## AQA

# Level 2 Certificate in Further Mathematics 

Practice Paper Set 4
Paper 1 8360/1

## Mark Schemes

Principal Examiners have prepared these mark schemes for practice papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

It is not possible to indicate all the possible approaches to questions that would gain credit in a 'live' examination. The principles we work to are given in the glossary on page 3 of this mark scheme.

- Evidence of any method that would lead to a correct answer, if applied accurately, is generally worthy of credit.
- Accuracy marks are awarded for correct answers following on from a correct method. The correct method may be implied, but in this qualification there is a greater expectation that method will be appropriate and clearly shown.


## Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M Method marks are awarded for a correct method which could lead to a correct answer.

A Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.

B Marks awarded independent of method.
ft Follow through marks. Marks awarded for correct working following a mistake in an earlier step.

SC Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.

Mdep A method mark dependent on a previous method mark being awarded.

B dep A mark that can only be awarded if a previous independent mark has been awarded.
oe
Or equivalent. Accept answers that are equivalent.
eg, accept 0.5 as well as $\frac{1}{2}$
$[\boldsymbol{a}, \boldsymbol{b}] \quad$ Accept values between $a$ and $b$ inclusive.
3.14... Allow answers which begin 3.14 eg 31.4, 3142, 3.149

Use of brackets It is not necessary to see the bracketed work to award the marks.

## Paper 2 - Calculator

| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 1 | $\left(1 \frac{1}{2}\right)^{2}-\left(1 \frac{1}{5}\right)^{2}$ | M1 | $\text { or } \frac{9}{4}-\frac{36}{25} \text { or } w^{2}+\left(1 \frac{1}{5}\right)^{2}=\left(1 \frac{1}{2}\right)^{2}$ |
|  | $\frac{225-144}{100}$ or $\frac{225}{100}-\frac{144}{100}$ | M1 |  |
|  | $\begin{aligned} & \sqrt{\left(1 \frac{1}{2}\right)^{2}-\left(1 \frac{1}{5}\right)^{2}} \text { or } \sqrt{\left(\frac{9}{4}-\frac{36}{25}\right)} \\ & \text { or } \sqrt{\text { their } \frac{81}{100}} \end{aligned}$ | M1 dep | Dep on 1st M1 <br> Use of $3,4,5 \Delta$, with $1 \frac{1}{2}=\frac{15}{10}$ or 1.5 and $1 \frac{1}{5}=\frac{12}{10}$ or 1.2 scores M3 |
|  | $\frac{9}{10}$ | A1 | oe |
|  | Alternative method |  |  |
|  | $1.5^{2}-1.2^{2}$ | M1 | or $w^{2}+1.2^{2}=1.5^{2}$ |
|  | $2.25-1.44$ or $(1.5+1.2)(1.5-1.2)$ | M1 | 0.81 |
|  | $\sqrt{(1.5)^{2}-(1.2)^{2}}$ or <br> $\sqrt{(2.25)^{2}-(1.44)^{2}}$ or $\sqrt{\text { their } 0.81}$ | M1 dep | Dep on 1st M1 <br> Use of $3,4,5 \Delta$, with $1 \frac{1}{2}=\frac{15}{10}$ or 1.5 and $1 \frac{1}{5}=\frac{12}{10}$ or 1.2 scores M3 |
|  | 0.9 | A1 | oe |


| 2 | $4 h x-4-3 x-3 h$ | M1 | Allow one error or $5 x+5 \mathrm{k}$ (no errors) |
| :---: | :--- | :---: | :--- |
|  | $4 h x-3 x=5 x$ or $-4-3 h=5 k$ | M1 dep | oe equating their $x$ terms or constant <br> terms |
|  | $h=2$ | A1 |  |
|  | $k=-2$ | A1 ft | ft their $h$ if $M$ marks gained |


| Q Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 3(a) | $\frac{x}{y}=\frac{3}{2}$ | M1 | oe |
| :---: | :--- | :--- | :--- |
|  | $x=\frac{3 y}{2}$ | A1 | oe |
| 3(b) | $2 x+y=3 y+y(=4 y)$ <br> or <br> $3 x-2 y=3\left(\frac{3 y}{2}\right)-2 y\left(=2 \frac{1}{2} y\right)$ | M1 | $(2 \times 3+2):(3 \times 3-2 \times 2)$ oe <br> using any values of $x$ and $y$ in the ratio <br> $3: 2$ |
|  | $8: 5$ | A1 |  |

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| $A D B=180-90-(90-2 x) \quad(=2 x)$ | B1 | Angle sum of triangle |
| :--- | :--- | :--- |
| $D A C=$ their $2 x-x$ | B1 | Exterior anglesum of interior opposite <br> angles |
| $D A C(=x)=A C B$ | B1 | Must have all reasons for full marks |
|  | Alternative method 1 |  |
| $A D C=90+90-2 x(=180-2 x)$ | B1 | Exterior angle $=$ sum of interior opposite |
| angles |  |  |


| $\mathbf{Q}$ | Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 5 | $10 x$ or $6 x^{2}$ | B1 |  |
| :--- | :--- | :--- | :--- |
|  | their $6 x^{2}=$ their $10 x$ | M1 | oe |
|  | $2 x(3 x-5)=0$ or $3 x=5$ | M1 | oe ft if a quadratic equation |
| $x=\frac{5}{3}$ | A1 | oe |  |


| 6 | $\frac{6 x^{5}-15 x^{2}}{x^{2}}$ | M1 |  |
| :--- | :--- | :---: | :--- |
|  | $6 x^{3}-15$ | A1 |  |
|  | $18 x^{2}$ | A1 ft | ft from a two-term polynomial |


| 7 | $\frac{2 k-3 h}{h k}$ or $2 k-3 h=4 h k$ | M1 | or $\frac{2}{h}=4+\frac{3}{k}$ |
| :---: | :--- | :---: | :--- |
|  | $2 k=4 h k+3 h$ | M 1 | or $\frac{2}{h}=\frac{4 k+3}{k}$ |
|  | $2 k=h(4 k+3)$ and $h=\frac{2 k}{4 k+3}$ | A 1 | $\frac{h}{2}=\frac{k}{4 k+3}$ and $h=\frac{2 k}{4 k+3}$ |


| 8 | $(y=) 3 x^{2}+2 x-8$ | B 1 |  |
| :---: | :--- | :---: | :--- |
|  | $\frac{\mathrm{~d} y}{\mathrm{~d} x}=6 x+2$ | B 1 ft | ft if 3 terms for their $y=\ldots \ldots$. |
|  | 14 | B 1 ft |  |


| Q Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |


| 9 | $4 x+5 x=180$ or $9 x=180$ | M1 | Opposite angles in cyclic quadrilateral are supplementary |
| :---: | :---: | :---: | :---: |
|  | $x=20$ | A1 |  |
|  | $2 y+2 \times($ their 20$)+5 y=180$ | M1 | Opposite angles in cyclic quadrilateral are supplementary |
|  | $y=20$ (so $x=y$ ) | A1 | Must state a reason for full marks |
|  | Alternative method |  |  |
|  | $4 x+5 x=180$ or $9 x=180$ | M1 | Opposite angles in cyclic quadrilateral are supplementary |
|  | $2 x+7 y=180$ | M1 | Opposite angles in cyclic quadrilateral are supplementary |
|  | $4 x+5 x=2 x+7 y$ | M1 |  |
|  | $7 x=7 y($ so $x=y)$ | A1 | Must state a reason for full marks |


| 10 | $\frac{\mathrm{~d} y}{\mathrm{~d} x}=-8-3 x^{2}$ | M1 |  |
| :---: | :--- | :--- | :--- |
|  | $x^{2} \geq 0$ for all values of $x$ <br> so $-8-3 x^{2}<0$ for all values of $x$ | A1 | oe |
|  | Gradient is always negative so $y$ is <br> a decreasing function for all values of <br> $x$ | A1 | Must make connection between $\frac{\mathrm{d} y}{\mathrm{~d} x}$ <br> gradient/decreasing function |

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| $p^{2}=m^{2}+(3 m)^{2}-2 \times m \times 3 m \times \cos 60$ | M1 | oe |
| :--- | :---: | :--- |
| $p^{2}=m^{2}+(3 m)^{2}-3 m^{2}$ | A 1 |  |
| $p^{2}=7 m^{2}$ so <br> $p=m \sqrt{7}$ | A 1 |  |


| Q Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 12 | $\frac{480--180}{-260-620}$ | M1 |  |
| :---: | :---: | :---: | :---: |
|  | $-\frac{3}{4}$ | A1 |  |
|  | $\begin{aligned} & 480=\text { their }-\frac{3}{4} \times(-260)+c \\ & \text { or }-180=\text { their }-\frac{3}{4} \times(620)+c \end{aligned}$ | M1 | oe |
|  | $B=(0,285)$ | A1 |  |
|  | $0=$ their $-\frac{3}{4} x+285$ | M1 | oe |
|  | $C=(380,0)$ | A1 |  |


| 13 | $(5,9)$ | B1 | Centre of circle |
| :---: | :---: | :---: | :---: |
|  | $D(1,9)$ or $B(9,9)$ | B1 |  |
|  | Midpoint of a side of the square eg midpoint of $A B$ $\frac{(5+\text { their } 9)}{2}, \frac{(13+\text { their } 9)}{2} \text { or }(7,11)$ | M1 | $\begin{aligned} & B C \rightarrow(7,7) \\ & C D \rightarrow(3,7) \\ & D A \rightarrow(3,11) \end{aligned}$ |
|  | radius ${ }^{2}=(\text { their } 7-\text { their } 5)^{2}+$ (their 11 - their 9 ) ${ }^{2}$ or 8 | M1 | oe |
|  | $(x-5)^{2}+(y-9)^{2}=8$ | A1 | oe |
|  | Alternative method 1 |  |  |
|  | $(5,9)$ | B1 | Centre of circle |
|  | $\cos 45=\frac{r}{4}$ or $\cos 45=\frac{A B}{8}$ | M1 | oe |
|  | $r=4 \times \frac{1}{\sqrt{2}} \text { or } A B=8 \times \frac{1}{\sqrt{2}}$ | M1 | oe |
|  | $r^{2}=\frac{16}{2} \text { or } 8$ | M1 |  |
|  | $(x-5)^{2}+(y-9)^{2}=8$ | A1 | Oe |


| Q Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |


|  | Alternative method 2 |  |  |
| :---: | :---: | :---: | :---: |
|  | $(5,9)$ | B1 | Centre of circle |
|  | Distance $A O=4$ | B1 | oe |
|  | $A M=r$ and $O M=r$ | M1 | where $M$ is the midpoint of $A B$ and $O$ is the centre of the circle $r$ is the radius of the circle oe |
|  | $r^{2}+r^{2}=4^{2}$ or $2 r^{2}=16$ or $r^{2}=8$ | M1 |  |
|  | $(x-5)^{2}+(y-9)^{2}=8$ | A1 | oe |


| 14(a) | $(2)^{3}+8(2)^{2}+(2)-42$ | M1 |  |
| :---: | :---: | :---: | :---: |
|  | $8+32+2-42=0$ | A1 |  |
| 14(b) | $\begin{aligned} & \left(x^{3}+8 x^{2}+x-42=\right) \\ & (x-2)\left(x^{2}+n x+21\right) \end{aligned}$ | M1 | oe <br> or <br> Substitutes another value into the expression and tests for ' $=0$ ' <br> or <br> Long division of polynomials getting as far as $x^{2}+10 x \ldots$ |
|  | $(x-2)\left(x^{2}+10 x+21\right)$ | A1 | $(x+3)$ is a factor |
|  | $(x-2)(x+3)(x+7)$ | A1 | $(x+7)$ is a factor |
|  | 2, -3 and -7 | A1 |  |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 15 | $\frac{(5 \sqrt{5}-2)(2 \sqrt{5}+3)}{(2 \sqrt{5}-3)(2 \sqrt{5}+3)}$ | M1 |  |
|  | $50-4 \sqrt{5}+15 \sqrt{5}-6$ <br> or $20-9$ | M1 | oe allow one error |
|  | $\begin{aligned} & 50-4 \sqrt{5}+15 \sqrt{5}-6 \\ & \text { and } \\ & 20-9 \end{aligned}$ | A1 | oe |
|  | $4+\sqrt{5}$ | A1 |  |


| 16 | $2\left(x^{2}-4 x\right)(+9)$ or $2\left(x^{2}-4 x+\frac{9}{2}\right)$ | M1 |  |
| :---: | :---: | :---: | :---: |
|  | Sight of ( $x-2)^{2}$ | M1 |  |
|  | $2\left[(x-2)^{2}-4\right]+9$ <br> or $2\left[(x-2)^{2}-4+\frac{9}{2}\right]$ | A1 |  |
|  | $2(x-2)^{2}+1$ | A1 |  |
|  | Squared terms are always $\geq 0$ so the expression is always $\geq 1$ or $>0$ | A1 | oe |
|  | Alternative method |  |  |
|  | $\frac{d y}{d x}=4 x-8 x$ | M1 | Starting with $y=2 x^{2}-8 x+9$ |
|  | Stationary point when $x=2$ | M1 |  |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=-4 \text { when } x=1$ <br> and $\frac{\mathrm{d} y}{\mathrm{~d} x}=4 \text { when } x=3$ <br> ... hence minimum point when $x=2$ | A1 |  |
|  | $y=1($ when $x=2)$ | A1 |  |
|  | Minimum point at $(2,1)$ hence expression must be $>0$ for all values of $x$ | A1 | oe |


| $\mathbf{Q}$ | Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 17 | $2 a+a b=-1$ | B1 | Allow one error in these two steps |
| :---: | :---: | :---: | :---: |
|  | $a-3 b=2$ | B1 |  |
|  | $2 a+\frac{a(a-2)}{3}=-1$ | M1 | $(3 b+2)(2+b)=-1$ |
|  | $a^{2}+4 a+3=0$ | A1 | $3 b^{2}+8 b+5=0$ |
|  | $(a+1)(a+3)(=0)$ | M1 | $(3 b+5)(b+1)(=0)$ |
|  | $a=-1, b=-1$ <br> and $a=-3, b=-\frac{5}{3}$ | A1 |  |

