POSSIBLE ANSWERS FOR: Additional Mathemati

1.1 When $a=2$: $f(2)=8$

$\lim_{x\to 2^+} f(x) = 8 = \lim_{x\to 2^-} f(x)$

$\therefore f(x)$ is continuous at $x=2$

$$f'(x) = \begin{cases} 3x^2 & \text{for } x \le 2 \\ 3 & \text{for } x > 2 \end{cases}$$

$\lim_{x\to 2^-} f'(x) = 12$

$\lim_{x\to 2^+} f'(x) = 3$

$\therefore f(x)$ is not differentiable at $x=2$

1.2 When $a=1$ $f(1)=1$

$\lim_{x\to 1^+} f(x) = 5$

$\therefore f(x)$ is neither continuous nor differentiable at $x=1$

1.3. When $a=-1$ $f(-1)=-1$

$\lim_{x\to -1^+} f(x) = -1 = \lim_{x\to -1^-} f(x)$

$\lim_{x\to -1^-} f'(x) = 3$

$\lim_{x\to -1^+} f'(x) = 3$

$\therefore f(x)$ is continuous and differentiable at $x=-1$.

(30)

2.1 $\lim_{x\to -3} \frac{x^3+27}{x+3} = \lim_{x\to -3} \frac{(x+3)(x^2-3x+9)}{x+3}$

$= 27$

(6)

2.2. $\lim_{x\to 0} \frac{\theta}{3\tan 2\theta} = \lim_{x\to 0} \frac{2\theta}{\sin 2\theta} \cdot \frac{\cos 2\theta}{6}$

$= \frac{1}{6}$

(8)

3.1 ~~Domain~~: $-1 \leq 3x+1 \leq 1$ ✓

$-2 \leq 3x \leq 0$

$-\frac{2}{3} \leq x \leq 0$ ✓✓✓

Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ ✓✓ (8)

3.2.1. When $x < 0$, $\arccos x \in \left(\frac{\pi}{2}; \pi\right]$ ✓✓✓ which is not part of the domain of $\arcsin(3x+1)$ ✓✓ (6)

3.2.2 Let $\arcsin(3x+1) = \arccos x = \alpha$

~~then~~ $\sin\alpha = 3x+1$ ✓✓ and $\cos\alpha = x$ ✓✓

But $\sin^2\alpha + \cos^2\alpha = 1$ for all α

$\therefore (3x+1)^2 + x^2 = 1$ ✓✓

$\therefore 9x^2 + 6x + 1 + x^2 - 1 = 0$

$\therefore 10x^2 + 6x = 0$

$\therefore 2x(5x+3) = 0$

$\therefore \underline{x = 0}$ ✓✓ or $x = -\frac{3}{5}$ (8)

Inadmissable (see 3.2.1)

4.1 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓✓✓

$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$ ✓✓

$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}.\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$ ✓✓✓✓

$= \lim_{h \to 0} \frac{x - x - h}{h\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$

$= \lim_{h \to 0} - \frac{1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$

(14)

$= -\frac{1}{2x^{3/2}}$ ✓✓✓✓

4.2.1 $\frac{d}{dx}\left(\arcsin x - \sqrt{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}} - \frac{(-2x)}{2\sqrt{1-x^2}}$

$$= \frac{1+x}{\sqrt{1-x^2}}$$

(5

4.2.2 $f(x) = \frac{(3x-5)^3}{2x^2}$

$$f'(x) = \frac{3(3x-5)^2 \cdot 3 \cdot 2x^2 - 4x(3x-5)^3}{4x^4}$$

(8)

$$= \frac{4x(3x-5)^2\left[3x^2 - 3x + 5\right]}{4x^4}$$

$$= \frac{(3x-5)^2(3x^2-3x+5)}{x^3}$$

4.2.3 $y = 2\tan^2\left(\frac{\pi}{2} - 3\theta\right)$

$$\frac{dy}{dx} = 4\tan\left(\frac{\pi}{2}-3\theta\right) \cdot \sec^2\left(\frac{\pi}{2}-3\theta\right) \cdot (-3)$$

When $\theta = \pi/4$

$$\text{Gradient} = 4\tan\left(-\frac{\pi}{4}\right) \cdot \sec^2\left(-\frac{\pi}{4}\right)(-3)$$

$$= -12(-1) \cdot 2$$

$$= 24.$$

(10)

5.1 $y = x^4 - 2x^3 + x$

$$\frac{dy}{dx} = 4x^3 - 6x^2 + 1$$

When $x = \frac{1}{2}$

$$\frac{dy}{dx} = 4 \cdot \frac{1}{8} - 6 \cdot \frac{1}{4} + 1$$

$$= \frac{1}{2} - \frac{3}{2} + 1$$

$$= 0$$

(6)

$\therefore$ $y = x^4 - 2x^3 + x$ has a t. pt. at $x = \frac{1}{2}$

5.2 $$x_n = x_{n-1} - \frac{4(x_{n-1})^3 - 6(x_{n-1})^2 + 1}{12(x_{n-1})^2 - 12(x_{n-1})}$$

When $x_{n-1} = 0$ and when $x_{n-1} = 1$,

$$12(x_{n-1})^2 - 12(x_{n-1}) = 0 \quad \text{(Zero denom.)}$$

$\therefore$ Newton's method fails. (10)

5.3 Let $f(x) = \frac{dy}{dx} = 4x^3 - 6x^2 + 1$

Then $f(1) = 4 - 6 + 1 < 0$
$f(2) = 32 - 24 + 1 > 0$

$\therefore$ Since $f(x)$ is continuous for all $x \in \mathbb{R}$
$y = x^4 - 2x^3 + x$ has a t. pt. between
$x = 1$ and $x = 2$

Let $x_0 = 2$

Then $x_1 = 2 - \frac{32 - 24 + 1}{48 - 24}$

$= 2 - \frac{9}{24}$

$= 1{,}625$

$x_2 = 1{,}625 - \frac{4(1{,}625)^3 - 6(1{,}625)^2 + 1}{12(1{,}625)^2 - 12(1{,}625)}$

$= 1{,}4346\ldots$

$x_3 = 1{,}3729\ldots$

$x_4 = 1{,}36610\ldots$ (14)

$x_5 = 1{,}3660\ldots$

$\therefore$ t. pt. at $x = 1{,}366$ (corr. to 3 dec. pl.)

<u>or</u>

$4x^3 - 6x^2 + 1 = (2x - 1)(2x^2 - 2x - 1)$

$= (2x - 1)\left(x - \frac{1 + \sqrt{3}}{2}\right)\left(x - \frac{1 - \sqrt{3}}{2}\right)$

$x = \frac{2 \pm \sqrt{4 + 4}}{4}$

$= \frac{2 \pm 2\sqrt{3}}{4}$

$= \frac{1 \pm \sqrt{3}}{2}$

When $\frac{dy}{dx} = 0$

$x = \frac{1}{2}$ or $x = \frac{1 + \sqrt{3}}{2}$ or $x = \frac{1 - \sqrt{3}}{2}$

$= 1{,}366$ (corr. to 3 dec. pl.)

(14)

6.1 $\displaystyle\int \frac{1}{x^2+2x+3}\,dx$

$$= \int \frac{1}{(x+1)^2+2}\,dx \quad ✓✓$$

$$✓✓\; = \frac{1}{2}\int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}\,dx$$

$$= \frac{\sqrt{2}}{2}\arctan\left(\frac{x+1}{\sqrt{2}}\right) + c \quad ✓✓ ✓✓ ✓✓$$

10 10

6.2 $\displaystyle\int_0^5 \frac{2}{\sqrt{x+4}}\,dx$.

Let $u = x+4$

Then Integral $= 2\displaystyle\int_4^9 u^{-\frac{1}{2}}\,du$ ✓✓ ✓✓

$$= 4\left[u^{\frac{1}{2}}\right]_4^9 \quad ✓✓$$

$$= 4[3-2]$$

$$= 4 \quad ✓✓$$

8

OR.

$$✓✓\; 4\int_0^5 \frac{1}{2\sqrt{x+4}}\,dx \quad ✓✓$$

$$= 4\left[\sqrt{x+4}\right]_0^5 \quad ✓✓$$

$$= 4[3-2]$$

$$= 4 \quad ✓✓$$

8

P

6.3 $\int_0^{\pi/2} \sin^5\theta \, d\theta$

$$= \int_0^{\pi/2} (1-\cos^2\theta)^2 . \sin\theta . d\theta$$

$$= \int_0^{\pi/2} (1-2\cos^2\theta + \cos^4\theta) \sin\theta \, d\theta .$$

$$= \left[-\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta \right]_0^{\pi/2}$$

$$= -1 + \frac{2}{3} - \frac{1}{5}$$

$$= \frac{-15+10-3}{15}$$

14

$$= -\frac{8}{15}$$

7.1 At pt(s) of intersection

$$\frac{14}{3} - \frac{2x^2}{3} = \frac{4}{x}$$

$\therefore \ 14x - 2x^3 = 12$

$\therefore \ x^3 - 7x + 6 = 0$

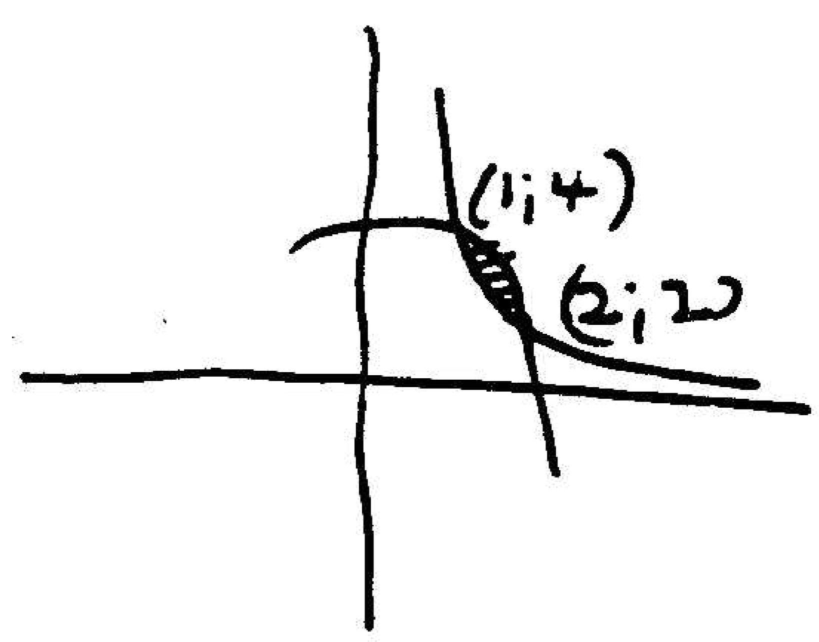
$(x-1)(x^2+x-6)=0$

$(x-1)(x+3)(x-2)=0$

In the first quadrant

$x=1$ and $x=2$ 8

$y=4$ $\quad$ $y=2$



7.2 Area $= \int_1^2 \left(\frac{14}{3} - \frac{2x^2}{3} - \frac{4}{x} \right) dx$

$$= \left[\frac{14x}{3} - \frac{2x^3}{9} + \frac{4}{x^2} \right]_1^2$$

$$= \frac{28}{3} - \frac{16}{9} + 1 - \frac{14}{3} + \frac{2}{9} - 4$$ 12

$$= \frac{1}{9}$$

Pk

8. $V = \pi \int_{-1}^{5} [36 - (x-2)^2] dx$

$= \pi \left[36x - \frac{(x-2)^3}{3} \right]_{-1}^{5}$

$= \pi [180 - 9 - 36 + 9]$

(12)

$= 144\pi$

SECTION B
FINANCE

9. $P = 15\,000\left(1+\frac{0,093}{4}\right)^{-8} + 15\,000(1,07)^{2}\left(1+\frac{0,093}{14}\right)^{-16}$ (14)

$= R24\ 370$ (correct to nearest rand)

10.1 $\int (0,2t+3)\,dt = 0,1t^2 + 3t + F = C(t)$

$C(5) = 2,5 + 15 + F = 20$

$\therefore F = 20 - 17,5$

$= 2,5$ (8)

$\therefore$ Fixed cost $= R2,5$ million

10.2 Max. profit when $C'(t) = R'(t)$

$\therefore 0,2t + 3 = 9 - 0,4t$

$\therefore 0,6t = 6$

$\therefore t = 10$ (6)

Max. profit after 10 years

10.3 Average profit $= \left[\frac{9t - 0,2t^2 - 0,1t^2 - 3t - 2,5}{6}\right]_4^{10}$

$= \left[\frac{-0,3t^2 + 6t - 2,5}{6}\right]_4^{10}$

$= \frac{-30 + 60 - 2,5 + 4,8 - 24 + 2,5}{6}$

$= \frac{10,8}{6}$ (6)

$= R1,8$ million

$$11.2\quad 2\,500\,000 + 10\,000\left(1+\frac{0{,}1075}{2}\right)^{2} + 10\,000\left(1+\frac{0{,}1075}{2}\right)^{4} + \dots + 10\,000\left(1+\frac{0{,}1075}{2}\right)^{14}$$

$$= x + x(1{,}00876\ldots) + x(1{,}00876\ldots)^{2} + \dots + x(1{,}00876\ldots)^{94} + x(1{,}00876\ldots)^{95}$$

$$\therefore\ x = \frac{2\,500\,000\left[\dfrac{10\,000\left(1+\frac{0{,}1075}{2}\right)^{2}\left[\left(1+\frac{0{,}1075}{2}\right)^{2\times 7}-1\right]}{\left(1+\frac{0{,}1075}{2}\right)^{2}-1}\right]}{\dfrac{\left[(1{,}00876\ldots)^{96}-1\right]}{0{,}00876\ldots}}$$

(20)

$= R17\ 440$ (correct to nearest rand)

$$12.1\qquad 120\,000 = \frac{3000\left[1-\left(1+\frac{0{,}145}{12}\right)^{-n}\right]}{\frac{0{,}145}{12}}$$

$$\therefore\ \left(1+\frac{0{,}145}{12}\right)^{-n} = 1 - \frac{120\,000\left(\frac{0{,}145}{12}\right)}{3\,000}$$

$$\therefore\ n = \frac{\log\left[1-\frac{120\,000\left(\frac{0{,}145}{12}\right)}{3\,000}\right]}{-\log\left(1+\frac{0{,}145}{12}\right)}$$

$= 54{,}979\ldots$ months

$\therefore$ Loan will take 55 months to amortise

(14)

12.2 Let last payment $= Rx$

$$120\,000 = \frac{3000\left[1-\left(1+\frac{0{,}145}{12}\right)^{-54}\right]}{\frac{0{,}145}{12}} + x\left(1+\frac{0{,}14}{12}\right.$$

$$\therefore\ x = \left\{120\,000 - \frac{3000\left[1-\left(1+\frac{0{,}145}{12}\right)^{-54}\right]}{\frac{0{,}145}{12}}\right\}\left(1+\frac{0{,}145}{12}\right)^{55}$$

$= R1\ 518{,}48$ (corr. to nearest cent). (12)

12.3 Balance at $T_{20} = 120\,000\left(1+\frac{0,145}{12}\right)^{20} - \frac{3\,000\left(1+\frac{0,145}{12}\right)\left[\left(1+\right.\right.}{\frac{0,145}{12}}$

$= R88\ 169,6868\ldots$

$$\text{Bal.}\left(1+\frac{0,145}{12}\right)^{3} = \frac{R\left[1-\left(1+\frac{0,145}{12}\right)^{-29}\right]}{\frac{0,145}{12}}$$

$$\therefore R = \frac{\text{Bal}\left(1+\frac{0,145}{12}\right)^{3} \cdot \frac{0,145}{12}}{\left[1-\left(1+\frac{0,145}{12}\right)^{-29}\right]}$$

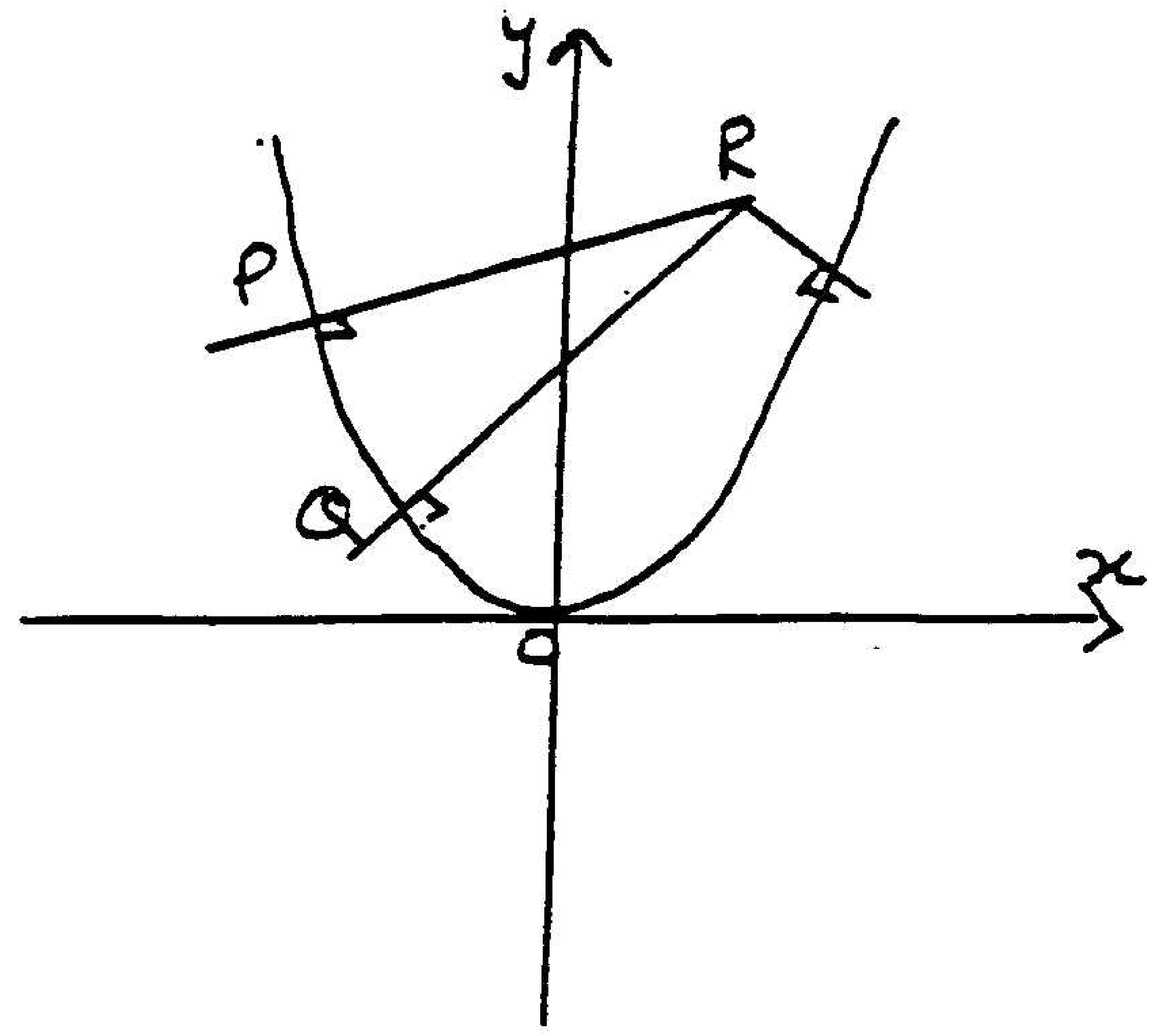
$= R3\ 755,12$ (corr. to nearest cent)

20

SECTION C:

ANALYTICAL GEOMETRY

13



13.1 $y = x^2$

$\frac{dy}{dx} = 2x$

When $x = t$, $y = t^2$ and $\frac{dy}{dx} = 2t$ ✓✓

$\therefore$ Gradient of normal $= -\frac{1}{2t}$ ✓✓

$\therefore$ Equation of normal is $\frac{y-t^2}{x-t} = -\frac{1}{2t}$ ✓✓✓✓

$(y-t^2)\,2t + x - t = 0$

$\therefore -2t^3 + 2ty - t + x = 0$ (10)

ie $2t^3 + (1-2y)t - x = 0$ ✓✓

13.2 Equ. of PR is $-16 - 2 + 4y - x = 0$ ✓✓

$x - 4y = -18$ ①

Equ. of QR is $-2 - 1 + 2y - x = 0$

$x - 2y = -3$ ② ✓✓

① − ② $\quad -2y = -15$

$\therefore y = 15/2$ ✓✓

$x = 2(15/2) - 3$

$= 12$ ✓✓

$\therefore$ R is the point $(12 ; 15/2)$

13.3 At R, $2t^3 + (1-15)t - 12 = 0$ ✓✓

$\therefore 2t^3 - 14t - 12 = 0$

ie. $t^3 - 7t - 6 = 0$

$(t+2)(t+1)(t-3) = 0$ ✓✓

$\therefore \quad t=-2 \quad$ or $\quad t=-1 \quad$ or $\quad t=3$ ✓✓

$\therefore$ Equ. of third normal is

$$2(27)+(1-2y)3-x=0 \checkmark\checkmark$$

$$54+3-6y-x=0$$

$$57=x+6y \checkmark\checkmark$$

10

14.1. $\dfrac{x^2}{9}-\dfrac{y^2}{16}=1$

$\therefore a^2=9$ and $b^2=16$

$$\therefore e^2=1+\frac{16}{9}$$

$$=\frac{25}{9}$$

$$\therefore e=\frac{5}{3} \checkmark\checkmark$$

Foci are $(3.\frac{5}{3};0)$ and $(-3.\frac{5}{3};0)$

$=(5;0)$ and $(-5;0)$ ✓✓

4

14.2. Directrices are $x=\pm\dfrac{3}{\frac{5}{3}}$

$$=\pm\frac{9}{5} \checkmark\checkmark\checkmark\checkmark$$

Asymptotes: $3y=4x$ ✓✓ and $3y=-4x$ ✓✓

8

14.3. Tangent at $(3\sec\theta;4\tan\theta)$ is

$$\frac{3\sec\theta . x}{9}-\frac{4\tan\theta . y}{16}=1 \checkmark\checkmark\checkmark\checkmark$$

$$\therefore \frac{x}{3\cos\theta}-\frac{y\sin\theta}{4\cos\theta}=1$$

8

$$\therefore 4x-3y\sin\theta=12\cos\theta \checkmark\checkmark\checkmark\checkmark$$

14.4 For $0<\theta<\frac{\pi}{2}$, P is in the first quad

$\therefore$ Applicable directrix is $x=\dfrac{9}{5}$

At pt. of int $4\left(\frac{9}{5}\right) - 3\sin\theta . y = 12\cos\theta$

$\therefore 36 - 15y\sin\theta = 60\cos\theta$

$\therefore y = \frac{12}{5\sin\theta} - \frac{20\cos\theta}{5\sin\theta}$ (8)

$= \frac{12 - 20\cos\theta}{5\sin\theta}$

14.5 Gradient of QF $= \dfrac{0 - \frac{12-20\cos\theta}{5\sin\theta}}{5 - \frac{9}{5}}$

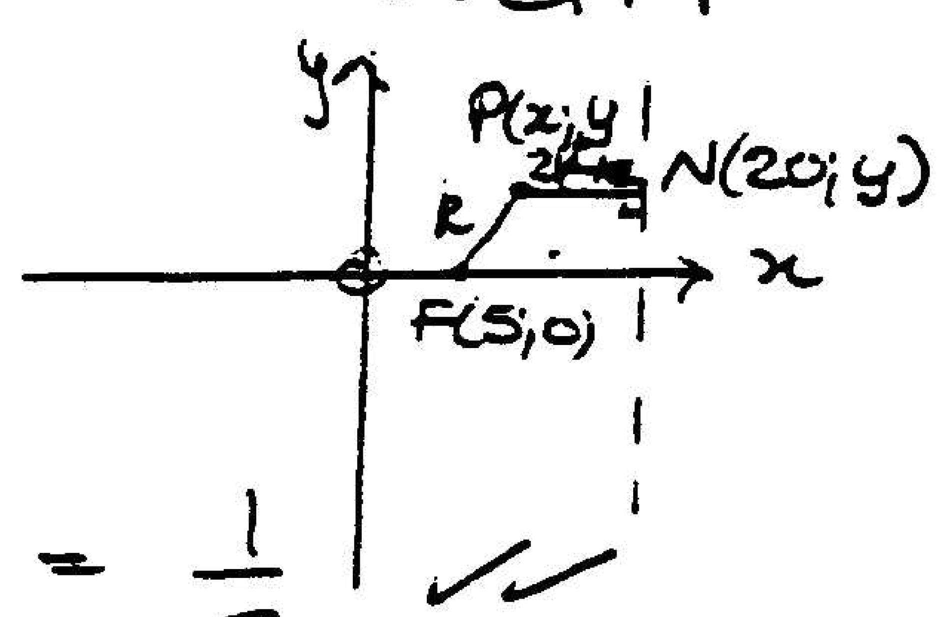
Gradient of PF $= \dfrac{4\tan\theta}{3\sec\theta - 5}$

Gradient QF × Gradient PF $= \frac{5}{16} \cdot \frac{4(5\cos\theta - 3)}{5\sin\theta} \cdot \frac{4\sin\theta}{\cos\theta} \cdot \frac{c\ldots}{3-\ldots}$

$= -1$ (10)

$\therefore Q\hat{F}P = 90°$

15.1



$\frac{PF}{PN} = \frac{1}{2}$

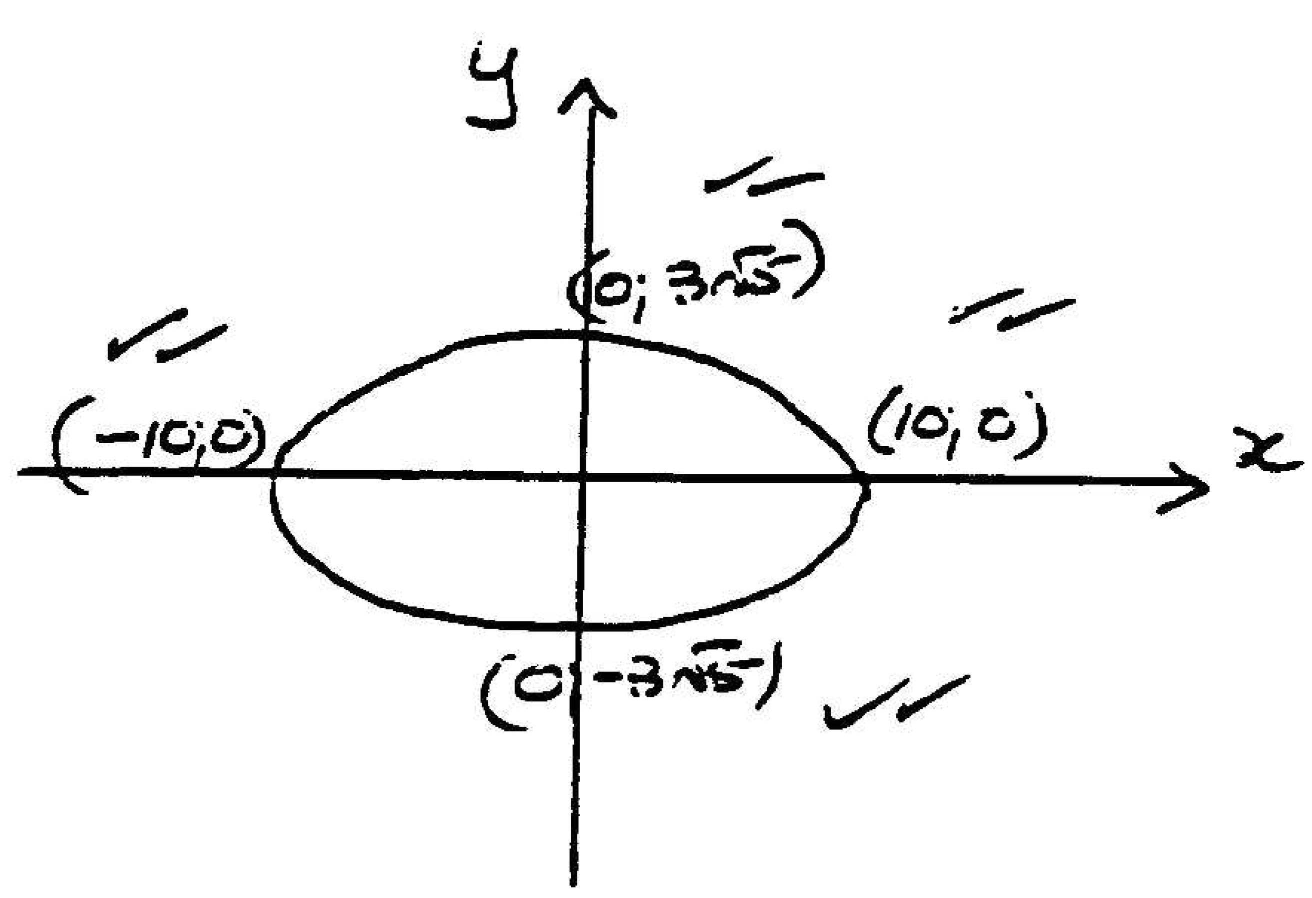
$\therefore 4PF^2 = PN^2$

$\therefore 4\left[(x-5)^2 + y^2\right] = (x-20)^2$

$\therefore 4(x^2 - 10x + 25) + 4y^2 = x^2 - 40x + 400$ (8)

$\therefore 3x^2 + 4y^2 = 300$

15.2



(8)

15.3. At $(5;7{,}5)$ $\quad 6x+8y.\frac{dy}{dx}=0$ ✓✓✓✓

$\therefore \frac{dy}{dx} = -\frac{6x}{8y}$ ✓✓

$= -\frac{3x}{4y}$

$= \frac{-3(5)}{4(\frac{15}{2})}$

$= -\frac{15}{30}$

$= -\frac{1}{2}$ ✓✓

$\therefore$ Gradient of radius $= 2$ ✓✓

$\therefore$ Centre is $(3{,}75;0)$ ✓✓

$\therefore$ Equ. of circle is

$\underbrace{(x-3{,}75)^2+y^2}_{✓✓✓✓} = (7{,}5-3{,}75)^2+5^2$ (18)

$= 3{,}75^2+5^2$

$= (39{,}0625 \text{ or } \frac{625}{16})$ ✓✓

Algebra memo:

1.1. $v^3+5=0$ Let $h(r) = r^3+5$ and $f(r) = r^2-r-1$

$$\begin{array}{r|l} & r+1 \\ \hline r^2-r-1 & r^3+0r^2+0r+5 \\ & \underline{r^3-r^2-r} \\ & \quad r^2+r+5 \\ & \quad \underline{r^2-r-1} \\ & \qquad 2r+6 \end{array}$$

(4)

$\therefore h(r) \equiv f(r)(r+1)+2(r+3)$

1	-1	-1	-3
x	-3	12	x
1	-4	11	x

(4)

$\therefore f(r) \equiv (r+3)(r-4)+11$

Now $11 \equiv f(r)-(r-4)(r+3)$ (2) $\therefore$ HCF $= 11$.

But $\frac{1}{2}h(r) - \frac{1}{2}f(r)(r+1) \equiv r+3$

$\therefore 11 \equiv f(r) - (r-4)\{\frac{1}{2}h(r) - \frac{1}{2}f(r)(r+1)\}$ (2)

$\equiv f(r) - \frac{1}{2}(r-4)h(r) + \frac{1}{2}f(r)(r+1)(r-4)$

$\equiv f(r)[1+\frac{1}{2}(r^2-3r-4)] - \frac{1}{2}(r-4)h(r)$

$\equiv f(r)[1+\frac{1}{2}r^2-\frac{3}{2}r-2] - \frac{1}{2}(r-4)h(r)$

$\equiv f(r)\left(\frac{r^2-3r-2}{2}\right) - \frac{1}{2}(r-4)h(r)$
(2) (1)

But $h(r) \equiv 0$

$\therefore \frac{1}{f(r)} \equiv \frac{r^2-3r-2}{22}$ (2)

$\therefore \frac{r}{f(r)} \equiv \frac{r^3-3r^2-2r}{22} \equiv \frac{-3r^2-2r-5}{22}$ (2)

[20]

(1.2) If $n=1$: $7+3-1=9$ /9 → True for $n=1$ (2)

Assume true for $n=k$: 7^k+3k-1 /9 (2)

Prove true for $n=k+1$: $7^{k+1}+3(k+1)-1$ /9 (2)

$7^{k+1}+3(k+1)-1 = 7.7^k+3k+3-1$

$= 7.7^k+21k-7-18k+9$ (2)

$= \underbrace{7(7^k+3k-1)}_{\text{div by 9 by assump}} - \underbrace{9(2k-1)}_{/9 \text{ naturally}}$ (1) (2)

$\therefore$ True for $n=k+1$ iff true for $n=k$.

$\therefore$ True $\forall n \in \mathbb{N}$ (2)

[14]

Q(2.1)

$\alpha + \beta = -b/a = -2b$ 2

$\alpha\beta = \frac{c}{a} = c$ 1

Now: $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ 1

$\therefore \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ 2

$= -8b^3 - 3c(-2b)$

$= -8b^3 + 6bc$ 3

$= 2b(3c - 4b^2)$

[10]

(2.2)

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{2b(3c-4b^2)}{c}$ 2, 2

$\frac{\alpha^2}{\beta} \cdot \frac{\beta^2}{\alpha} = \alpha\beta = c$. 2

$\therefore x^2 - \frac{2b(3c-4b^2)}{c}x + c = 0$

$\therefore cx^2 - 2b(3c-4b^2)x + c^2 = 0$ 2 any one eq.

[8]

Q(3)

3.1. If $\alpha + b\sqrt{v}$ is an irrational zero of $a(x) \in Z[x]$, then $\alpha - b\sqrt{v}$ is also a zero of $a(x)$. 2 2 2

[6]

3.2. $(x-1)^2 - (\sqrt{5})^2 = x^2 - 2x - 4$ is a factor 4

$$
\begin{array}{r|l}
 & 2x^3 + x^2 - 4x - 3 \\
\hline
x^2 - 2x - 4 & 2x^5 - 3x^4 - 16x^3 + x^2 + 22x + 12 \\
 & 2x^5 - 4x^4 - 8x^3 \\
 & x^4 - 6x^3 + x^2 \\
 & x^4 - 2x^3 - 4x^2 \\
 & -4x^3 + 5x^2 + 22x \\
 & -4x^3 + 8x^2 + 16x \\
 & -3x^2 + 6x + 12 \\
 & -3x^2 + 6x + 12 \\
 & \cdot \quad \cdot \quad \cdot
\end{array}
$$

4

Alternative 1

Let $p(x) = 2x^3 + x^2 - 4x - 3$

$p(-1) = -2 + 1 + 4 - 3 = 0$

2	1	-4	-3	-1
x	-2	1	3	x
2	-1	-3	0	x

4

or

Alternative 2

$(x+1)^2$ is a factor of $p(x)$

$$
\begin{array}{r|l}
 & 2x - 3 \\
\hline
x^2 + 2x + 1 & 2x^3 + x^2 - 4x - 3 \\
 & 2x^3 + 4x^2 + 2x \\
 & -3x^2 - 6x - 3 \\
 & -3x^2 - 6x - 3 \\
 & \cdot \quad \cdot \quad \cdot
\end{array}
$$

4

$\therefore h(x) \equiv (x+1)^2(x^2 - 2x - 4)(2x - 3)$ 2

[16]

Q(4) $f(x) = \dfrac{x^2 - 4x + 4}{x^2 + x - 6} = \dfrac{(x-2)^2}{(x+3)(x-2)} \left(= \dfrac{x-2}{x+3} \right)$

(4.1) **Vertical:** $(x+3)(x-2) = 0$

$\therefore x = -3$ or $x = 2$ (4)

Horizontal: $y = \lim\limits_{x \to \infty} \dfrac{x^2 - 4x + 4}{x^2 + x - 6} = 1$ (2)

Oblique: None (2)

[8]

(4.2) $\dfrac{(2x-4)(x^2+x-6) - (x^2-4x+4)(2x+1)}{(x^2+x-6)^2} > 0$ (4)

$\therefore \dfrac{2(x-2)(x-2)(x+3) - (x-2)^2(2x+1)}{(x+3)^2(x-2)^2} > 0$

$\therefore \dfrac{(x-2)^2(2x+6-2x-1)}{(x+3)^2(x-2)^2} > 0$

(2) $\therefore \dfrac{(x-2)^2}{(x+3)^2(x-2)^2} > 0$

$+ \quad -3 \quad + \quad 2 \quad +$

True $\forall x \in \mathbb{R} \setminus \{-3; 2\}$ (2) (2)

[10]

or $\dfrac{(x+3) - (x-2)}{(x+3)^2} > 0$ (4)

$\therefore \dfrac{5}{(x+3)^2} > 0$ (2)

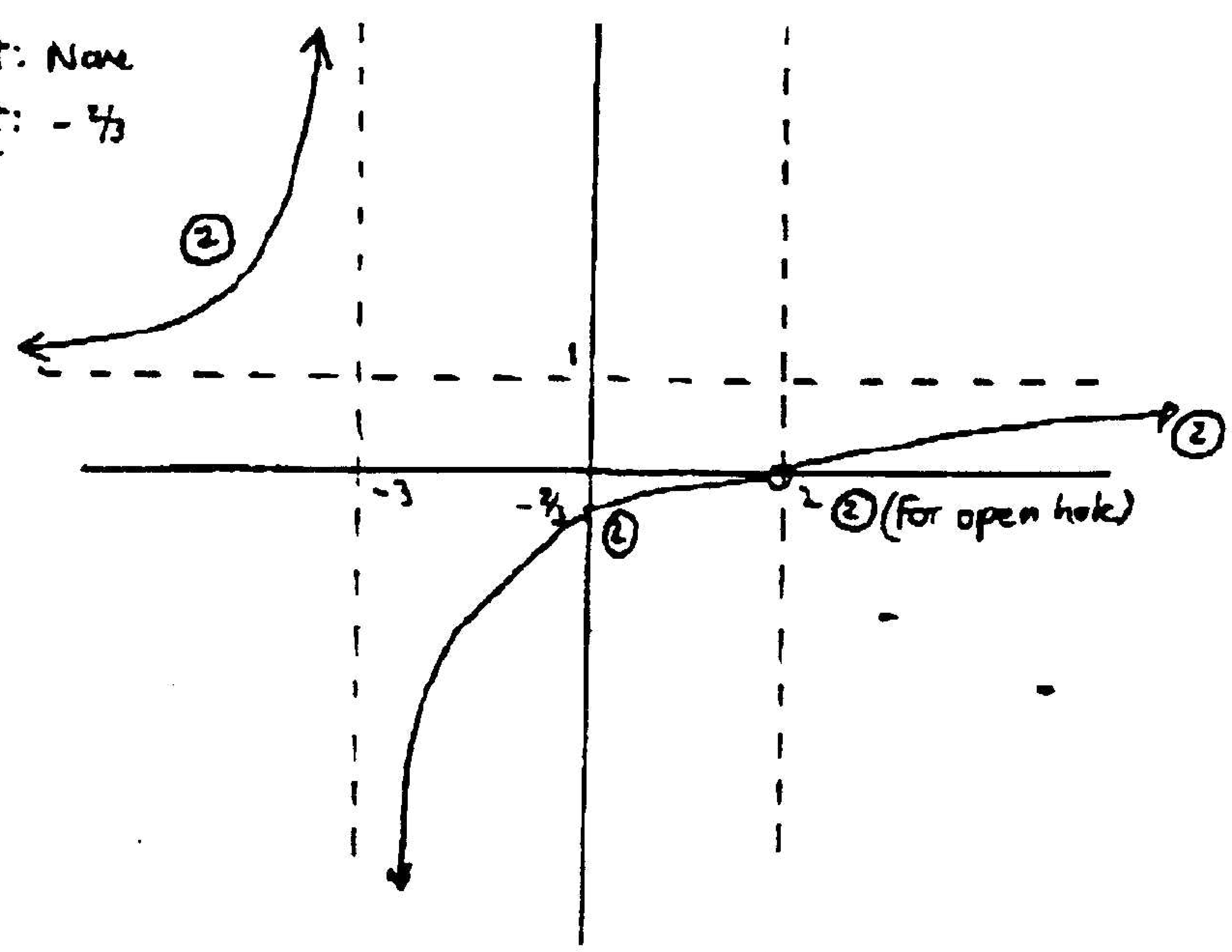
$+ \quad -3 \quad + \quad 2 \quad +$

True $\forall x \in \mathbb{R} \setminus \{-3; 2\}$ (2) (2)

[10]

(4.3) X_{int}: None

Y_{int}: $-\frac{2}{3}$



[8]

SECTION E

STATISTICS

20.1 No. of "words" $= \dfrac{9!}{3!\,2!}$

$= 30\ 240$ 6

20.2 Prob. "word" chosen at random starts and ends with "L" $= \dfrac{7!}{3!} \times \dfrac{3!\,2!}{9!}$

$= \dfrac{1}{36}$ (or $0{,}2\dot{7}$)

8

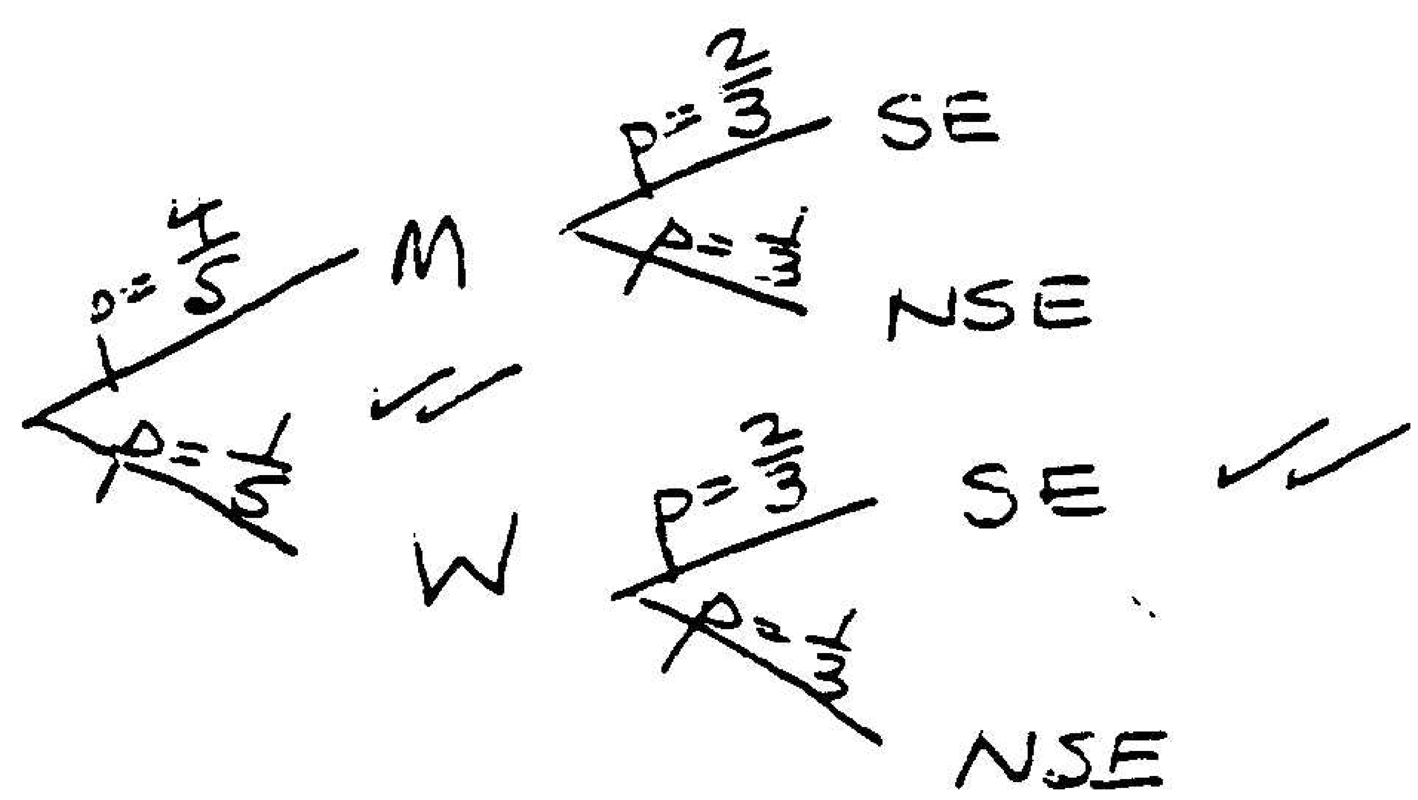
21.1 $P(C=6 + T=4) = \binom{10}{6}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$

$= 0{,}251$ (corr. to 3 dec. pl.)

21.2 $P(C>4) = 1 - \sum_{c=0}^{2} \binom{10}{c}\left(\frac{3}{5}\right)^c\left(\frac{2}{5}\right)^{10-c}$

$= 0{,}988$ (corr. to 3 dec. pl.)

22.1



6

P(woman who experienced side effects) $= \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$

22.2 P(patient experiences side effects)

$= \frac{2}{3} \times \frac{4}{5} + \frac{1}{4} \times \frac{1}{5}$

$= \frac{35}{60}$ 6

$= \frac{7}{12}$

22.3 That side effects are more likely in male patients 2 p18

2.3 $P\left(\pi \in \left(0{,}48 - 1{,}64\sqrt{\frac{0{,}48 \times 0{,}52}{50}}\ ;\ 0{,}48 + 1{,}64\sqrt{\frac{0{,}48 \times 0{,}52}{50}}\right)\right) =$

$\therefore$ 90% confidence interval $= (36{,}4\%\ ;\ 59{,}6\%)$ 10

(corr. to 1 dec. pl.)

2.4.1 $P(X < 30) = P\left(Z < \frac{30-52}{12}\right)$

$= P(Z < 1{,}83)$

$= 0{,}5 - 0{,}46638$

$= 0{,}03362$ 10

$= 3{,}3\%$ (corr. to 1 dec. pl.)

2.4.2 $P(X > x) = 0{,}2$

$\therefore P(X < x) = 0{,}5 + 0{,}3$

$\therefore \frac{x-52}{12} = 0{,}84$

$\therefore x - 52 = 10{,}08$

$\therefore x = 52 + 10{,}08$ 10

$\therefore$ Min. mark $= 63$

2.5.1.1 $\left(\frac{3}{r+3}\right)^2 + \left(\frac{r}{r+3}\right)^2 = \frac{5}{8}$

$\therefore 8\left[9 + r^2\right] = 5(r+3)^2$

$\therefore 72 + 8r^2 = 5r^2 + 30r + 45$

$\therefore 3r^2 - 30r + 27 = 0$

$\therefore r^2 - 10r + 9 = 0$

$(r-9)(r-1) = 0$ 8

$r = 9$ or $r = 1$

25.1.2 $$\frac{\binom{3}{2}\binom{r}{0}+\binom{3}{0}\binom{r}{2}}{\binom{3+r}{2}}=\frac{1}{2} \quad \checkmark\checkmark\checkmark\checkmark$$

$$\therefore 2\left[3+\frac{r!}{(r-2)!\,2!}\right]=\frac{(3+r)!}{(1+r)!\,2!}$$

$$\therefore 6+(r-1).r=\frac{(3+r)(2+r)}{2}$$

$$\therefore 12+2r^2-2r=6+5r+r^2$$

$$\therefore r^2-7r+6=0$$

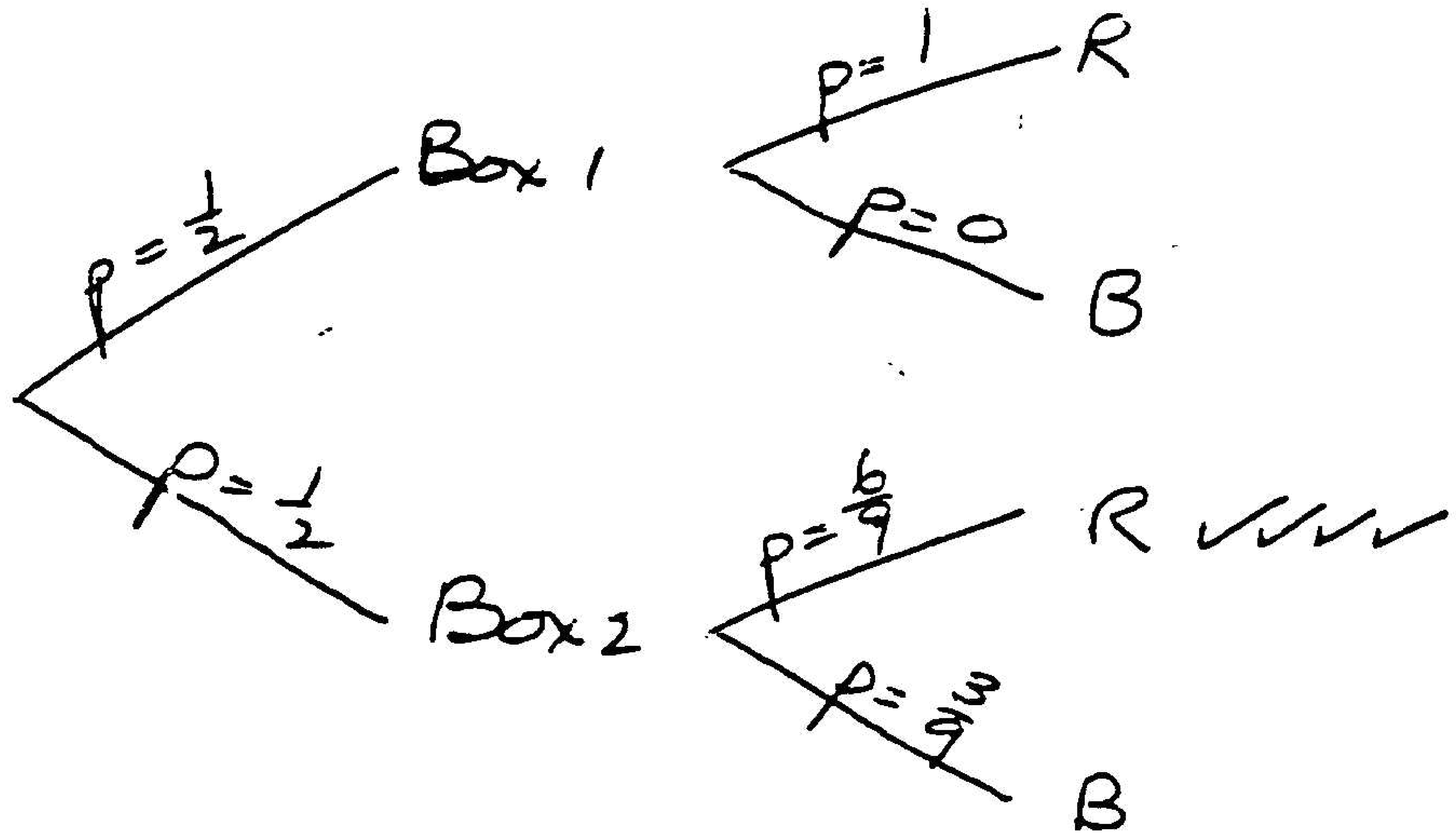
$$(r-6)(r-1)=0 \quad \checkmark\checkmark$$

$$\therefore r=6 \quad \text{or} \quad r=1 \qquad (8)$$

When $\checkmark\checkmark$ $r=1$ both conditions (Mpho and Piet) ε

25.2.1 Put one $\checkmark\checkmark$ red ball in the one box and all $\checkmark$ the other balls in the other box (4)

25.2.2



$$P(R)=\frac{1}{2}\times 1+\frac{1}{2}\times\frac{6}{9}$$

$$=\frac{5}{6} \quad \checkmark\checkmark \qquad (6)$$