AQA Qualifications

# AQA CERTIFICATE
# FURTHER MATHEMATICS

Paper 2 83602
Mark scheme

83602
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## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

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|---|---|
| **M** | Method marks are awarded for a correct method which could lead to a correct answer. |
| **M dep** | A method mark dependent on a previous method mark being awarded. |
| **A** | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| **B** | Marks awarded independent of method. |
| **B dep** | A mark that can only be awarded if a previous independent mark has been awarded. |
| **ft** | Follow through marks. Marks awarded following a mistake in an earlier step. |
| **SC** | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth. |
| **oe** | Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$ |
| **[*a*, *b*]** | Accept values between *a* and *b* inclusive. |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 1 | $x^2$<br>4<br>5 | B3 | All in appropriate boxes<br>B1 for each correct box<br>In the left hand boxes ignore inclusion of $y$ = and/or f($x$) = |
| 2 | **Alternative method 1** | | |
| | $A$ (6, 0) or $x$ = 6 (for $A$) | B1 | May be on diagram or be implied |
| | $\frac{1}{2}$ × their 6 × $y$ = 24 | M1 | |
| | $y$ = 8 | A1ft | Only ft B0 M1 |
| | their 8 = 12 – 2$x$ | M1 | |
| | $x$ = 2 | A1ft | ft their $y$<br>SC2 Answer (8, 2) with no valid working<br>SC1 $B$ (0, 12) or $y$ = 12 (for $B$) |
| | **Alternative method 2** | | |
| | $A$ (6, 0) or $x$ = 6 (for $A$) | B1 | May be on diagram or be implied |
| | $B$ (0, 12) or $y$ = 12 (for $B$)<br>and<br>(area $OAB$ =) $\frac{1}{2}$ × their 6 × 12<br>or 36<br>and<br>$\frac{1}{2}$ × 12 × $x$ = their 36 – 24 | M1 | |
| | $x$ = 2 | A1ft | Only ft B0 M1 |
| | $y$ = 12 – 2 × their 2 | M1 | |
| | $y$ = 8 | A1ft | ft their $y$<br>SC2 Answer (8, 2) with no valid working<br>SC1 $B$ (0, 12) or $y$ = 12 (for $B$) |

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| **2** | **Alternative method 3** | | |
| | $A$ (6, 0) or $x = 6$ (for $A$) | B1 | May be on diagram or be implied |
| | $\frac{1}{2} \times$ their $6 \times y = 24$ | M1 | |
| | $y = 8$ | A1ft | Only ft B0 M1 |
| | $B$ (0, 12) or $y = 12$ (for $B$)<br>and<br>(area $OAB$ =) $\frac{1}{2} \times$ their $6 \times 12$<br>or 36<br>and<br>$\frac{1}{2} \times 12 \times x =$ their 36 – 24 | M1 | |
| | $x = 2$ | A1ft | Only ft B0 with 2nd M1 gained<br>SC2 Answer (8, 2) with no valid working<br>SC1 $B$ (0, 12) or $y = 12$ (for $B$) |
| | **Alternative method 4** | | |
| | $A$ (6, 0) or $x = 6$ (for $A$) | B1 | May be on diagram or be implied |
| | $B$ (0, 12) or $y = 12$ (for $B$)<br>and<br>(area $OAB$ =) $\frac{1}{2} \times$ their $6 \times 12$<br>or 36<br>and<br>$\frac{1}{2} \times 12 \times x =$ their 36 – 24 | M1 | |
| | $x = 2$ | A1ft | Only ft B0 M1 |
| | $\frac{1}{2} \times$ their $6 \times y = 24$ | M1 | |
| | $y = 8$ | A1ft | Only ft B0 with 2nd M1 gained<br>SC2 Answer (8, 2) with no valid working<br>SC1 $B$ (0, 12) or $y = 12$ (for $B$) |

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| **2** | **Alternative method 5** | | |
| | $A$ (6, 0) or $x = 6$ (for $A$) | B1 | May be on diagram or be implied |
| | $B$ (0, 12) or $y = 12$ (for $B$)<br>and<br>(area $OAB$ =) $\frac{1}{2}$ × their 6 × 12<br>or 36<br>and<br>$\frac{24}{\text{their } 36} \times 12$ | M1 | |
| | $y = 8$ | A1ft | Only ft B0 M1 |
| | $B$ (0, 12) or $y = 12$ (for $B$)<br>and<br>(area $OAB$ =) $\frac{1}{2}$ × their 6 × 12<br>or 36<br>and<br>$\frac{\text{their } 36 - 24}{\text{their } 36} \times \text{their } 6$ | M1 | |
| | $x = 2$ | A1ft | Only ft B0 with 2$^{nd}$ M1 gained<br>SC2 Answer (8, 2) with no valid working<br>SC1 $B$ (0, 12) or $y = 12$ (for $B$) |

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| **3(a)** | Valid reason<br>e.g.1 Triangle $OTS$ is isosceles<br>e.g.2 $OT = OS$<br>e.g.3 $OT$ and $OS$ are radii | B1 | |

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| **3(b)** | Correct equation<br>e.g.1 $5x = 2(x + 30)$<br>e.g.2 $2.5x = x + 30$<br>e.g.3 $(180 - 2x) + 120 + 5x = 360$<br>e.g.4 $x + 30 + x + 30 + 360 - 5x = 360$ | M1 | oe<br><br><br>Brackets not needed in e.g.3 |
| | Collects terms for their initial equation<br>e.g.1 $5x - 2x = 60$<br>e.g.2 $2.5x - x = 30$<br>e.g.3 $-2x + 5x = 360 - 180 - 120$ | M1 | oe<br>their initial equation must have ≥ 2 terms in $x$<br>Any brackets must be expanded correctly |
| | 20 | A1 | |

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| **4(a)** | $x^3 - 2x^2$ | B2 | B1 for $x^3$<br>B1 for $-2x^2$ |

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| **4(b)** | $3x^2$ or $-4x$ | M1 | At least one term of their $x^3 - 2x^2$ differentiated correctly |
| | $3(3)^2 - 4(3)$ or $27 - 12$ | M1dep | oe<br>Substitutes $x = 3$ in their $\frac{dy}{dx}$<br>their $\frac{dy}{dx}$ must be an expression in $x$<br>Allow even if their (a) has only one term |
| | 15 | A1ft | ft M2 and their (a)<br>Only ft if their (a) has at least two terms of different order and all of their terms are differentiated correctly |

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| 4(c) | $y - 9 =$ their 15$(x - 3)$<br>or<br>$y =$ their 15$x + c$ and substitutes (3, 9) | M1 | oe e.g. $\frac{9-y}{3-x} =$ their 15<br>their 15 from (b)<br>Allow $y - 9 = \frac{-1}{\text{their 15}}(x - 3)$<br>or<br>$y = \frac{-1}{\text{their 15}}x + c$ and substitutes (3, 9)<br>for M1 A0 only |
| | $y = 15x - 36$ | A1ft | ft their 15 from (b)<br>$15x - 36$ is M1 A0 unless $y = 15x - 36$ seen in working |

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| **5** | $5(4c + 3)$ and $2(c - 8)$<br>or<br>$20c + 15$ and $2c - 16$ | M1 | oe e.g. $10(4c + 3) + 4(c - 8)$<br>Allow one error in expansion if not showing brackets<br>e.g. Allow $20c + 3$ and $2c - 16$<br>Equation or fractions not necessary |
| | Correct equation with no unexpanded brackets<br>e.g.1 $20c + 15 + 2c - 16 = 10$<br>e.g.2 $22c - 1 = 10$<br>e.g.3 $\frac{(20c+15)}{10} + \frac{(2c-16)}{10} = 1$<br>e.g.4 $\frac{44c-2}{20} = 1$ | A1 | |
| | Eliminates denominators correctly and collects terms for their equation<br>e.g.1 $20c + 2c = 10 - 15 + 16$<br>e.g.2 $22c = 11$ | M1dep | dep on first M1<br>Do not award this mark if the denominator has been eliminated incorrectly at any time in the working<br>Allow one sign error when collecting terms |
| | $\frac{1}{2}$ or $\frac{11}{22}$ | A1ft | oe<br>Only ft from M1 A0 M1 with a maximum of one error in expansions and collecting terms<br>SC2 Answer $\frac{15}{22}$ oe |

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| **6** | (radius =) $\sqrt{289}$ or 17<br>or<br>(radius =) $\sqrt{121}$ or 11 | B1 | |
| | $(\frac{1}{4}\times)\ 2 \times \pi \times$ their 17 or $34\pi$ or $\frac{17\pi}{2}$<br>or [106.76, 107] or [26.69, 26.71]<br>or<br>$(\frac{1}{4}\times)\ 2 \times \pi \times$ their 11 or $22\pi$ or $\frac{11\pi}{2}$<br>or [69.08, 69.124] or [17.27, 17.3] | M1 | oe<br>their 17 can be 289<br>their 11 can be 121 |
| | their 17 – their 11 or 6 | M1 | their 17 can be 289<br>their 11 can be 121<br>May be implied by 12 seen in next method mark |
| | $\frac{1}{4} \times 2 \times \pi \times$ their 17 +<br>$\frac{1}{4} \times 2 \times \pi \times$ their 11 +<br>2 × their 6 | M1 | their 17 can be 289<br>their 11 can be 121 |
| | $14\pi + 12$ or [55.96, 56(.0)] | A1 | SC2 $42\pi$ or [131.88, 132] |

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| **7(a)** | $x^7$ | B2 | B1 $\sqrt{x^{14}}$ or $(x^{14})^{\frac{1}{2}}$ or $\sqrt{x^{5+9}}$<br>or $(x^{5+9})^{\frac{1}{2}}$ or $x^{\frac{14}{2}}$ or $x^{\frac{5+9}{2}}$<br>or $x^{\frac{5}{2}} \times x^{\frac{9}{2}}$ or $x^{2.5} \times x^{4.5}$ |

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| **7(b)** | 0.2 or $\frac{1}{5}$ or $5^{-1}$ | B2 | B1 $125^{-\frac{1}{3}}$ or $\sqrt[-3]{125}$ or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{125}}$ or $\frac{1}{125^{\frac{1}{3}}}$ or $\frac{1}{\sqrt[3]{125}}$ or $\left(\frac{1}{5^3}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{5^3}}$ or $\frac{1^{\frac{1}{3}}}{5}$ or $\frac{\sqrt[3]{1}}{5}$ or $\frac{1}{y^3} = 125$ or $y^3 = \frac{1}{125}$ or $\frac{1}{y} = 5$ or $\frac{1}{y} = \sqrt[3]{125}$ or $\frac{1}{y} = 125^{\frac{1}{3}}$ |

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| **8** | $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$ | B2 | B1 2 by 2 matrix with at least two elements correct |
| | their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}(\times)\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ | M1 | Multiplication can be in either order if their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$ is a 2 by 2 matrix<br>Do not award if their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$ is **M** |
| | $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ | A1 | Must have B2 with M1 seen |

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| **9** | **Alternative method 1** | | |
| | angle $ACD$ = 180 – 78 or 102 | M1 | |
| | angle $ECD$ = 360 – 115 – their 102<br>or 143 | M1 | angle $ECD$ = 143 implies M1 M1 |
| | (143 + 32 =) 175 and No<br>or<br>143 + 32 ≠ 180 (and No) | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |
| | **Alternative method 2** | | |
| | angle $ACD$ = 180 – 78 or 102 | M1 | |
| | (Assumes $CD$ is parallel to $EF$)<br>angle $DCE$ = 180 – 32 or 148 | M1 | |
| | (102 + 148 + 115 =) 365 and No<br>or<br>102 + 148 + 115 ≠ 360 (and No) | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |
| | **Alternative method 3** | | |
| | Extends $DC$ to $X$<br>angle $XCA$ = 78 | M1 | $X$ may be a different letter or not labelled |
| | angle $XCE$ = 115 – their 78 or 37 | M1 | angle $XCE$ = 37 implies M1 M1 |
| | 37 and No | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |
| | **Alternative method 4** | | |
| | Extends $DC$ to $X$<br>angle $XCA$ = 78 | M1 | $X$ may be a different letter or not labelled |
| | (Assumes $CD$ is parallel to $EF$)<br>angle $XCE$ = 32 | M1 | |
| | (32 + 78 =) 110 and No<br>or<br>32 + 78 ≠ 115 (and No) | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |

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| 9 | **Alternative method 5** | | |
| | Extends $AC$ to meet $EF$ at $Y$<br>angle $ECY$ = 180 – 115 or 65 | M1 | $Y$ may be a different letter or not labelled |
| | angle $EYC$ = 180 – their 65 – 32<br>or 83 | M1 | angle $EYC$ = 83 implies M1 M1 |
| | 83 and No | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |
| | **Alternative method 6** | | |
| | Extends $AC$ to meet $EF$ at $Y$<br>angle $ECY$ = 180 – 115 or 65 | M1 | $Y$ may be a different letter or not labelled |
| | (Assumes $AB$ is parallel $EF$)<br>angle $EYC$ = 78 | M1 | |
| | (32 + 78 + 65 =) 175 and No<br>or<br>32 + 78 + 65 ≠ 180 (and No) | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |
| | **Alternative method 7** | | |
| | Draws a line from $X$ on $AB$ to $Y$ on $EF$ passing through $C$ with right angles marked at $AXC$ and $CYE$<br>(Assumes $CD$ is parallel to $EF$)<br>angle $ACX$ = 180 – 90 – 78 or 12 | M1 | $X$ and $Y$ may be different letters or not labelled |
| | angle $ECY$ = 180 – 90 – 32 or 58 | M1 | |
| | (12 + 115 + 58 =) 185 and No<br>or<br>12 + 115 + 58 ≠ 180 (and No) | A1 | oe<br>SC3 32 + 78 = 110 and No<br>or 32 + 78 ≠ 115 (and No) |

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| **10(a)** | $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ | B2 | B1 Rotation 180° (about/centre $O$)<br>or<br>indication that $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$<br>or<br>indication that $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$<br>or<br>$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(\times)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$<br>or<br>$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(\times)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$<br>or<br>reflection in $y = -x$ and $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ |

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| **10(b)** | Correct square (vertices $O$, $A''$ (–3, 0) $B''$ (–3, –3) and $C''$ (0, –3)) with correct labelling | B3 | B2 Correct square with incorrect or no labelling<br>or<br>correct points plotted with correct labelling<br><br>B1 3 by 3 square in wrong position (ignore labelling)<br>or<br>correct points plotted with incorrect or no labelling<br>or<br>enlargement scale factor –3 (centre $O$)<br>or<br>$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ or<br>$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ or<br>$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ |

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| **11(a)** | $\frac{4c^5}{9d^3}$ or $\frac{4c^5d^{-3}}{9}$ or<br><br>$\frac{0.\dot{4}c^5}{d^3}$ or $0.\dot{4}c^5d^{-3}$ | B3 | B2 Any two of these three components<br>• numerator having $c^5$ (no $c$ in denominator)<br>• denominator having $d^3$ (no $d$ in numerator) or numerator having $d^{-3}$ (no $d$ in denominator)<br>• number $\frac{4}{9}$ or $0.\dot{4}$<br><br>B1 Any one of these three components<br>• numerator having $c^5$ (no $c$ in denominator)<br>• denominator having $d^3$ (no $d$ in numerator) or numerator having $d^{-3}$ (no $d$ in denominator)<br>• number $\frac{4}{9}$ or $0.\dot{4}$<br><br>or<br>$\frac{40c^7d^3}{90d^6c^2}$ or $\frac{20c^7d^3}{45d^6c^2}$ or $\frac{8c^7d^3}{18d^6c^2}$<br>or $\frac{1.\dot{3}c^7d^3}{3d^6c^2}$ or $\frac{\frac{4}{3}c^7d^3}{3d^6c^2}$<br><br>SC1 $\frac{9d^3}{4c^5}$ or $\frac{2.25d^3}{c^5}$<br>Always award SC1 if this is their final answer even if $\frac{4c^5}{9d^3}$ seen in working |

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| **11(b)** | $(m + 1)(m - 4)$ or $m^2 - 3m - 4$<br>seen as a common denominator | B1 | oe |
| | $5(m - 4) + 6(m + 1)$ | M1 | Allow one error in expansion if not showing brackets<br>e.g. Allow $5m - 20 + m + 6$ |
| | $\dfrac{5m - 20 + 6m + 6}{\text{their common denominator}}$<br>or<br>$\dfrac{5m - 20}{\text{their common denominator}} + \dfrac{6m + 6}{\text{their common denominator}}$ | M1 | Allow one error in expansion of numerator(s)<br><br>their common denominator must be a quadratic |
| | $\dfrac{11m - 14}{(m + 1)(m - 4)}$ or $\dfrac{11m - 14}{m^2 - 3m - 4}$ | A1 | |

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| **12** | **Alternative method 1** | | |
| | $x^2 + (2x)^2 = 20$ or $\sqrt{20 - x^2} = 2x$ | M1 | oe Condone absence of brackets |
| | $5x^2 = 20$ or $5x^2 - 20$ (= 0) | M1 | oe e.g $x^2 = 4$<br>Collects terms for their quadratic to $ax^2 = b$ or $ax^2 - b$ (= 0)<br>$a$ and $b$ both non-zero<br>This mark implies the first M1 |
| | $\sqrt{\dfrac{20}{\text{their } 5}}$ or $x = \sqrt{4}$ or<br>$5(x + 2)(x - 2)$ (= 0) | M1 | Correct attempt to solve their quadratic<br>oe e.g. $(x + 2)(x - 2)$ (= 0)<br>If using formula must substitute correctly<br>If using completing the square must correctly obtain<br>$(px + q)^2 = r$ or $(px + q)^2 - r$ (= 0)<br>$p$, $q$ and $r$ non-zero |
| | $x = 2$ and $x = -2$<br>or<br>$x = 2$ and $y = 4$<br>or<br>$x = -2$ and $y = -4$ | A1 | Allow $x = \pm 2$ |
| | $D$ (2, 4) and $E$ (–2 , –4 ) | A1 | Correct letter must be linked to correct point<br>SC2 Both points correct by T & I<br>SC1 One point correct by T & I |

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| **12** | **Alternative method 2** | | |
| | $\left(\frac{y}{2}\right)^2 + y^2 = 20$ or $\sqrt{20 - y^2} = \frac{y}{2}$ | M1 | oe Condone absence of brackets |
| | $5y^2 = 80$ or $\frac{5}{4}y^2 = 20$ or<br>$5y^2 - 80 = 0$ | M1 | oe e.g $y^2 = 16$<br>Collects terms for their quadratic to $ay^2 = b$ or $ay^2 - b$ (= 0)<br>$a$ and $b$ both non-zero<br>This mark implies the first M1 |
| | $\sqrt{\frac{80}{\text{their } 5}}$ or $y = \sqrt{16}$ or<br>$5(y + 4)(y - 4)$ (= 0) | M1 | Correct attempt to solve their quadratic<br>oe e.g. $(y + 4)(y - 4)$ (= 0)<br>If using formula must substitute correctly<br>If using completing the square must correctly obtain<br>$(py + q)^2 = r$ or $(py + q)^2 - r$ $(= 0)$<br>$p$, $q$ and $r$ non-zero |
| | $y = 4$ and $y = -4$<br>or<br>$y = 4$ and $x = 2$<br>or<br>$y = -4$ and $x = -2$ | A1 | Allow $y = \pm 4$ |
| | $D$ (2, 4) and $E$ (–2 , –4 ) | A1 | Correct letter must be linked to correct point<br>SC2 Both points correct by T & I<br>SC1 One point correct by T & I |

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| **13(a)** | $C$ | B1 | |

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| **13(b)** | $D$ | B1 | |

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| **13(c)** | $A$ | B1 | |

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| **14** | $x(5 - 3w) = 2w + 1$ | M1 | |
| | $5x - 3xw = 2w + 1$<br>or<br>$5 - 3w = \frac{2w}{x} + \frac{1}{x}$ | M1dep | oe e.g. $5x - 3xw - 2w = 1$<br>Expands brackets correctly<br>or<br>divides each term by $x$ |
| | $5x - 1 = 2w + 3xw$<br>or<br>$5 - \frac{1}{x} = \frac{2w}{x} + 3w$ | M1dep | oe e.g. $-3xw - 2w = 1 - 5x$<br>Collects terms in $w$ (must have ≥ 2 terms containing $w$)<br>Allow one sign error only<br>dep on first M1 only |
| | $\frac{5x-1}{2+3x} = w$ | A1 | oe e.g. $w = \frac{1-5x}{-3x-2}$<br>Must have $= w$ or $w =$ |

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| **15(a)** | 29 and 23 identified | B2 | B1 $(n + 9)(n + 3)$ or 667 or 29 or 23 |

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| **15(b)** | **Alternative method 1** | | |
| | $(n-3)^2$ | M1 | Allow $(n-3)(n-3)$ for $(n-3)^2$ |
| | $(n-3)^2 - 9 + 14$<br>or<br>$(n-3)^2 + 5$ | A1 | Allow $(n-3)(n-3)$ for $(n-3)^2$ |
| | $(n-3)^2 \geq 0$ then adding 5 so always positive<br>or<br>States minimum value is 5<br>or<br>States (3, 5) is minimum point | A1ft | oe Allow $(n-3)(n-3)$ for $(n-3)^2$<br>ft M1 A0<br>Must see M1 and attempt $(n-3)^2 + k$<br>ft $(n-3)^2 + k$ where $k > 0$<br>SC2 States minimum value is 5<br>or<br>States (3, 5) is minimum point |
| | **Alternative method 2** | | |
| | Quadratic curve sketched in first quadrant with minimum point above the $x$-axis | M1 | Labelling on axes not required |
| | (discriminant =) –20 | A1 | |
| | States no (real) roots | A1ft | oe Allow roots → solutions<br>ft M1 A0<br>Must see M1 and attempt a discriminant<br>ft discriminant < 0<br>SC2 States minimum value is 5<br>or<br>States (3, 5) is minimum point |

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| **15(b)** | **Alternative method 3** | | |
| | $2n - 6 = 0$ | M1 | oe equation<br>e.g. $2n = 6$ or $n = 3$ |
| | (second derivative =) 2 | A1 | |
| | States minimum value is 5<br>or<br>States (3, 5) is minimum point | A1ft | oe<br>ft M1 A0<br>Must see M1 and attempt a second derivative<br>ft (second derivative ) > 0<br>SC2 States minimum value is 5<br>or<br>States (3, 5) is minimum point |

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| **16(a)** | $a - 2$ | B1 | |

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| **16(b)** | **Alternative method 1** | | |
| | $C$ (0, 4) or $y = 4$ (for $C$) | B1 | May be on diagram |
| | $\frac{\text{their } 4 - 0}{0 - 2}$ or $-2$<br>or<br>$\frac{(a-2)^2 - 0}{a - 2}$ or $a - 2$ | M1 | gradient $BC$<br>or<br>gradient $AB$ |
| | $\frac{\text{their } 4 - 0}{0 - 2}$ or $-2$<br>and<br>$\frac{(a-2)^2 - 0}{a - 2}$ or $a - 2$ | M1dep | gradient $BC$<br>or<br>gradient $AB$ |
| | their $a - 2 = \frac{-1}{\text{their } -2}$ | M1dep | |
| | $2\frac{1}{2}$ | A1 | oe |
| | **Alternative method 2** | | |
| | $C$ (0, 4) or $y = 4$ (for $C$) | B1 | May be on diagram |
| | $\frac{\text{their } 4 - 0}{0 - 2}$ or $-2$ | M1 | gradient $BC$ |
| | $y = -\frac{1}{\text{their } -2}(x - 2)$<br>or $y = \frac{1}{2}(x - 2)$ or $y = \frac{1}{2}x - 1$ | M1dep | Equation $AB$ |
| | their $\frac{1}{2}(x - 2) = (x - 2)^2$ | M1dep | oe e.g. $\frac{1}{2}x - 1 = x^2 - 4x + 4$ |
| | $2\frac{1}{2}$ | A1 | oe |

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| **16b** | **Alternative method 3** | | |
| | $C$ (0, 4) or $y = 4$ (for $C$) | B1 | May be on diagram |
| | $a^2 + ((a-2)^2 -$ their $4)^2$<br>or<br>$(a-2)^2 + ((a-2)^2)^2$ | M1 | $AC^2$<br>or<br>$AB^2$<br>oe |
| | $a^2 + ((a-2)^2 -$ their $4)^2$ =<br>$(a-2)^2 + ((a-2)^2)^2 + 2^2 +$ their $4^2$ | M1dep | $AC^2 = AB^2 + BC^2$<br>oe e.g. $AC^2 - AB^2 = BC^2$<br>Only ft their 4 |
| | their $8a^2 - 36a + 40$ (= 0) | M1dep | oe<br>their quadratic $pa^2 + qa + r$ (= 0)<br>$p$, $q$ and $r$ all non-zero |
| | $2\frac{1}{2}$ | A1 | oe |
| | **Alternative method 4** | | |
| | $C$ (0, 4) or $y = 4$ (for $C$) | B1 | May be on diagram |
| | tan $OBC$ = $\frac{\text{their 4}}{2}$ | M1 | oe |
| | angle $ABD$ =<br>180 – 90 – their $\tan^{-1} \frac{\text{their 4}}{2}$ | M1dep | oe<br>D on the $x$-axis such that angle $BDA = 90^\circ$ |
| | tan their angle $ABD$ = $\frac{(a-2)^2}{a-2}$ | M1dep | oe |
| | $2\frac{1}{2}$ | A1 | oe |

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| **17(a)** | $3d(4c^2 - 3d)$ | B2 | B1 $d(12c^2 - 9d)$ or $3(4c^2d - 3d^2)$ |

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| **17(b)** | **Alternative method 1** | | |
| | $(w + 4)^2$ as a factor | M1 | Allow $(w + 4)(w + 4)$ |
| | $(w + 4)^2(w + 4 – (w + 1))$<br>or<br>$(w + 4)^2(w + 4 – w + 1)$<br>or<br>$(w + 4)^2(w + 4 – w – 1)$ | M1dep | Allow $(w + 4)(w + 4)$ for $(w + 4)^2$ |
| | $3(w + 4)^2$ | A1 | Allow $3(w + 4)(w + 4)$ |
| | **Alternative method 2** | | |
| | $(w + 4)[(w + 4)^2 – (w + 4)(w + 1)]$ | M1 | |
| | $(w + 4)(aw + b)$ | M1dep | $a$ and $b$ both non-zero |
| | $3(w + 4)^2$ | A1 | Allow $3(w + 4)(w + 4)$ |
| | **Alternative method 3** | | |
| | $w^3 + 12w^2 + 48w + 64$<br>or<br>$w^3 + 9w^2 + 24w + 16$<br>or<br>$–w^3 – 9w^2 – 24w – 16$<br>or<br>$–w^3 + 9w^2 + 24w + 16$<br>or<br>$3w^2 + 24w + 48$<br>or<br>$3(w^2 + 8w + 16)$ | M1 | Must collect terms |
| | $(3w + 12)(w + 4)$ | M1dep | Correctly factorises their three term quadratic |
| | $3(w + 4)^2$ | A1 | Accept $3(w + 4)(w + 4)$ |

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| **18** | **Alternative method 1** | | |
| | $\sqrt{14^2+8^2}$ or $\sqrt{260}$<br>or $2\sqrt{65}$ or [16.1, 16.125] | M1 | $AC$ |
| | tan $(x)$ = $\frac{7}{\text{their } AC}$ | M1dep | oe |
| | [23.4667, 23.5] | A1 | |
| | **Alternative method 2** | | |
| | $\sqrt{14^2+8^2+7^2}$ or $\sqrt{309}$<br>or [17.578, 17.6] | M1 | $EC$<br><br>May be seen in stages<br>e.g. Work out $AC$ with correct method then work out their $AC^2$ + $7^2$ then square roots<br><br>Condone use of $2\sqrt{65}\ ^2$ for $AC^2$ |
| | sin $(x)$ = $\frac{7}{\text{their } EC}$ (× sin 90)<br><br>or<br><br>cos $(x)$ = $\frac{\sqrt{8^2+14^2}}{\text{their } EC}$ | M1dep | cos $(x)$ = $\frac{8^2+14^2+\text{their } EC^2 - 7^2}{2\times\text{their }\sqrt{8^2+14^2}\times\text{their } EC}$<br><br>Condone use of $2\sqrt{65}\ ^2$ for $AC^2$ |
| | [23.4667, 23.5] | A1 | |

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| **19(a)** | $2\pi r(r + 5)$ seen | M1 | oe e.g. $2 \times \pi \times r(r + 5)$ |
| | $\frac{9\pi r^2}{2}$ | M1 | oe e.g. $\pi \times r \times \frac{9r}{2}$ |
| | $\pi r^2 + 2\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or<br>$\frac{2\pi r^2 + 4\pi r^2 + 20\pi r + 9\pi r^2}{2}$ or<br>$3\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or<br>$\frac{6\pi r^2 + 20\pi r + 9\pi r^2}{2}$ | A1 | Correct unsimplified expression with brackets $2\pi r(r + 5)$ expanded<br>May still contain multiplication signs |
| | $\frac{15\pi r^2}{2} + 10\pi r = \frac{5\pi r}{2}(3r + 4)$<br>or<br>$\frac{15\pi r^2 + 20\pi r}{2} = \frac{5\pi r}{2}(3r + 4)$ | A1 | Must see M2 A1 |

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| **19(b)** | $\frac{5\pi r}{2}(3r + 4) = 1200\pi$ | M1 | oe<br>Allow $1200\pi \to 1200$ |
| | Correct equation or 3 term expression with no unexpanded brackets<br>e.g.1 $3r^2 + 4r - 480$ (= 0)<br>e.g.2 $15r^2 + 20r = 2400$<br>e.g.3 $\frac{15\pi}{2}r^2 + 10\pi r = 1200\pi$ | A1 | oe |
| | Attempt to factorise their 3 term quadratic<br>e.g. for $3r^2 + 4r - 480$<br>$(3r + a)(r + b)$<br>where $ab = \pm480$ or $3b + a = \pm4$<br>or<br>Attempt to substitute in the formula for their 3 term quadratic (allow one sign error)<br>e.g. for $3r^2 + 4r - 480$<br>$\frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$ or<br>$\frac{4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$ (1 sign error) | M1dep | oe<br>Attempt to complete the square for their 3 term quadratic<br>e.g. for $3r^2 + 4r - 480$<br>$(3)\ [(r + \frac{2}{3})^2 \ldots\ldots.]$ |
| | Correctly factorises their 3 term quadratic<br>e.g. for $3r^2 + 4r - 480$ (= 0)<br>$(3r + 40)(r - 12)$ (= 0)<br>or<br>Correct substitution in formula for their 3 term quadratic<br>e.g. for $3r^2 + 4r - 480$ (= 0)<br>$\frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$ | A1ft | ft M1 A0 M1dep<br>oe<br>Correct completion of square for their 3 term quadratic<br>e.g. for $3r^2 + 4r - 480$<br>$(3)\ [(r + \frac{2}{3})^2 - (\frac{2}{3})^2 - 160\ ]$ oe |
| | 12 | A1 | Do not award if negative solution also included |

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| **20** | $(8x - y)^2 = (6x)^2 + (x + y)^2$ | M1 | oe<br>Allow $(8x - y)\ (8x - y)$ and $(x + y)\ (x + y)$<br>Condone $6x^2$ |
| | Expands $(8x - y)^2$ to 4 terms with 3 correct from<br>$64x^2 - 8xy - 8xy + y^2$ | M1 | oe<br>If going straight to 3 terms must be<br>$64x^2 - 16xy + ky^2 \quad (k \neq 0)$ or<br>$ax^2 - 16xy + y^2 \quad (a \neq 0)$ |
| | Expands $(x + y)^2$ to 4 terms with 3 correct from<br>$x^2 + xy + xy + y^2$ | M1 | oe<br>If going straight to 3 terms must be<br>$x^2 + 2xy + ay^2 \quad (a \neq 0)$ or<br>$bx^2 + 2xy + y^2 \quad (b \neq 0)$ |
| | $27x^2 - 18xy\ (= 0)$ or $27x^2 = 18xy$<br>or better<br>e.g.1 $9x^2 - 6xy\ (= 0)$<br>e.g.2 $3x^2 = 2xy$ | A1 | $64x - 16y = 36x + x + 2y$<br>or equivalent linear equation<br>e.g.1 $64x - 16y - 36x = x + 2y$<br>e.g.2 $64x - 16y - x - 2y = 36x$ |
| | Any correct factorisation of their $px^2 + qxy$ or correct division of their $px^2 = qxy$ by a multiple of $x$<br>($p$ and q non zero)<br>e.g.1 $9x\ (3x - 2y)\ (= 0)$<br>e.g.2 $3x\ (9x - 6y)\ (= 0)$<br>e.g.3 $27x = 18y$<br>e.g.4 $9x = 6y$ | M1 | Correct collection and correct simplification of terms for their linear equation in $x$ and $y$<br>e.g. $27x = 18y$<br>To gain this mark there must have been some expansion of brackets seen |
| | $3x = 2y$ or $\frac{x}{y} = \frac{2}{3}$ or $\frac{y}{x} = \frac{3}{2}$<br>or $x = \frac{2}{3}y$ or $y = \frac{3}{2}x$ or<br>$\frac{x}{2} = \frac{y}{3}$ or $\frac{2}{x} = \frac{3}{y}$ | A1 | Must see M1 M1 M1 A1<br>Do not allow if a contradictory statement is also seen |

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| **21** | **Alternative method 1** | | |
| | $\sin(x) = \sqrt{\frac{1}{16}}$ or $\sin(x) = \frac{1}{4}$ | M1 | |
| | [14.4775, 14.5] | A1 | Do not award if another solution in range $0 \leq x < 90$ is given |
| | $\sin x = -\sqrt{\frac{1}{16}}$ or $\sin x = -\frac{1}{4}$<br>or –[14.4775, 14.5]<br>or 180 + their [14.4775, 14.5] | M1 | their [14.4775, 14.5] must be a positive acute angle |
| | [194.4775, 194.5] | A1 | Do not award if another solution in range $180 \leq x \leq 270$ is given |
| | [165.5, 165.5225] | B1ft | ft 180 – their [14.4775, 14.5]<br>their [14.4775, 14.5] must be a positive acute angle<br>Do not award if another solution in range $90 \leq x < 180$ is given |

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| **21** | **Alternative method 2** | | |
| | $\cos(x) = \sqrt{1-\frac{1}{16}}$ or $\cos(x) = \sqrt{\frac{15}{16}}$<br>or $\cos(x) = \frac{\sqrt{15}}{4}$<br>or $\cos(x) = [0.968, 0.97]$ | M1 | |
| | [14.4775, 14.5] | A1 | Do not award if another solution in range $0 \le x < 90$ is given |
| | $\cos(x) = -\sqrt{1-\frac{1}{16}}$ or<br>$\cos(x) = -\sqrt{\frac{15}{16}}$ or $\cos(x) = -\frac{\sqrt{15}}{4}$<br>or $\cos(x) = -[0.968, 0.97]$<br>or 180 + their [14.4775, 14.5] | M1 | their [14.4775, 14.5] must be a positive acute angle |
| | [194.4775, 194.5] | A1 | Do not award if another solution in range $180 \le x \le 270$ is given |
| | [165.5, 165.5225] | B1ft | ft 180 – their [14.4775, 14.5]<br>their [14.4775, 14.5] must be a positive acute angle<br>Do not award if another solution in range $90 \le x < 180$ is given |

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| **22** | **Alternative method 1** | | |
| | Substitutes a value $0 < x < 3$ and obtains a correct expression in $k$<br>e.g. $x = 2 \rightarrow 2k(2-3)^3$ or $2k(-1)^3$<br>and<br>substitutes a value $x > 3$ and obtains a correct expression in $k$<br>e.g. $x = 4 \rightarrow 4k(4-3)^3$ or $4k(1)^3$ | M1 | oe |
| | Obtains correct expressions for both and correctly indicates whether they are positive or negative<br>e.g. $-2k$ positive and $4k$ negative | M1dep | |
| | Max(imum point) | A1 | Must see the working for M1 M1 |
| | **Alternative method 2** | | |
| | Correct second derivative with $x = 3$ substituted in leading to 0<br>i.e. $4kx^3 - 27kx^2 + 54kx - 27k$<br>and $x = 3 \rightarrow 0$ | M1 | oe<br>e.g. $3kx(x-3)^2 + k(x-3)^3$<br>and $x = 3 \rightarrow 0$ |
| | Correct third derivative with $x = 3$ substituted in leading to 0<br>and<br>correct fourth derivative with $x = 3$ substituted in leading to $< 0$<br>i.e. $12kx^2 - 54kx + 54k$<br>and $x = 3 \rightarrow 0$<br>and<br>$24kx - 54k$<br>and $x = 3 \rightarrow 18k$ negative | M1dep | |
| | Max(imum point) | A1 | Must see the working for M1 M1 |