interest on the quarterly payments as well as the R5 000 instalments is 12% per annum compounded quarterly. Determine what the quarterly payments must be if they are made at the end of every 3 months. The payments start in three months' time and will end with the purchase of the new machine.

(20)
[26]

QUESTION 12

The Honeymoon couple decide to buy a Labrador puppy who consumes vast quantities of dog biscuits, produced by a company called "Happy Pet". The cost and revenue functions in Rands for this company are given by:

$$C(x) = 10x^3 - 30x^2 + 40x + 1000$$

$$R(x) = -45x^2 + 130x$$

where x represents the number of tons of dog biscuits produced per week.

- 12.1 Calculate the weekly profit if 5 tons of dog biscuits are produced per week.

(6)

- 12.2 Hoeveel ton hondebeskuitjies moet “Happy Pet” weekliks vervaardig sodat hulle wins 'n maksimum sal wees? Gee jou antwoord korrek tot twee desimale syfers. (12)
[18]

TOTAAL VIR AFDELING B: [100]

AFDELING C
ANALITIESE MEETKUNDE

VRAAG 13

$f(x)$ is die kurwe met parametriese vergelykings $x = \sec \theta$ en $y = 2 \tan \theta$.

- 13.1 Skryf die vergelyking van $f(x)$ in Cartesiese (kanoniese) vorm neer. (4)

- 13.2 Skets $f(x)$ en benoem alle afsnitte en asimptote. (6)
[10]

VRAAG 14

$$\frac{(x - 1)^2}{25} + \frac{(y + 2)^2}{9} = 1 \text{ is 'n ellips.}$$

Bereken die

- 14.1 eksentriteit. (6)

- 14.2 fokus van hierdie ellipse. (6)
[12]

VRAAG 15

Die parabool $y^2 = 8x$ word gegee.

- 15.1 Punt $P(2t^2 ; 4t)$ lê op die parabool. Bereken waar die raaklyn aan die parabool by P die direktrijs sny. (10)

- 15.2 15.2.1 Toon aan dat die vergelyking van die normaal aan die parabool by $M(2 ; 4)$ $y + x = 6$ is. (10)

- 15.2.2 Bereken waar hierdie normaal weer die parabool sny. (8)

- 12.2 How many tons of dog biscuits must be produced per week in order for “Happy Pet’s” profit to be maximised? Give your answer correct to two decimal digits. (12)
[18]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY

QUESTION 13

$f(x)$ is the curve with parametric equations $x = \sec \theta$ and $y = 2 \tan \theta$.

- 13.1 Write down the equation of $f(x)$ in Cartesian (canonical) form. (4)
- 13.2 Sketch $f(x)$, labelling any intercepts and asymptotes. (6)
[10]

QUESTION 14

$\frac{(x - 1)^2}{25} + \frac{(y + 2)^2}{9} = 1$ is an ellipse.

Calculate

- 14.1 the eccentricity. (6)
- 14.2 the foci of this ellipse. (6)
[12]

QUESTION 15

The parabola $y^2 = 8x$ is given.

- 15.1 Point $P(2t^2 ; 4t)$ lies on the parabola. Calculate where the tangent to the parabola at P meets its directrix. (10)
- 15.2 15.2.1 Show that the equation of the normal to the parabola at $M(2 ; 4)$ is $y + x = 6$. (10)
- 15.2.2 Calculate where this normal meets the parabola again. (8)

15.3 Die lyn deur $A(2 ; 0)$ loodreg op die raaklyn by $P(2t^2 ; 4t)$, sny die raaklyn by Q .

15.3.1 Toon aan dat die koördinate van Q $(0 ; 2t)$ is. (10)

15.3.2 As R die middelpunt van PQ is, bepaal die vergelyking van die lokus van R in Cartesiese vorm. (6)
[44]

VRAAG 16

16.1 Bepaal die vergelyking van die vlak deur $A(1 ; 4 ; 6)$, $B(2 ; 7 ; 5)$ en $C(-3 ; 8 ; 7)$. (14)

16.2 Bereken die skerp hoek tussen die vlakke $3x - y - 4z = 7$ en $2x + 3y - z = 11$, korrek tot twee desimale syfers. (10)
[24]

VRAAG 17

Toon aan dat die lyne $\begin{cases} x = 1 + 2s \\ y = 1 + s \\ z = s \end{cases}$ en $\begin{cases} x = 3 + t \\ y = 2 - t \\ z = 4 + 2t \end{cases}$ nooit sny nie. [10]

TOTAAL VIR AFDELING C: [100]

AFDELING D ALGEBRA

VRAAG 18

18.1 Stel die toegevoegde (irrasionale) wortelstelling. (6)

18.2 As $f(\sqrt{2} - 3) = 0$, faktoriseer $f(x) = x^4 + 6x^3 + 3x^2 - 24x - 28$ volledig oor $\mathbb{Q}[x]$ (14)
[20]

15.3 The line through $A(2 ; 0)$ perpendicular to the tangent at $P(2t^2 ; 4t)$, intersects the tangent at Q .

15.3.1 Show that the co-ordinates of Q are $(0 ; 2t)$. (10)

15.3.2 If R is the midpoint of PQ , determine the equation of the locus of R in Cartesian form. (6)
[44]

QUESTION 16

16.1 Determine the equation of the plane through $A(1 ; 4 ; 6)$, $B(2 ; 7 ; 5)$ and $C(-3 ; 8 ; 7)$. (14)

16.2 Calculate the acute angle between planes $3x - y - 4z = 7$ and $2x + 3y - z = 11$, correct to 2 decimal places. (10)
[24]

QUESTION 17

Show that the lines $\begin{cases} x = 1 + 2s \\ y = 1 + s \\ z = s \end{cases}$ and $\begin{cases} x = 3 + t \\ y = 2 - t \\ z = 4 + 2t \end{cases}$ do not intersect. [10]

TOTAL FOR SECTION C: [100]

SECTION D ALGEBRA

QUESTION 18

18.1 State the conjugate surds theorem. (6)

18.2 If $f(\sqrt{2} - 3) = 0$, factorise $f(x) = x^4 + 6x^3 + 3x^2 - 24x - 28$ fully over $\mathbb{Q}[x]$ (14)
[20]

VRAAG 19

19.1 Bepaal die GGD van $f(x) = x^2 + 2x + 1$ en $g(x) = x^3 - 5$. (10)

19.2 Skryf die GGD in terme van $f(x)$ en $g(x)$ neer. (12)

19.3 Rasionaliseer vervolgens die noemer van $\frac{12}{\alpha^2 + 2\alpha + 1}$ as $\alpha = \sqrt[3]{5}$ (8)

19.4 Skryf een ander waarde van α (behalwe $\alpha = \sqrt[3]{5}$) neer waarvoor
 $\frac{12}{\alpha^2 + 2\alpha + 1} = \alpha^2 + \alpha - 3$ (2)
[32]

VRAAG 20

Gebruik wiskundige induksie en bewys dat $g(x) = 7^{2p} - 2^{2p}$ deelbaar is deur 5 as $p \in \mathbb{N}$ [16]

VRAAG 21

$$f(x) = \frac{4x^2 + 2}{3x^4 + 4x^2 + 1}$$

21.1 Skryf die y - afsnit van $f(x)$ neer. (2)

21.2 Toon aan dat $f(x)$ geen x -afsnitte het nie. (4)

21.3 $f(x)$ het geen vertikale of skuins asimptote nie, maar dit het een horisontale asimptoot. Bepaal die vergelyking van hierdie horisontale asimptoot. (4)

21.4 $f(x)$ het 'n maksimum draaipunt by $(0 ; 2)$. Maak 'n netjiese skets van die grafiek van $f(x)$, en toon al die afsnitte, asimptote en draaipunte aan. (6)
[16]

VRAAG 22

Ontbind $\frac{2x^3 + 3x^2 + x + 2}{x^4 + x^2}$ in parsieële breuke. [16]

TOTAAL VIR AFDELING D: [100]

QUESTION 19

19.1 Determine the HCF of $f(x) = x^2 + 2x + 1$ and $g(x) = x^3 - 5$. (10)

19.2 Write down the HCF in terms of $f(x)$ and $g(x)$. (12)

19.3 Hence rationalise the denominator of $\frac{12}{\alpha^2 + 2\alpha + 1}$ if $\alpha = \sqrt[3]{5}$ (8)

19.4 Write down one other value of α (besides $\alpha = \sqrt[3]{5}$) for which
 $\frac{12}{\alpha^2 + 2\alpha + 1} = \alpha^2 + \alpha - 3$ (2)
[32]

QUESTION 20

Prove by mathematical induction that $g(x) = 7^{2p} - 2^{2p}$ is divisible by 5 if $p \in \mathbb{N}$ [16]

QUESTION 21

$$f(x) = \frac{4x^2 + 2}{3x^4 + 4x^2 + 1}$$

21.1 Write down the y -intercept of $f(x)$. (2)

21.2 Show that $f(x)$ has no x -intercepts. (4)

21.3 $f(x)$ has no vertical or oblique asymptotes, but it does have one horizontal asymptote. Determine the equation of this horizontal asymptote. (4)

21.4 $f(x)$ has a maximum turning point at $(0; 2)$. Make a neat sketch of the graph of $f(x)$, showing all intercepts, asymptotes and turning points. (6)
[16]

QUESTION 22

Decompose $\frac{2x^3 + 3x^2 + x + 2}{x^4 + x^2}$ into partial fractions. [16]

TOTAL FOR SECTION D: [100]

AFDELING E
STATISTIEK

Rond alle antwoorde af tot vier desimale syfers, tensy anders aangedui.

VRAAG 23

By Vulindlela Hoërskool is daar 220 Graad 12-leerders. In 'n ondersoek word 3 vakke gekies: Wiskunde (M), Addisionele Wiskunde (A) en Wetenskap (S). Daar word gevind dat:

- 139 leerders neem Wiskunde
- 27 leerders neem Addisionele Wiskunde
- Alle Addisionele Wiskunde kandidate neem ook Wiskunde
- 12 Addisionele Wiskunde kandidate neem nie Wetenskap nie
- 56 Wetenskap kandidate neem nie Wiskunde nie
- 49 leerders neem Wiskunde en Wetenskap
- 25 leerders neem nie een van die drie vakke nie

- 23.1 Teken 'n Venn Diagram om bogenoemde situasie voor te stel. (10)
- 23.2 Skryf neer hoeveel leerders slegs Wiskunde neem. (2)
- 23.3 Watter verhouding van Graad 12-leerders neem ten minste 2 van die 3 vakke? (6)
- 23.4 Bepaal $n(M \cap A')$ (4)
- [22]**

VRAAG 24

'n Houer bevat elf groen en ses rooi balle. Vyf balle word getrek.

- 24.1 Die balle word getrek sonder terugplasing. Bepaal die waarskynlikheid dat
- 24.1.1 daar vier groen en een rooi bal getrek word. (8)
- 24.1.2 daar hoogstens vier groen balle getrek word. (8)
- 24.2 Gestel die balle word getrek met terugplasing.
- 24.2.1 Skryf die waarskynlikheidsfunksie neer vir die waarskynlikheid dat daar x groen balle getrek word. (8)
- 24.2.2 Bepaal die waarskynlikheid dat daar vier groen en een rooi bal getrek word. (6)

SECTION E
STATISTICS

Round all answers off to four decimal digits, unless stated otherwise.

QUESTION 23

At Vulindlela High School there are 220 Grade 12 learners. In a survey 3 subjects are selected: Mathematics (M), Additional Mathematics (A) and Science (S). It is found that:

- 139 learners take Mathematics
- 27 learners take Additional Mathematics
- All Additional Mathematics candidates also take Mathematics
- 12 Additional Mathematics candidates do not take Science
- 56 Science candidates do not take Mathematics
- 49 learners take Mathematics and Science
- 25 learners take none of the three subjects

- 23.1 Draw a Venn Diagram to illustrate the above situation. (10)
- 23.2 Write down how many learners take only Mathematics. (2)
- 23.3 What ratio of Grade 12 learners take at least 2 of the 3 subjects? (6)
- 23.4 Determine $n(M \cap A')$. (4)
- [22]**

QUESTION 24

A jug contains eleven green and six red balls. Five balls are drawn.

- 24.1 The balls are drawn without replacement.
- 24.1.1 Determine the probability that four green and one red ball are drawn. (8)
- 24.1.2 Calculate the probability that at the most four green balls are drawn. (8)
- 24.2 Suppose the balls are drawn with replacement.
- 24.2.1 Write down the probability function for the probability that x green balls are drawn. (8)
- 24.2.2 Determine the probability that four green and one red ball are drawn. (6)

- 24.3 Onder watter omstandighede behoort die antwoorde van Vrae 24.1.1 en 24.2.2 baie naby aan mekaar te wees? (2)
[32]

VRAAG 25

Mvundla moet Woensdag om 17:30 tuis wees, maar hy moet aan 'n krieketwedstryd deelneem. Die weervoorspelling sê dat daar 'n 20% kans op reën is. Indien dit reën, is die waarskynlikheid dat die wedstryd nie gespeel sal word nie, 90%. Andersins, as dit nie reën nie, is daar 'n 90% kans dat die wedstryd wel sal voortgaan. Die krieketwedstryd het 'n 60% kans om voor 17:30 te eindig, indien dit plaasvind. Teken 'n boomdiagram en gebruik dit, of andersins, om die waarskynlikheid te bereken dat Mvundla laat (na 17:30) by die huis sal kom. [16]

VRAAG 26

Mnr. Groenvingers kweek baie groot wortels wat hy by die oggendmark verkoop. Hulle het 'n gemiddelde lengte van 25 cm en 'n standaardafwyking van 3,4.

- 26.1 Watter persentasie (tot die naaste heelgetal) sal langer as 30 cm wees? (10)
- 26.2 Die wortels wat Mnr Groenvingers wil verkoop moet langer as 'n sekere afsnylengte wees. Wat is hierdie afsnylengte (tot die naaste cm) indien hy 80% van sy wortels kan verkoop? (10)
[20]

VRAAG 27

Daar word beweer dat “Lingalonga” batterye 'n gemiddelde leeftyd van 24 uur het. 'n Steekproef van 60 van hierdie batterye word getoets en daar word gevind dat die gemiddelde leeftyd 25 ure en die standaardafwyking 2,7 ure is.

- 27.1 Bereken 'n 99% betroubaarheidsinterval vir die gemiddelde leeftyd van “Lingalonga” batterye. (8)
- 27.2 Bevestig hierdie antwoord hulle bewering? (2)
[10]

TOTAAL VIR AFDELING E: [100]

TOTAAL: 400

- 24.3 Under what circumstances will the answers to Questions 24.1.1 and 24.2.2 be very close? (2)
[32]

QUESTION 25

Mvundla needs to be home by 17:30 on Wednesday but is due to play a cricket match. The weather report says that there is a 20% chance of rain. If it does rain, the probability that the match will be cancelled is 90%. If it is fine, on the other hand, there is a 90% chance the match will go ahead. The cricket match has a 60% chance of finishing before 17:30, if it is played. Draw a tree diagram and use it, or otherwise, to calculate the probability that Mvundla will be home late (after 17:30). [16]

QUESTION 26

Mr Greenfingers grows very large carrots which he sells at the morning market. They have an average length of 25 cm with a standard deviation of 3,4.

- 26.1 What percentage (to the nearest whole number) are over 30 cm? (10)
- 26.2 To be sold, Mr Greenfinger's carrots have to be over a certain length. What is this cut-off length (to the nearest cm) if he sells 80% of his carrots? (10)
[20]

QUESTION 27

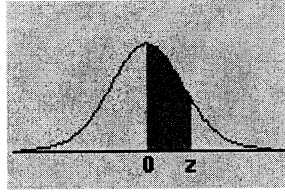
"Lingalonga" batteries are claimed to have a "mean life of 24 hours". A sample of 60 of these batteries are tested and it is found that the mean is 25 hours and the standard deviation is 2,7 hours.

- 27.1 Calculate a 99% confidence interval for the mean life of "Lingalonga" batteries. (8)
- 27.2 Does the result support the claim? (2)
[10]

TOTAL FOR SECTION E: [100]

TOTAL: 400

Normal Distribution/ Normalverspreiding



$$P(X \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}} dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

FORMULA SHEET/ FORMULEBLAD

Differential en Integraal Calculus

Differensiaal- en Integraalrekenen

$s = r\theta$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin A \cdot \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$\sin A \cdot \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A \cdot \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$\text{Riemann Sum} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$$

$F(x)$	$F'(x)$
$a \cdot x^n$	$na \cdot x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\cot x$	$-\text{cosec}^2 x$
$\text{cosec } x$	$-\text{cosec } x \cdot \cot x$
$\arcsin x$ $\text{bgsin } x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$ $\text{bgcos } x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$ $\text{bgtan } x$	$\frac{1}{x^2 + 1}$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$

Finance/ Finansies

$F = P(1+i)^n$ $F = P(1-i)^n$

$F = P(1+in)$ $F = P(1-in)$

$$P = x \cdot \frac{1 - (1+i)^{-n}}{i} \quad F = x \cdot \frac{(1+i)^n - 1}{i}$$

Analytical Geometry/ Analitiese Meetkunde

$y = 4ax^2$ $yy_1 = 2a(x+x_1)$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Algebra

$$\alpha + \beta = -\frac{b}{a} \quad \alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha \dots \beta = \frac{c}{a} \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha \cdot \beta \cdot \gamma = -\frac{d}{a}$$

Statistics / Statistiek

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$${}^n P_r = \frac{n!}{(n-r)!} \quad {}^n C_r = \frac{n!}{(n-r)! r!}$$

$$P(R = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(R = x) = \frac{\binom{p}{x} \binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z = \frac{\bar{X} - \mu}{\sigma}$$

$$P(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}) = 0.95$$

$$P\left(p - 1.96 \sqrt{\frac{p(1-p)}{n}} < \pi < p + 1.96 \sqrt{\frac{p(1-p)}{n}}\right) = 0.95$$

Wiskunde Formuleblad/ Mathematics Formula Sheet

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}(a + l) \quad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad S_\infty = \frac{a}{1 - r}$$

$$A = P\left(1 + \frac{r}{100}\right)^n \quad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y^2 + x^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$