4-9

2008 12

100286

MATHEMATICS (Optional)

गणित (वैकल्पिक)

Time : 3 hours **Maximum Marks : 200**

Note :

(i) In all attempt ***Five*** *questions.*

(ii) Question No. 1 is ***compulsory.***

(iii) Of the remaining questions, Attempt ***Any Four*** *by selecting* ***one question*** *from each section.*

(iv) Number of optional questions upto the prescribed number in the order in which questions have been solved, will only be assessed and excess answers of the question/s will not be assessed.

(v) Candidate should not write roll number, any names (including his/her own), signature, address or any indication of his/her identity anywhere inside the answer book otherwise he/she will be penalised.

1. Answer **any four** of the following :

(a) Define (i) Feasible solution to LPP. 2

(ii) Basic feasible solution to LPP. 2

(iii) Convex set. 2

Show that the set of all feasible solution to a LPP (if a feasible solution exists) is a convex set. 4

(b) If $\{a_n\}$ is a non-increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n a_{2^n}$ converges,

then show that $\sum_{n=0}^{\infty} a_n$ converges. 10

(c) Determine components of velocity and acceleration of a moving particle along radial and transverse directions. 5 + 5

(d) Write a programme in C/recent computer language to evaluate the roots of a quadratic equation $ax^2+bx+c = 0$ requesting the user to input the values of a, b, c and to output real roots root1 and root2. 10

(e) If $\{f_n\}$ is a sequence of continuous real valued functions on the metric space X that converges uniformly to 'f' on X, then show that 'f' is also continuous on X. 10

P.T.O.

SECTION - A

2. (a) Show that every group is isomorphic to a subgroup of a permutation group A(S) for some appropriate S. 18

State the name of this theorem. 2

(b) If $\{v_1, v_2, \ldots, v_n\}$ is a basis of a vector space V and if $\{w_1, w_2, \ldots, w_m\}$ is linearly independent in V, then show that $m \leq n$. 20

3. (a) Define (i) Euclidean Domain. 3

(ii) Principal Ideal Domain (PID). 3

Show that every Euclidean Domain is PID. 14

(b) State Cayley Hamilton theorem and using it find inverse of the matrix A if it exists. 2

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 4 \\ 3 & 4 & 8 \end{pmatrix}$$ 18

SECTION - B

4. (a) If 'f' is a real-valued continuous function on a compact metric space X, then show that $f(X)$, range of 'f' is compact and 'f' attains a maximum and minimum at points of X. 10 + 10

(b) Define absolute convergence and conditional convergence for improper integrals of the type $\int_a^\infty f(x)\,dx$ for continuous function $f(x)$. 2 + 2

Show that $\int_a^\infty \frac{\sin x}{x}\,dx$ is convergent but not absolutely. 8 + 8

5. (a) Define differentiability of a function of two variables at a point. 2

Let $f: E \rightarrow \mathbb{R}$ be defined on a neighbourhood E of $(a,b) \in \mathbb{R} \times \mathbb{R}$ such that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are continuous at (a,b). Show that 'f' is differentiable at (a, b). Is the converse of this is true ? Justify your answer. 10

8

(b) Define a Riemann integral for a bounded real function on [a, b].
Show that a bounded real function 'f' is Riemann integrable on [a,b] if and only if for every $\epsilon > 0$, there is a partition P of [a,b] such that $U(P, f) - L(P, f) < \epsilon$. 7 + 7

SECTION - C

6. (a) Show that the general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ of second degree represents a conic section. 20

(b) Define (i) Divergence of a vector point function. 2
(ii) Curl of a vector point function. 2

(c) Show that $\bar{b}.\nabla(\bar{a}.\nabla\frac{1}{r}) = \frac{3(\bar{a}.\bar{r})(\bar{b}.\bar{r})}{r^5} - \frac{\bar{a}.\bar{b}}{r^3}$ 10

(d) Show that $\text{curl}(\bar{r} \times \bar{a}) = -2\bar{a}$. 6

7. (a) (i) Find the length of perpendicular from a point (x_1, y_1, z_1) to the plane $ax+by+cz = d$. 10
(ii) Find equation of a sphere for which the circle $x^2+y^2+z^2+7y-2z+2 = 0$, $2x+3y+4z = 8$ is a great circle. 10

(b) Verify Gauss divergence theorem for $\bar{f} \equiv (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. 20

SECTION - D

8. (a) (i) Explain when $M(x,y)+N(x,y)y' = 0$ is said to be exact in some rectangle R. 2
(ii) Define integrating factor of the equation $M(x,y)dx+N(x,y)dy = 0$, where M and N have continuous partial derivatives in some rectangle R. 2
Show that the equation $M(x,y)+N(x,y)y' = 0$ is exact in

$R: |x - x_o| \le a, |y - y_o| \le b$ if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R. 8 + 8

(b) Find the inverse of the coefficient matrix of the system.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

by Gauss - Jordan method with partial pivoting and hence solve the system. 20

P.T.O.

9. (a) If ϕ_1 is a solution of $y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, and $\phi_1(x) \neq 0$ on I, describe a method to determine a second linearly independent solution ϕ_2 of this differential equation on I. 12

Hence or otherwise find second linearly independent solution of $y'' - 4xy' + (4x^2 - 2)y = 0$ after verifying that $\phi_1(x) = e^{x^2}$ is a solution of this differential equation. 8

(b) Describe Trapezoidal and Simpson $\frac{1}{3}rd$ rule to find $\int_a^b f(x)dx$ numerically. 5 + 5

Find an approximate value of $\int_0^1 \frac{dx}{1+x}$ by using Trapezoidal and Simpson $\frac{1}{3}rd$ rule and compare with exact solution. 5 + 5

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