

2008

100102

STATISTICS (Optional)

सांख्यिकी (वैकल्पिक)

Time : 3 hours

Maximum Marks : 200

Note :

- (i) In all attempt Five Questions.
- (ii) Question No. 1 is **Compulsory**.
- (iii) Of the remaining Questions, Attempt Any four by selecting one Question from each section.
- (iv) Numbers of optional questions upto the prescribed number in the order in which questions have been solved, will only be assessed and excess answers of the question/s will not be assessed.
- (v) Statistical and logarithmic tables will be supplied on request.
- (vi) Use of your own simple electronic calculator is allowed.
- (vii) Candidate should not write roll number, any names (including his/her own), signature, address or any indication of his/her identity anywhere inside the answer book otherwise he/she will be penalised.

1. Answer **any four** of the following :

- (a) X_1, X_2, \dots, X_{11} are independent random variables (r.v.s.) such that X_i has the exponential distribution with mean $(1/i)$ $i=1, 2, \dots, 11$ respectively. Obtain the distribution of

10

(i) $X = \sum_{i=1}^{10} i \cdot X_i$ (ii) $Y = \sum_{i=2}^{11} i \cdot X_i$

Also find the coefficient of correlation between X and Y.

- (b) In the analysis of variance (ANOVA) of one-way classified data, prove that :

10

Total sum of squares = within group sum of squares + Between group sum of squares.

Obtain the computational formula for each of the above sum of squares.

- (c) X_1, X_2, \dots, X_n is a random sample of size n from an uniform distribution defined over the interval (a, b). Obtain the moment estimators of a and b.

10

P.T.O.

- (d) Five artists have submitted bids to complete 4 sets of plates and the costs are given in the following table :

10

Artists	Cost per plate (in Rs.)			
	P ₁	P ₂	P ₃	P ₄
A	245	295	210	360
B	220	300	155	300
C	250	220	190	320
D	240	250	180	390
E	235	245	200	370

One set of plates is to be commissioned per artist so that the total cost is minimum. Which artist will not receive a commission ? What is the minimum total cost ?

- (e) If a random variable X has beta distribution of first kind with parameters $\frac{m}{2}$ and $\frac{n}{2}$.

Find the distribution of $Y = \frac{nX}{m(1-X)}$. Hence or otherwise state the mean and variance of Y.

10

SECTION - A

2. Answer the following sub-questions :

- (a) A two-dimensional random variable (X, Y) has the following joint probability density function (p.d.f.).

15

$$f(x, y) = x + y \quad \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

$$= 0 \quad \text{elsewhere.}$$

Find (i) $P(X \leq Y)$

(ii) $P\left(X^2 + Y^2 \leq \frac{1}{4}\right)$

(iii) $P\left(Y \geq \frac{1}{2}\right)$

- (b) A random variable (r.v.) X has a normal distribution with parameters μ and σ^2 . Obtain the expressions for the odd ordered and even ordered central moments of X . Hence obtain the values of β_1 and β_2 . Comment on the nature of the distribution. 15

- (c) If X_1, X_2 and X_3 are the independent observations from an univariate normal population with mean zero and unit variance, state giving reasons the sampling

distribution of $U = \frac{\sqrt{2} X_3}{\sqrt{X_1^2 + X_2^2}}$. 10

By using an appropriate table find

- (i) C such that $P[U \leq C] = 0.995$ and

- (ii) $P[|U| \leq 4.3]$.

3. Answer the following sub-questions :

- (a) A two-dimensional discrete random variable (X, Y) has the joint probability distribution as follows : 15

X \ Y	1	2	3	4	Total
0	1/24	1/12	1/12	1/24	1/4
1	1/12	1/6	1/6	1/12	1/2
2	1/24	1/12	1/12	1/24	1/4
Total	1/6	1/3	1/3	1/6	1

- Obtain
- (i) marginal distribution of X ,
 - (ii) marginal distribution of Y ,
 - (iii) conditional distribution of X given $Y=3$,
 - (iv) $E[X|Y=3]$ and $V[X|Y=3]$.

Are X and Y independent r.v.s. ?

P.T.O.

- (b) The joint probability mass function (p.m.f.) of X_1, X_2, X_3 is :

$$P(x_1, x_2, x_3) = \frac{n! p_1^{x_1} \cdot p_2^{x_2} \cdot p_3^{x_3}}{x_1! x_2! x_3!}$$

where $x_i = 0, 1, \dots, n$

For $i = 1, 2, 3;$

$0 < p_i < 1$ for $i = 1, 2, 3;$

$x_1 + x_2 + x_3 = n, p_1 + p_2 + p_3 = 1.$

Derive the joint moment generating function of X_1, X_2, X_3 . Hence or otherwise determine :

- (i) The value of coefficient of correlation between X_1 and X_2 .
 (ii) The value of coefficient of partial correlation between X_1 and X_2 with respect to X_3 .
 (c) Show that a chi-square distribution having n degrees of freedom is positively skewed.

10

SECTION - B

4. Answer the following sub-questions :

- (a) With two strata, a sampler would like to have $n_1 = n_2$ for administrative convenience instead of using the values that are obtained by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given by equal and Neyman's allocation respectively, show that ignoring finite population correction,

15

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \frac{(r-1)^2}{(r+1)^2}$$

where $r = \frac{n_1}{n_2}$ as given by Neyman's allocation.

- (b) Describe the technique of analysis of covariance (ANCOVA) in Randomised Block Design (R.B.D.).
 (c) Let n triplets of observations be available on X_1, X_2 and X_3 . Obtain the equation of the regression plane of X_3 on X_1 and X_2 by using the method of least squares.

10

15

5. Answer the following sub-questions :

- (a) State the estimators of the population mean in ratio and regression methods. Are these estimators unbiased? Comment on the relative efficiency of these estimators with respect to that in simple random sampling without replacement (SRSWOR). Justify your comment. 15
- (b) The following table gives the plan of an experiment involving three factors N, P, K in blocks of size 4 plots each : 10

Replication	Blocks	Factorial Effects
1	I	(1) pk nk np
	II	n p k npk
2	III	(1) k npk np
	IV	p nk pk n
3	V	p npk (1) nk
	VI	n k pk np

- (i) Identify the type of the experiment.
- (ii) Construct the ANOVA table for the experiment.
- (c) Find the distribution function and the p.d.f. of the r th order statistic $X_{(r)}$ when $X_{(1)} < X_{(2)} < \dots < X_{(n)}$ are the order statistics of a random sample of size n from a continuous distribution with p.d.f. $f(x)$. 15

SECTION - C

6. Answer the following sub-questions :

- (a) X_1, X_2, \dots, X_n is a random sample (r.s.) from a geometric distribution with following p.m.f. : 15
- $$p(x, \theta) = \begin{cases} \theta(1-\theta)^{x-1} & \text{for } 0 < \theta < 1; x = 1, 2, 3, \dots \\ 0 & \text{elsewhere.} \end{cases}$$

Show that the sample mean \bar{X} is an unbiased estimator of $\frac{1}{\theta}$. Determine variance

of \bar{X} and show that \bar{X} attains the lower bound given by Crammer - Rao inequality.

- (b) Let X_1, X_2, \dots, X_n be a r.s. from a gamma distribution with parameters (α, λ) . Find the asymptotic variance of the maximum likelihood estimator (m.l.e.) of α when λ is known. 10
- (c) X_1, X_2, \dots, X_{10} is a r.s. from an exponential distribution with mean θ . Find the most powerful test of size $\alpha = 0.05$ to test the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 4$. 15

P.T.O.

7. Answer the following sub-questions :

- (a) X is a r.v. having uniform distribution on the interval $(0, \theta)$. Let X_1, X_2, \dots, X_n be a r.s. from this distribution. Obtain a sufficient statistic for θ . Is it unbiased for θ ? If not, obtain an unbiased estimator of θ from it. Further, if $T = 2 \sum_{i=1}^n X_i/n$, evaluate the relative efficiency of T with respect to the unbiased estimator of θ obtained above. 15
- (b) A r.s. of 100 boys and 100 girls from a certain college were asked whether they attend any coaching classes. It was observed that 36 boys and 64 girls attend the coaching classes. Find the 95% confidence interval for the difference between the proportion of boys and girls who attend coaching classes. 10
- (c) Describe Mann - Whitney test. 15

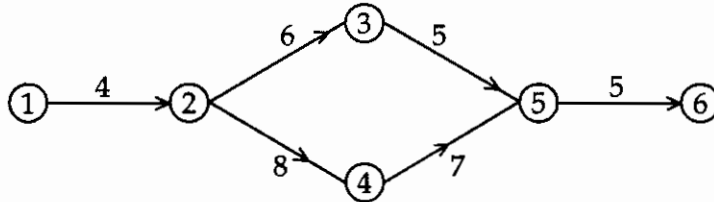
SECTION - D

8. Answer the following sub-questions :

- (a) Solve the following linear programming problem (L.P.P.) by Big M method : 10
 Maximize $Z = 3 X_1 + 2 X_2$
 Subject to
 $2 X_1 + X_2 \leq 2$
 $3 X_1 + 4 X_2 \geq 12$
 $X_1, X_2 \geq 0$
- (b) Explain the following methods of estimating the trend values in a time series : 15
 (i) Method of moving averages.
 (ii) Curve fitting by least squares method.
 (iii) Graphical method.
- (c) Suppose a large number of lots, each of size 2000 and 0.5% defective are submitted for inspection. Given three single sampling plans (i) $n = 40, c = 0$, (ii) $n = 80, c = 1$ and (iii) $n = 120, c = 2$; state giving reasons which sampling plan involves the minimum total inspection, considering both sampling inspection and screening of the rejected lots. 15

9. Answer the following sub-questions :

- (a) Calculate the slack times for the activities in the following network. Hence or otherwise find the critical path in this network. 10



- (b) Let $L(p)$ and $P(q)$ represent Laspeyre's index number for prices and Paasche's index number for quantities. Show that $L(p) \times P(q) = V_{01}$ where V_{01} is the value index. Hence or otherwise show that 15

$$\frac{L(p)}{L(q)} = \frac{P(p)}{P(q)}$$

where $L(q)$ and $P(p)$ represent Laspeyre's index number for quantities and Paasche's index number for prices respectively.

Also show that if (i) $A(p) = \frac{L(p) + P(p)}{2}$ then $A(p) >$ Fisher's ideal index number for prices.

(ii) $H(p) = \frac{2L(p)P(p)}{L(p) + P(p)}$ then

Fisher's ideal index number for prices $> H(p)$.

- (c) Explain the following procedures of setting limits on a control chart for fraction defective in a production process when the sample size is not fixed and standard of the process is not specified : 15

- (i) Method of variable control limits.
- (ii) Method of stabilised control limits.
- (iii) Method based on average sample size.

- o O o -