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Level 2 Certificate in Further Mathematics June 2012

Paper 1 8360/1



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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

- M Method marks are awarded for a correct method which could lead to a correct answer.
- A Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
- B Marks awarded independent of method.
- **M Dep** A method mark dependent on a previous method mark being awarded.
- **B Dep** A mark that can only be awarded if a previous independent mark has been awarded.
- ft Follow through marks. Marks awarded following a mistake in an earlier step.
- **SC** Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
- oe Or equivalent. Accept answers that are equivalent.

eg, accept 0.5 as well as $\frac{1}{2}$

Paper 1 - Non-Calculator

Q	Answer	Mark	Comments
			Γ
1(a)	9	B1	
1(b)	$f(x) \ge 7$	B1	Allow $y \ge 7$
L		1	1
2	(10)	B2	B1 For each component
	(17)		$\begin{pmatrix} 10+0\\5+12 \end{pmatrix} \text{ scores B1}$
3	3x < -9 or $x < -3$	M1	oe
	-4	A1	SC1 For $x \le -4$
	· · · · · · · · · · · · · · · · · · ·		
4(a)	$2(x^2 - x - 20)$	M1	Common factor might be removed later
	(2x + a)(x + b) or	M1	$ab = \pm 40$ or $2b + a = \pm 2$
	2(x+c)(x+d)		$cd = \pm 20$ or $c + d = \pm 1$
	2(x + 4)(x - 5)	A1	(2x+8)(x-5) and $(x+4)(2x-10)$ and $(x+4)(x-5)$ all score SC2
			(x - 4)(x + 5) scores SC1
4(b)	(x + y)[(x + y) + (2x + 5y)]	M1	
	(x+y)(3x+6y)	A1	
	3(x+y)(x+2y)	A1	(x + y)(x + 2y) scores SC2
Alt 4(b)	$x^{2} + xy + xy + y^{2} + 2x^{2} + 2xy + 5xy + 5y^{2}$	M1	Condone two errors
	or		
	$3x^2 + 9xy + 6y^2$		
	(x + y)(3x + 6y) or	A1	
	(3x+3y)(x+2y) or		
	$3(x^2+3xy+2y^2)$		
	3(x+y)(x+2y)	A1	(x+y)(x+2y) scores SC2
			· · · · · · · · · · · · · · · · · · ·
5	$8c^{3}d^{12}$	B2	B1 For two out of three components correct

Q	Answer	Mark	Comments
		I	
6	2y - 3x = 4 $3y + 2x = -7$	M1	oe Rearrange into suitable form for elimination Allow one error
	6y - 9x = 12 $4y - 6x = 8$ andor $6y + 4x = -14$ $9y + 6x = -21$	M1	oe Attempting to equate <i>x</i> or <i>y</i> coefficients Allow one error
	13x = -26 or $13y = -13$	M1	oe Correct elimination from their equations only award if ≤ 1 error on first two M marks
	x = -2 and $y = -1$	A1	
Alt 1 6	2y = 3x + 4 $3x = 2y - 4$ and or and $3y = -2x - 7$ $2x = -3y - 7$	M1	oe Rearrange into suitable form for elimination Allow one error
	6y = 9x + 12 $6x = 4y - 8$ andor $6y = -4x - 14$ $6x = -9y - 21$	M1	oe Attempting to equate <i>x</i> or <i>y</i> coefficients Allow one error
	0 = 13x + 26 or $0 = 13y + 13$	M1	oe Correct elimination from their equations Only award if ≤1 error on first two M marks
	x = -2 and $y = -1$	A1	
Alt 2 6	x = -1.5y - 3.5	M1	oe Rearrange into suitable form for substitution Allow one error
	2y = 3(-1.5y - 3.5) + 4	M1	oe Substitution Allow one error
	6.5 <i>y</i> = -6.5	M1	oe Correct simplification from their equation Only award if ≤1 error on first two M marks
	y = -1 and $x = -2$	A1	

Q	Answer	Mark	Comments
1		1	
Alt 3 6	y = 1.5x + 2	M1	oe Rearrange into suitable form for substitution Allow one error
	2x = -3(1.5x + 2) - 7	M1	oe Substitution Allow one error
	6.5x = -13	M1	oe Correct simplification from their equation Only award if ≤1 error on first two M marks
	x = -2 and $y = -1$	A1	

7	Angle $CAD = 46$ or Angle $ACD = 37$ or Angle $CDE = 83$ or $(37 + 46)$ or Angle $ADC = 97$ or $180 - (37 + 46)$	M1	Any of these angles correctly marked or named could be on diagram
	Angle $DCE = 46$ or Angle $ACE = 83$ or $(37 + 46)$	M1	
	51	A1	

8(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 10x$	M1	Allow one error
	3 – 10 (= –7)	A1	$3 \times 1 + 10 \times -1$ is sufficient
8(b)	$(y =) (-1)^3 + 5 (-1)^2 + 1$	M1	
	(y =) 5	A1	
	Use of ' m ' = -7 seen or implied	M1	Must be used in an equation
	y - their 5 = -7 ($x + 1$)	A1 ft	oe eg. $y = -7x - 2$

91:2:5B3B2 For any ratio that is one step away the answer eg $\sqrt{12}: 2\sqrt{12}: 5\sqrt{12}$	Q	Answer	Mark	Comments
91:2:5B3B2 For any ratio that is one step away to the answer eg $\sqrt{12:2}\sqrt{12:5}\sqrt{12}$				
$\sqrt{1} : \sqrt{4} : \sqrt{25}$ $2 : 4 : 10$ B1 For at least two of the three terms in their simplest form ie two of $2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3}$ B1 For any correct equivalent ratio eg $\sqrt{2} : \sqrt{8} : \sqrt{50}$	9	1:2:5	B3	B2 For any ratio that is one step away from the answer eg $\sqrt{12}: 2\sqrt{12}: 5\sqrt{12}$ $\sqrt{1}: \sqrt{4}: \sqrt{25}$ 2: 4: 10 B1 For at least two of the three terms in their simplest form ie two of $2\sqrt{3}: 4\sqrt{3}: 10\sqrt{3}$ B1 For any correct equivalent ratio eg $\sqrt{2}: \sqrt{8}: \sqrt{50}$

Q	Answer	Mark	Comments
10	$(5n-3)^2 + 1$	M1	
	$25n^2 - 15n - 15n + 9 + 1$	M1	Allow 1 error
			Must have an n^2 term
	$25n^2 - 30n + 10$	A1	
	$5(5n^2-6n+2)$	B1 ft	ое
			eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5
Alt 1 10	Use of $an^2 + bn + c$ for terms of quadratic sequence	M1	
	ie, any one of		
	a+b+c=5 $4a+2b+c=50$		
	9a + 3b + c = 145		
	3a + b = 45	M1	For eliminating c
	5a + b = 95		
	$25n^2 - 30n + 10$	A1	
	$5(5n^2-6n+2)$	B1 ft	ое
			eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5
Alt 2	5 50 145 290	M1	25 <i>n</i> ²
10	45 95 145		
	2nd difference of $50 \div 2 (= 25)$		
	Subtracts their $25n^2$ from terms of sequence	M1	-30 <i>n</i>
	-20 -50 -80		
	$25n^2 - 30n + 10$	A1	
	$5(5n^2-6n+2)$	B1ft	oe eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5

Q	Answer	Mark	Comments
11(a)	Gradient $AC = \frac{4-0}{0-12}$ or $-\frac{1}{3}$	M1	ое
	$y = -\frac{1}{3}x + 4$	A1	oe eg $x + 3y = 4$ Must be an equation
11(b)	Gradient <i>OB</i> = 3	B1 ft	ft Their gradient in (a) using $m_1 \times m_2 = -1$
	Equation of <i>OB</i> : $y = 3x$	M1	ft Their gradient OB
	$3x = -\frac{1}{3}x + 4$	M1	ft Their equations
	$x = \frac{6}{5}$ or 1.2	A1 ft	oe (<i>x</i> coordinate of midpoint of <i>OB</i>) ft From their linear equations
	$y = \frac{18}{5}$ or 3.6	A1	oe (y coordinate of midpoint of OB)
	$\left(\frac{12}{5},\frac{36}{5}\right)$ or (2.4, 7.2)	B1 ft	oe ft Their <i>x</i> and <i>y</i> values for the midpoint
		1	
12(a)	Line $y = \frac{1}{2}x$ drawn	B1	Between $x = 0$ and $x = 4$
12(b)	Line $y = 2$ drawn	B1	Between $x = 0$ and $x = 4$
12(c)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 6x^2 + a$	M1	Allow one error
	x = -1 6 + a	A1	
	x = 2 24 + a	A1	

M1

A1

equation in a

Must follow from their $\frac{dy}{dx}$ and must be an

a = -3 from $\frac{dy}{dx} = 6x^2 + ax$ scores SC3

Their $(24 + a) = 2 \times \text{their } (6 + a)$

a = 12

Q	Answer	Mark	Comments
13	(x + 6)(x - 2)	B1	
	(x+5)(x-5)	B1	
	x(x-5)	B1	
	$\frac{\text{their } (x+6)(x-2)}{\text{their } (x+5)(x-5)} \times \frac{\text{their } x(x-5)}{x+6}$	M1	Must have attempted to factorise at least two of the above
	$\frac{x(x-2)}{x+5}$ or $\frac{x^2-2x}{x+5}$	A1	A0 if incorrect further work seen

14	$x = 8^{\frac{2}{3}}$ or $x = \sqrt[3]{64}$ or $x^3 = 64$ or	M1	
	$\sqrt{x} = 2$ or $x = 2^2$		
	<i>x</i> = 4	A1	
	$y^2 = \frac{4}{25}$ or $\frac{1}{y^2} = \frac{25}{4}$ or	M1	
	$y^{-1} = \sqrt{\frac{25}{4}}$ or $\frac{1}{y} = \sqrt{\frac{25}{4}}$		
	$y = \frac{2}{5}$ or $y^{-1} = \frac{5}{2}$ or $\frac{1}{y} = \frac{5}{2}$	A1	Accept $y = \pm \frac{2}{5}$ or $y^{-1} = \pm \frac{5}{2}$ or $\frac{1}{y} = \pm \frac{5}{2}$
	10	A1	

Q	Answer	Mark	Comments
15(a)	Correct use of Pythagoras' Theorem eg YZ = $\sqrt{2^2 - 1^2}$	M1	ое
	$X = 60^\circ$ and sin $X = \frac{\sqrt{3}}{2}$	A1	$X = 60^{\circ}$ stated or 60° marked on diagram
15(b)	Correct use of Sine Rule $\frac{2 - \sqrt{3}}{\sin A} = \frac{(4\sqrt{3} - 6)}{\sin B}$	M1	oe
	sin $B = \frac{(4\sqrt{3} - 6)}{(2 - \sqrt{3})} \times \frac{1}{4}$	M1	oe eg $\frac{(4\sqrt{3}-6)}{8-4\sqrt{3}}$ or $\frac{\sqrt{3}-1.5}{2-\sqrt{3}}$
	$=\frac{(4\sqrt{3}-6)(2+\sqrt{3})}{4(2-\sqrt{3})(2+\sqrt{3})}$	M1	For multiplying both numerator and denominator by conjugate of the form $a + \sqrt{b}$
			ft their expression for sin B eg $\frac{(4\sqrt{3}-6)(8+4\sqrt{3})}{(8-4\sqrt{3})(8+4\sqrt{3})}$ or $\frac{(\sqrt{3}-1.5)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$
	Numerator = $8\sqrt{3} - 12 + 12 - 6\sqrt{3}$	A1 ft	eg $32\sqrt{3} - 48 + 48 - 24\sqrt{3}$ or $2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$
	Denominator = 4	A1 ft	eg 16 or 1
	sin $B = \frac{\sqrt{3}}{2}$ and $B = 60^{\circ}$	A1	Clearly shown
Alt 1 15 (b)	$\frac{CD}{4\sqrt{3}-6} = \frac{1}{4}$ or $CD = \frac{1}{4}(4\sqrt{3}-6)$	M1	oe where <i>D</i> is the foot of the perpendicular from <i>C</i> to <i>AB</i>
	$\sin B = \frac{\frac{1}{4}(4\sqrt{3} - 6)}{2 - \sqrt{3}}$	M1	
	$\frac{\frac{1}{4} (4\sqrt{3} - 6)(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$	M1	For multiplying both numerator and denominator by conjugate of the form $a + \sqrt{b}$ ft their expression for sin <i>B</i>
		A1 ft	
	$\frac{1}{2\sqrt{3}} = 2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$ Denominator = 1	A1 ft	
	sin $B = \frac{\sqrt{3}}{2}$ and $B = 60^{\circ}$	A1	Clearly shown

Q	Answer	Mark	Comments
			· ·
Alt 2	Correct use of Sine Rule	M1	oe
15(b)	$\frac{\sin A}{2-\sqrt{3}} = \frac{\sin B}{4\sqrt{3}-6}$		
	$\frac{\sin A (2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{\sin B}{4\sqrt{3} - 6}$	M1	
	$\frac{\sin A (2 + \sqrt{3})}{1} = \frac{\sin B}{4\sqrt{3} - 6}$	A1	
	sin $B = \frac{1}{4}(2 + \sqrt{3})(4\sqrt{3} - 6)$	M1	
	Numerator = $2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$	A1	
	sin $B = \frac{\sqrt{3}}{2}$ and $B = 60^{\circ}$	A1	Clearly shown
		1	1
16	$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\frac{1}{\tan \theta} \equiv \frac{\cos \theta}{\sin \theta}$	M1	oe

16	$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\frac{1}{\tan \theta} \equiv \frac{\cos \theta}{\sin \theta}$	M1	oe
	Denominator = sin $\theta \cos \theta$	M1 Dep	oe
	$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$	A1	All steps clearly shown
	$(\sin^2 \theta + \cos^2 \theta \equiv 1)$ and $\frac{1}{\sin \theta \cos \theta}$		