Test Booklet No.
प्रश्नपत्रिका क्र.
F

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....
   (Name) ....
2. (Signature) ....
   (Name) ....

Seat No.
(In figures as in Admit Card)

Seat No. ....
(In words)

OMR Sheet No.
(To be filled by the Candidate)

DEC - 30213

Time Allowed : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : **24** Number of Questions in this Booklet : **50**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **50** objective type questions. Each question will carry *two* marks. *All* questions of Paper-II will be compulsory, covering entire syllabus (including all electives, without options).
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत **50** बहुपर्याय प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

DEC-30213/II

Mathematical Science

Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Note : This paper contains **50** multiple choice questions, each carrying **Two (2)** marks. Attempt *All* questions.

---

1. If

$$f : (a, b) \to \mathbf{R}$$

is differentiable, then the derivative of f at is given by :

$f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c-h)}{h}$

(A)

(B) $f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c-h)}{2h}$

(C) $f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c-h)}{3h}$

(D) $f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c-h)}{4h}$

2. Let

$$f(x) = x^3 - 3x + 1 \text{ for } x \in [-1, 1].$$

Then, by the mean value theorem, the value of such that the tangent to at $x = c$ is parallel to the chord joining the points and is given by :

(A) 1

(B) 0

(C)

(D) $\pm \frac{1}{\sqrt{3}}$

3. If $f(x) = (x-1)(x-3)$ on [1, 3], then the suitable c in (1, 3) for the Rolle's theorem such that is :
   (A) $c = 2$
   (B)
   (C) $\sqrt{2}$
   (D) $\frac{3}{2}$

4. If $f(z)$ is an entire function such that $\mathrm{Re}(f(z))$ is bounded, then which one of the following is *true* ?
   (A) f is not a constant
   (B) f is the identity function
   (C) f is a constant
   (D) for some entire function

5. For any two complex numbers z_1 and z_2 if $|z_1 + z_2| = |z_1| + |z_2|$ holds, then, which one of the following is *true* ?
   (A) $z_1 = t z_2$, where $t \geq 0$
   (B) where $t \leq 0$
   (C) z_1 and z_2 are linearly independent over **R**
   (D)

6. Let A and B be non-empty subsets of a vector space V. Suppose that $A \subseteq B$. Then :
   (A) If B is linearly independent, then so is A
   (B) If B is linearly dependent, then so is A
   (C) If A is linearly independent, then so is B
   (D) If B is a generating set, then so is A

7. Let

   be the linear transformation defined by :

   $$T(x, y) = (x, x + y)$$

   Then the matrix of T with respect to the standard basis is :
   (A)
   (B) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$
   (C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
   (D) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

8. Let A be an $m \times n$ real matrix. The space of solutions of the linear system AX = 0 has dimension :
   (A) m
   (B) at most minimum of m and n
   (C) $m + n$
   (D) at least $m - n$

9. Let A be an $n \times n$ matrix over $\mathbf{R}$ such that the rows of A form an orthonormal basis of $\mathbf{R}^n$. Then :
   (A) the columns of A are linearly dependent
   (B) the rows of A are linearly dependent
   (C) A is singular
   (D) the columns of A form an orthonormal basis of $\mathbf{R}^n$

10. Let A and B be similar matrices. Then :
   (A) rank A = rank B but the nullity of A need not be the same as the nullity of B
   (B) rank A = rank B and nullity of A = nullity of B
   (C) the nullity of A need not be the same as the nullity of B and the rank of A need not be the same as the rank of B
   (D) nullity of A = nullity of B but the rank of A need not be the same as the rank of B

11. The amount of calories in a chocolate bar is normally distributed with an average of 250 calories. If 99.7% of all the bars have between 205 and 295 calories, then the standard deviation (in calories) is :
   (A) 90
   (B) 45
   (C) 15
   (D) 10

12. Which of the following cases has $s = \{a, b, c\}$ constitute a probability space ?
   (A) $p(a) = 0.5, p(b) = 0.2, p(c) = 0.3$
   (B) $p(a) = 0.5, p(b) = 0.3, p(c) = 0.3$
   (C) $p(a) = 0.5, p(b) = 0.7, p(c) = -0.2$
   (D) $p(a) = 0.5, p(b) = 0.2, p(c) = 0.2$

13. If X and Y are independent, then which one of the following is *not* true ?
   (A) E(X + Y) = E(X) + E(Y)
   (B) E(XY) = E(X) E(Y)
   (C) E(X – Y) = E(X) – E(Y)
   (D) E(XY) = E(X) + E(Y)

14. Which of the following is the solution to the LP problem of maximizing $z = x_1 + x_2$ subject to :

$$x_1 + 2x_2 \le 7, \quad 3x_1 + x_2 \le 6,$$
$$x_1, x_2 \ge 0 \ ?$$

(A) (0, 7/2)

(B) (2, 0)

(C) (1, 3)

(D) (2, 3)

15. If P and Q are dual of each other, find values of a, b and c.

P : Max $2x_1 + ax_2$ subject to :

$$2x_1 + x_2 \le 3$$
$$bx_1 + 2x_2 \le c$$
$$x_1, x_2 \ge 0.$$

Q : Min $3u_1 + 4u_2$ subject to :

$$2u_1 + 4u_2 \ge 2$$
$$u_1 + 2u_2 \ge 5$$
$$u_1, u_2 \ge 0.$$

(A) $(a, b, c) = (5, 4, 4)$

(B) $(a, b, c) = (4, 4, 5)$

(C) $(a, b, c) = (5, 5, 4)$

(D) $(a, b, c) = (4, 5, 4)$

16. Which of the following statements is *false* ?

(A) If the primal is a minimization problem, its dual will be a maximization problem.

(B) For an unrestricted primal variable, the associated dual constraint is an equality.

(C) If a constraint in a maximization type of primal problem is "less-than-or-equal-to" type, the corresponding dual variable is non-negative.

(D) Columns of the constraint coefficients in the primal problem become columns of the constraint coefficients in the dual

17. If $\{I_n\}$ is a decreasing sequence of closed and bounded intervals in $\mathbf{R}$ such that $\inf\{b_n - a_n\} = 0$, where

$$I_n = [a_n, b_n]$$

then which one of the following is *true* ?

(A) $\cap I_n$ is empty

(B) $\cap I_n$ has exactly one point

(C) $\cap I_n$ has exactly two points

(D) $\cap I_n$ has infinitely many points

Or

If the AM and GM of a set of two observations are 9 and 6 respectively, then HM of this set is :

(A) 3

(B) 4

(C) 1.5

(D) $3\sqrt{6}$

18. If $\{a_n\}$ is a sequence of positive real numbers such that $a_{n+1} < r\, a_n$ for some $r \in (0, 1)$ and for all $n \in \mathbf{N}$, then which of the following is *true* ?

(A) $\{a_n\}$ is divergent

(B) $\{a_n\}$ converges to 0

(C) $\{a_n\}$ is strictly increasing

(D) $\{a_n\}$ converges to a non-zero real number

Or

If a constant 10 is subtracted from each observation of X and Y, then the regression coefficeint of X on Y is :

(A) Reduced by 10 percent

(B) Unchanged

(C) Increased by 10 percent

(D) Reduced by 20 percent

19. Let $\mathbf{R}$ be the set of real numbers. Let $f : \mathbf{R} \to \mathbf{R}$ be such that $|f(x) - f(y)| \le |x - y|^2$, for all $x, y \in \mathbf{R}$. Then $f(x)$ is :

(A) a constant

(B) the identity map

(C) x^2

(D) f is not differentiable on $\mathbf{R}$

Or

Let X be a random variable on whose cumulative distribution function (cdf) F_X has a Lebesgue probability density function F_X and $F_X(C) < 1$, where C is a fixed constant. Let $Y = \min\{X, C\}$. Then, the cdf of Y is given by :

(A)

(B) $F_Y(x) = \begin{cases} c & \text{if} \quad x < c \\ F_X(x) & \text{if} \quad x \geq c \end{cases}$

(C) $F_Y(x) = 1 - [1 - F_X(x)]^c$

(D) $F_Y(x) = F_X(x)$

20. Let A and B be non-empty subsets of $\mathbf{R}$, such that $A \subset B$. Then which one of the following is *true* ?

(A)

(B)

(C)

(D)

Or

Let X_1 and X_2 be independent random variables each assuming values +1 and –1 with probability ½. Let $X_3 = X_1 X_2$. Which of the following statements is *true* ?

(A) X_1 and X_3 are dependent

(B) X_2 and X_3 are dependent

(C) X_i and X_3 are independent for $i = 1, 2$

(D) X_1, X_2 and X_3 are independent

21. Let for all $n \in \mathbf{N}$. Then which one of the following is *true* ?

(A)

(B) $\lim\sup a_n \leq \lim\inf a_n$

(C) $\lim\sup a_n$ and $\lim\inf a_n$ do not exist

(D) $\lim\sup a_n = 1$ and $\lim\inf a_n = -1$

Or

Let $X_1, X_2, ..., X_n$ be independent random variables having common probability density function . Then the characteristic function of $\frac{S_n}{n} = \frac{(X_1 + X_2 + ... + X_n)}{n}$ converges to :

(A) $e^{-t^2/2}$

(B) $e^{t^2/2}$

(C) $e^{|t|}$

(D) $e^{-|t|}$

$|z| < 1$ $f(x) = (1 - \cos x) / (\pi x^2)$ $\sum_{i=1}^{n} \text{var}(X_i) \geq 2$ $z = \frac{1}{n}$ $\to 0$ $\to \infty$

22. Let Q be the set of rational numbers and **R** be the set of real numbers. Then which one of the following statements is *true* ?

(A) Q is not an ordered field

(B) Q is a complete ordered field

(C) **R** is not a complete ordered field

(D) **R** is a complete ordered field

Or

Let $X_1, X_2, ...$ be independent random variables with X_i distributed as Bernoulli (p_i, 1), $0 < p_i < 1$, $i = 1, 2, ...$ Which of the following statements is *false* ?

(A) $\sum_{i=1}^{n} E|X_i - E(X_i)|^3 \leq 2$

$\sum_{i=1}^{n} \text{var}(X_i)$

(B) $\sum_{i=1}^{n} (X_i - p_i) \Big/ \sqrt{\sum_{i=1}^{n} \text{var}(X_i)}$

$\xrightarrow{d} Z$, where $Z \sim N(0, 1)$

(C) as $n \to \infty$

(D) as $n \to \infty$

23. Let f be an analytic function in , having zeros at , (an integer). Then which one of the following is *true* ?

(A) f is a non-zero constant

(B) f is the identity map

(C) $f(z) = z^2$

(D) $f \equiv 0$ on

Or

Let X be a random variable with finite expectation μ and finite variance σ^2 and distribution function F_X. What will be the value of $\int [F(x+a)-F(x)]\, dx$?

(A) μ

(B) 0

(C) a + μ

(D) a

24. If a Mobius transformation has three fixed points, then which one of the following is *true* ?

(A) $f(z) = -z$

(B)

(C)

(D) $f(z)$ is a constant

Or

If X_i, are independent Poisson variates, then the conditional distribution of X_1 given is :

(A) Poisson

(B) Binomial

(C) Negative Binomial

(D) Geometric

25. If C is the circle $|z| = 1$, then :

$$\int_C \frac{dz}{z-2} =$$

(A) 0

(B) πi

(C)

(D)

Or

If X_1 and X_2 be a random sample of size two from exponential distribution with parameter λ, then the distribution of would be :

(A) U (0, 1)

(B) G (2 λ)

(C) Exp (λ)

(D) None of the above

26. Let f be an entire function such that $|f(z)| > M$, for all for some M > 0. Then, which one of the following is *true* ?

(A) f is an identity map

(B) $f(z) = z^2$

(C) f is a constant

(D) $f(z) = z^3$

Or

If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ then the distribution of $(\underset{\sim}{X} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{X} - \underset{\sim}{\mu})$ will be :

(A) $N_p(\underset{\sim}{O}, I)$

(B) $N_p(\underset{\sim}{O}, \Sigma)$

(C) χ^2_p

(D) None of the above

27. Let f be an entire function such that $f(z) = f\left(\frac{1}{2}z\right)$, for all $z \in \mathbb{C}$. Then $f(z)$ =

(A) z

(B) z^2

(C) const

(D) e^z

Or

If $X \sim F(m, n)$ and $Y \sim F(n, m)$, then for any $a > 0$,

$$P[X > a] + P\left[Y > \frac{1}{a}\right]$$

should be equal to :

(A) 0

(B) ∞

(C) 1

(D) None of the above

28. In any neighbourhood of an isolated essential singularity, f assumes :

(A) no complex number

(B) all complex numbers with at most one exception

(C) only finitely many complex numbers

(D) all complex numbers

Or

Let $X_1, X_2, \ldots, X_n$ be a random sample from :

$$F(x) = e^{-(x-\theta)}, \quad x \geq \theta$$

Consider the statements :

(*i*) $X_{(1)}$ is complete sufficient for θ

(*ii*) $X_{(1)}$ is the UMVUE of θ

(*iii*) $X_{(1)} - 1/n$ is the MLE of θ

Which of the following is *true* ?

(A) (*i*) and (*ii*)

(B) (*i*) only

(C) (*iii*) only

(D) (*ii*) and (*iii*)

29. Let f be a conformal self map of the unit disc with $f(0) = 0$. Then :
   (A) f is a rotation
   (B) f is an identity map
   (C) $f(z) = z^2$
   (D) $f(z) = z^3$

Or

Let X and Y be two random variables such that Y has Binomial (π, N), $0 < \pi < 1$ and given $Y = y$, X has Binomial (p, y), $0 < p < 1$, N-known. Then :
   (*i*) (X, Y) is minimal sufficient for (p, π)
   (*ii*) X is sufficient for p when π is known
   (*iii*) Y is sufficient for p when π is known
   (*iv*) (X, Y) has a distribution from exponential family
   (A) (*i*) and (*iv*) are true
   (B) (*i*) and (*iii*) are true
   (C) (*ii*) and (*iv*) are true
   (D) (*iii*) and (*iv*) are true

30. Let . Then $z = 0$ is a :
   (A) removable singularity of f
   (B) isolated essential singularity of f
   (C) non-isolated singularity of f
   (D) pole of f

Or

Let $X_1, X_2, \ldots, X_n$ be a random sample from Binomial (n, p), $0 < p < 1$, n known. Then :
   (A) $1/\bar{X}$ is an unbiased estimator of $1/p$
   (B) $n/\bar{X}$ is an unbiased estimator of $1/p$
   (C) $1/X_1$ is an unbiased estimator of $1/p$
   (D) $1/p$ is not an estimable parametric function

31. Let G be a group. Then G :
   (A) is isomorphic to a permutation group only if G is finite
   (B) is always isomorphic to a subgroup of a group of permutations
   (C) is never isomorphic to a permutation group
   (D) is isomorphic to a permutation group only if G is infinite

Or

Let X_1, X_2, ..., X_n be a random sample from N(θ, 1). For $|a| < 1$, define,

$$T_n = \begin{cases} \bar{X} & \text{if } |\bar{X}| > n^{-1/4} \\ a\bar{X} & \text{if } |\bar{X}| \le n^{-1/4} \end{cases}$$

Then :

(A) The asymptotic variance of $\sqrt{n}\, T_n$ is equal to one for all θ

(B) The asymptotic variance of $\sqrt{n}\, T_n$ is equal to a^2 for θ = 0

(C) The asymptotic variance of T_n is smaller than the Cramer-Rao lower bound

(D) The asymptotic variance of T_n is always larger than the Cramer-Rao lower bound

32. Let

$$G = \{1, -1, i, -i\}$$

Then G with multiplication as a binary operation is :

(A) not a group

(B) is a group isomorphic to $(\mathbf{Z}_2 \times \mathbf{Z}_2, +)$

(C) is isomorphic to a subgroup of $(\mathbf{Z}_8, +)$

(D) is a subgroup of $(\mathbf{C}, +)$

Or

Consider the problem of testing $H_0 : \theta = 0$ *Vs.* $H_1 : \theta > 0$ when (X_1, X_2) is a random sample from $U(\theta, \theta + 1)$. Let

$$\phi_1(X_1) = \begin{cases} 1 & \text{if } X_1 > 0.95 \\ 0 & \text{if otherwise} \end{cases}$$

$$\phi_2(X_1, X_2) = \begin{cases} 1 & \text{if } X_1 + X_2 > C \\ 0 & \text{if otherwise} \end{cases}$$

If ϕ_1 and ϕ_2 has the same size α = 0.05, what will be the value of C ?

(A) $1 - \sqrt{2}$

(B) $2 + \sqrt{0.1}$

(C) $2 - \sqrt{0.1}$

(D) $1 + \sqrt{2}$

33. Let R be a ring with unity. For $S \subseteq R$ $M_n(S)$ denotes the set of all $n \times n$ matrices with entries from S :

(A) If A is an ideal in the ring $M_n(R)$, then there exists an ideal I of R such that $A = M_n(I)$

(B) If S is a subring of R, then $M_n(S)$ need not be a subring of $M_n(R)$

(C) If R is commutative, then $M_n(R)$ is commutative

(D) $M_n(S)$ is always non-commutative

Or

A major automobile manufacturer has had to recall several models from its 2012 line due to quality control problems that were not discovered with its random final inspection procedures. This is an example of :

(A) Type-II error

(B) Type-I error

(C) Both Type-I error and Type-II error

(D) Neither Type-I error nor Type-II error

34. Let F be a field and

.

Then the set :

$$M = \left\{ \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} / \, b \in F \right\}$$

(A) is isomorphic to F

(B) is an ideal of R but not a maximal ideal

(C) is a field

(D) is a maximal ideal of R

Or

To test hypothesis about mean of a normal population with a known standard deviation, we can compare :

(A) The observed value of $\overline{x}$ with the critical value of z

(B) The observed value of z with the critical value of

(C) Both (A) and (B)

(D) The observed value of z with the critical value of z

35. Consider the ring R[x], where R is the field of real numbers. Let I be the principal ideal in R[x] generated by the polynomial $x^2 + 1$. Then :

(A) I is a maximal ideal in R[x]

(B) I = R[x]

(C) I is not a prime ideal in R[x]

(D) I = (0)

Or

Suppose we wish to test whether a population mean is significantly larger or smaller than 10. We take a sample and find . What should our alternative hypothesis be ?

(A)

(B)

(C)

(D) cannot be determined from the information given

36. Consider the groups Z_{12} and Z_{30}. The number of homomorphisms from $\mathbf{Z}_{12}$ to $\mathbf{Z}_{30}$ is :
   (A) 6
   (B) 12
   (C) 30
   (D) 3

Or

The sequence of C, D, C, D, C, D, C, D, C, D would probably be rejected by a test of runs as not being truly random, because :
   (A) The sequence contains too many runs
   (B) The sequence contains too few runs
   (C) The sequence contains only two symbols
   (D) The pattern C, D occurs only five times

37. Which of the following is a basis for the vector space of polynomials in $\mathbf{R}[x]$ of degree ≤ 3 ?
   (A) $1 + x + x^2$, $4 - x$, $11 + x + x^2 + 4x^3$
   (B) 1, $3 + x$, $x^2 + x^3$, $4 + x + 2x^2 + 2x^3$
   (C) $x^3 + x$, $x^2 + x$, $x + 1$, $x^3 + x^2 + 3x + 1$
   (D) 3, $4 - x$, $2 + 3x + x^2 + x^3$, $4 + 7x + x^3$

Or

In Mann-Whitney U test, a particular sampling distribution for U has a mean of 15. One value of U is calculated as $n_1 n_2 + n_1(n_1 + 1)/2 - R_1$ which equals 22.5. Can we immediately conclude that the value of $n_1 n_2 + n_2(n_2 + 1) - R_2$ in this situation, is :
   (A) 10 ?
   (B) 15 ?
   (C) 12.5 ?
   (D) 7.5 ?

38. Let X and Y be similar matrices. Which of the following statements is *not* true ?
   (A) X and Y have the same eigenvalues
   (B) X and Y have the same characteristic polynomials
   (C) X and Y have the same nullity
   (D) X and Y have the same eigenvectors

Or

Operations Research techniques are *not* applicable in the following situation :

(A) Objective can be defined for maximization or minimization

(B) Sufficient input data is available for formulating the problem

(C) Scientific methods, techniques and tools may be applied

(D) Resources available are unlimited

39. The characteristic equation of a matrix is,

$$x^3 - 2x^2 - x + 1 = 0.$$

If $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues of the matrix, then

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

is equal to :

(A) 3

(B) 5

(C) 6

(D) 8

Or

Neon lights in an industrial park are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs Rs. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 0.02 per day. The lead time between placing and receiving an order is 12 days. What should be the order quantity economically per order of neon lights ?

(A) 1,00,000

(B) 10,000

(C) 1,000

(D) 100

40. Let $v_1, v_2, ..., v_r$ be eigen vectors corresponding to eigen values $c_1, ..., c_r$, respectively, of a linear transformation.

(A) If $v_1, ..., v_r$ are distinct, then $c_1, ..., c_r$ are distinct

(B) If $c_1, ..., c_r$ are distinct then $v_1, ..., v_r$ are linearly independent

(C) If $v_1, ..., v_r$ are linearly independent, then $c_1, ..., c_r$ are distinct

(D) The vectors $v_1, ..., v_r$ are linearly independent iff $c_1, ..., c_r$ are distinct

Or

In the context of queueing theory, which of the following is *not* correct ?

(A) In the generalized queueing model, an arrival can be considered as births, whereas a departure can be looked upon as a death

(B) The distribution of waiting time is not related to queue discipline used in selecting the waiting customers for service

(C) When the waiting customer becomes impatient and decides to leave the queue, the customer is said to have renege

(D) The probability of a n-customers arriving during a time interval t, according to Poisson law is $p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$; $n = 0, 1, 2, \ldots.$

41. Let W be a subspace of the Euclidean space E. Then :

(A) $W = W^{\perp}$

(B) $W = W^{\perp\perp}$

(C) $W^{\perp\perp}$ is a proper subset of W

(D) $W \cap W^{\perp\perp} = \{0\}$

Or

Five jobs J_1, J_2, J_3, J_4 and J_5 are to be processed on three machines E, F and G in the order EFG. The timings of the jobs are :

(30, 40, 70), (80, 50, 90), (70, 10, 50), (50, 20, 60) and (40, 30, 100).

The optimum sequence would be :

(A) $J_4 \to J_1 \to J_3 \to J_2 \to J_5$

(B) $J_4 \to J_1 \to J_5 \to J_2 \to J_3$

(C) $J_1 \to J_4 \to J_5 \to J_2 \to J_3$

(D) $J_1 \to J_4 \to J_5 \to J_2 \to J_3$

42. Which of the following matrices is diagonalisable ?

(A) $\begin{bmatrix} 5 & 6 & 7 \\ 0 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Or

Dual Simplex method is applicable to those linear programming problems that starts with :

(A) An infeasible but optimum solution

(B) A feasible and optimum solution

(C) A feasible solution

(D) An infeasible solution

43. Let A be a Hermitian matrix. Then :

(A) det A is a real number

(B) A is positive definite

(C) A is unitary

(D) A is symmetric

Or

In a Game theory which of the following is *correct* ?

(1) Every two-person zero-sum game cannot be represented by a pair of linear programming problems with primal dual relationship

(2) The constant value, if added to each element of the pay-off matrix to formulate the given problem as a linear programming problem, should be substracted from the value of the game, determined from the solution to the linear programming problem

(3) Any two person game can be formulated and solved as a linear programming problem

(4) In a 2×n or m×2 game, each of the player's can mix at most two strategies if the multiple optimum solution do not exist

(A) (3), (4) and (2)

(B) (3), (4) and (1)

(C) (1), (2) and (3)

(D) (1), (2) and (4)

44. Consider the differential equations :

$$xy'' + (\sin x)\, y = 0 \quad \text{......} \quad (1)$$

$$x^2 y'' + (\sin x)\, y = 0 \quad \text{.....} \quad (2)$$

then $x = 0$ is the :

(A) Singular point of both the equations (1) and (2)

(B) Ordinary point of both the equations (1) and (2)

(C) Ordinary point of the equation (1) and regular singular point of the equation (2)

(D) Regular singular point of the equation (1) and irregular singular point of the equation (2)

Or

In the assignment problem which of the following is *correct* ?

(1) For every prohibited assignment, the given cost element is replaced by M, which is a very large value

(2) Multiple optimal assignment schedules may have different objective function values

(3) The dual variables for an assignment problem are given by u_i and v_j values of the optimum solution to the transformed transportation problem

(4) The variables of the dual problem of a given assignment problem may be unrestricted in sign.

(A) (1), (2) and (3)

(B) (2), (3) and (4)

(C) (3), (4) and (1)

(D) (4), (1) and (2)

45. Which one of the following is *not* true ?

(A) The eigen functions of Sturm-Liouville problem belong to two different eigen values are orthogonal

(B) The eigen values of Sturm-Liouville problem are real

(C) The two eigen functions corresponding to the same eigen value are linearly independent

(D) The eigen functions of Sturm-Liouville problem can be normalized

Or

The probability of a specified unit being included in the sample is :

(A) $\dfrac{1}{N}$

(B) $\dfrac{1}{{}^{N}C_{n}}$

(C) $\dfrac{n}{N}$

(D) $\dfrac{1}{N^{n}}$

46. Let ϕ_1, ϕ_2 be two linearly independent solutions of the equation $y'' + a_2(x)\,y = 0$. Then the Wronskian of ϕ_1 and ϕ_2 at any point in the interval I :
(A) is zero for all x
(B) is constant for all x
(C) is a linear function of x
(D) is a quadratic polynomial in x

Or

The systematic sampling is more precise than simple random sampling if :
(A) The variance within the systematic samples is less than the total variation in the population
(B) The variance within the systematic samples is more than the total variation in the population
(C) The variation between the systematic samples is less than the total variation in the population
(D) Both (A) and (C)

47. The initial value problem $y' = 3y^{2/3}$, $y(0) = 0$ in has :
(A) unique solution
(B) two solutions
(C) infinitely many solutions
(D) no solution

Or

The bias in the ratio estimator becomes zero if :
(A) The line of regression is a straight line
(B) The line of regression passes through the origin
(C) Both (A) and (B)
(D) None of the above

48. The complete integral of the partial differential equation

is :

(A) $F(x, y, z, a, b) = 0$, a, b are constants

(B) $F(u, v) = 0$, where $u = a$, $v = b$ are independent solutions of

(C) The eliminant of a and b from the equations $z = F(x, y, a, b)$, $F_a = 0$, $F_b = 0$

(D) The envelope of the one parameter family of solutions of $f(x, y, z, p, q) = 0$

$f(x, y, z, p, q) = 0$ $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

Or

F ratio contains :

(A) Two estimates of the population variance

(B) Two estimates of the population mean

(C) Two estimates of the population median

(D) One estimate of the population mean and one estimate of population variance

49. A function $f(x, y)$ on a region R satisfies Lipschitz condition provided :

(A) $f(x, y)$ is bounded and continuous and R is closed

(B) $f(x, y)$ and $\frac{\partial f}{\partial y}$ are continuous and R is an open disc in $\mathbf{R}^2$

(C) $\left|\frac{\partial f}{\partial y}\right| \le K, \forall (x, y) \in R$:

$$|x - x_0| \le a, |y - y_0| \le b,$$

$$a, b > 0$$

(D) $f(x, y)$ and are continuous and R is a closed bounded region

Or

If we have large enough sample sizes, we can discard which of the assumptions associated with ANOVA testing ?

(A) Each population has the same variance

(B) Each population has the same median

(C) Each population has the same mode

(D) The samples are drawn from a normal population

50. Let ϕ_1, ϕ_2 be two differentiable functions on an interval I, then :

(A) $W(\phi_1, \phi_2)(x) = 0 \quad \forall\, x \in I \Rightarrow$ are linearly dependent

(B) ϕ_1, ϕ_2 are linearly independent then $W(\phi_1, \phi_2)(x) \neq 0$

(C) $W(\phi_1, \phi_2)(x_0) \neq 0$ for some x_0 in I iff ϕ_1, ϕ_2 are linearly independent on I

(D) ϕ_1, ϕ_2 are linearly dependent functions on an interval I, then $W(\phi_1, \phi_2)(x) = 0 \;\forall\, x \in I$

Or

In a single-factor ANOVA problem involving five populations, with a random sample of four observations from each one, it is found that SSTr = 16.1408 and SSE = 37.3801. Then the value of the test statistic is :

(A) 2.316

(B) 0.432

(C) 1.522

(D) 1.619

ROUGH WORK

ROUGH WORK