

2007

Mathematics (Optional)

000614

गणित (वैकल्पिक)

Time : 3 hours

Maximum Marks : 200

- Note :
- (i) In all attempt Five Questions.
 - (ii) Question No. 1 is Compulsory.
 - (iii) Of the remaining Questions. Attempt Any four by selecting one Question from each section.
 - (iv) Numbers of optional questions upto the prescribed number in the order in which questions have been solved, will only be assessed and excess answers of the questions/s will not be assessed.
 - (v) Candidate should not write roll number, any names (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.

1. Answer any four of the following :

- (a) Complete one iteration of Simplex method, given the problem :

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$$\begin{aligned} \text{minimize} \quad & 2x_1 + 4x_2 + x_3 \\ \text{subject to} \quad & x_1 + 2x_2 - x_3 \leq 5 \\ & 2x_1 - x_2 + 2x_3 = 2 \\ & -x_1 + 2x_2 + 2x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Is the solution at the end of first iteration optimal ? Give reasons.

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- (b) If $f(n), g(n), n \in \mathbb{N}$ are such that -

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$$f(n) > 0, g(n) > 0, \lim_{n \rightarrow \infty} g(n) = \infty, \lim_{n \rightarrow \infty} f(n) = \infty \text{ and } \lim_{n \rightarrow \infty} \frac{g(n)+3}{f(n)} = \frac{1}{32}.$$

Prove that the series -

$$\sum_1^{\infty} \left(\frac{g(n)}{f(n)} \right)^{\frac{2n}{5}}$$

is convergent.

- (c) Derive the equations :

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$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_{\theta} = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

and $F_z = m\ddot{z}$

for a body moving under influence of force \vec{F} .

Here (r, θ, z) are the cylindrical co-ordinates.

- (d) Consider the Algorithm :

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ALGORITHM XYZ (m, n)

1. $r \leftarrow m \bmod n$
2. if $r = 0$ GOTO 6
3. $m \leftarrow n$
4. $n \leftarrow r$
5. GOTO STEP 1.
6. return n

for inputs m, n as positive integers and $m > n$.

Provide a DRY RUN for $m = 51, n = 33$. What does the function actually compute?

Provide adequate justification for your answer.

- (e) Show that the following alternating series is conditionally convergent.

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$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

Section - A

2. (a) Define a Group. When do you say it is abelian? 20
 If G is a group, prove that $(a.b)^2 = a^2.b^2$ for $\forall a, b \in G$ iff G is abelian.
- (b) If $V = \mathbb{R}^3 = \{ (x, y, z) : x, y, z \in \mathbb{R} \}$ and 20
 $W =$ subset of V such that $x - 2y - 3z = 0$,
 then show that
 (i) V is a 3-dim vector space with \mathbb{R} as scalars.
 and (ii) W is a subspace of V with $\dim(W) = 2$.

Further, if the equation defining W , i.e., $x - 2y - 3z = 0$ is changed to $x - 2y - 3z = 5$, will it still remain a subspace.

3. (a) (i) Define a Ring, Ideal and a Euclidian domain. 20
 (ii) Prove that a non empty subset H of a group G is a subset iff
 $a \in H, b \in H \Rightarrow a_0 b^{-1} \in H$.
- (b) If $S = \{ v_1, v_2, \dots, v_n \}$ is a set of n -linearly independent vectors belonging to vector space $V(F)$ and $\dim V = n$ then show that - 20
 (i) any $u \in V$ can be expressed as a linear combination of elements of S and
 (ii) such a representation is unique.

Section - B

4. (a) (i) Evaluate the limits - 20

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$
 and

$$\lim_{x \rightarrow 0} x^{n-3} \ln x^{x^3} \quad (n > 3, \text{ integer})$$

 (ii) State and prove Cauchy's MVT.
- (b) Test the convergence of the integrals - 20
 (i) $\int_a^{\infty} \frac{dx}{x^n}$ $a > 0, n \in \mathbb{R}$ and
 (ii) $\int_a^{\infty} \frac{dx}{n^x}$ $a > 0, n > 0$

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- (b) (i) Evaluate using $h = \frac{1}{6}$ and Simpson's $\frac{3}{8}$ th Rule $\int_0^1 \frac{1}{1+x} dx$. 20
- (ii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ if

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	1.280	0.554	1.296	2.432	4.0

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5. (a) (i) Show that $(|x| < 1)$ 20

$$\sqrt{1+x} = f(x) + \frac{x^3}{16(1+\theta x)^{5/2}}$$

with $0 < \theta < 1$ and $f(x)$ a quadratic expression in x .

- (ii) Find maximum of

$$v = xyz \quad x > 0, y > 0, z > 0.$$

provided that $xy + yz + zx - a = 0$.

- (b) If P, Q are partitions of $[a, b]$, then show that : 20

(i) $L(f, Q) \leq L(f, P)$ and

(ii) $U(f, Q) \geq U(f, P)$

whenever P is finer than Q , f is a Riemann integrable function over $[a, b]$ and L, U are the lower and upper Darboux sums.

Section - C

6. (a) Show that in general two tangents can be drawn to a circle from a point lying on the exterior of the circle, and lying in the plane of the circle. 20

Derive the equation of the Director Circle.

- (b) Show that area bounded by a simple closed curve C is given by - 20

$$\frac{1}{2} \oint_C (x dy - y dx)$$

Demonstrate its applicability by calculating the area bound by the curve :

$$x = a \cos \theta, y = b \sin \theta, 0 \leq \theta \leq 2\pi$$

7. (a) If $P(t_1)$ and $Q(t_2)$ are the end points of a focal chord of the standard parabola $y^2 = 4ax$, show that $t_1 t_2 = -1$. Here t is the parameter in the parametric representation. 20

$x = at^2$ and $y = 2at$ of the conic.

Further show that slopes of OP and OQ is constant where O is the origin.

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- (b) Consider the following double integral

$$\int_0^1 \int_y^{y+1} x^2 y \, dx dy = \iint_R x^2 y \, dy$$

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- (i) Describe the region R
 (ii) Change the order of integration
 (iii) Evaluate the integral

Section - D

8. (a) Find the complete solution of -

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$$y^2 \left(1 + \left(\frac{dy}{dx} \right)^2 \right) = R^2$$

Show that solution is a family of circles.

Equation has singular solutions as well. Find them.

- (b) (i) Find the value of x for which $\log_e x = \cos x$
 correct upto 5 decimal places.

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- (ii) Population of a town in a census is recorded as :

Year	1950	1960	1970	1980	1990
Population in '000'	46	66	81	93	101

Find the population (estimate) for the year 1985 by interpolation.

9. (a) (i) Given $F(D) y = (aD^2 + 2bD + c) y$ with a, b, c constants and $D \equiv d/dx$ establish following identities.

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- $F(D) e^{kx} = e^{kx} F(K)$
- $F(D^2) \sin ax = \sin ax F(-a^2)$
- $F(D) e^{kx} u(x) = e^{kx} F(D+K) u(x)$

- (ii) Solve $\frac{dy}{dx} + xy = x^3 y^3$

P.T.O.