# JUNIOR LYCEUMS FINAL EXAMINATIONS - 

Name $\qquad$ Mark

Class $\qquad$

- ANSWER ALL QUESTIONS.
- EACH QUESTION CARRIES 1 MARK.
- CALCULATORS, RULERS, PROTRACTORS AND OTHER MATHEMATICAL INSTRUMENTS ARE NOT ALLOWED
- WRITE DOWN YOUR ANSWER ONLY IN THE SPACE PROVIDED.


## DO NOT

 WRITE IN THIS SPACE|  | QUESTION | ANSWER |
| :---: | :---: | :---: |
| 1. | Write down the size of angle $x$. |  |
| 2. | Write down the value of $2^{6}-2^{5}$. |  |
| 3. | The value of $\cos 45^{\circ}$ is $\frac{1}{\sqrt{\mathrm{p}}}$. Write down the value of p . |  |
| 4. | Write down an expression for the length of the rectangle in terms of $x$. |  |
| 5. | Write down the value of $\left(5 \times 10^{3}\right)^{2}$ in standard form. |  |
| 6. | A school has 400 pupils of whom 250 are boys. Write down in its simplest form the ratio of boys to girls. |  |
| 7. | The volume, in $\mathbf{c m}^{\mathbf{3}}$, of the cylinder is: <br> A) $9 \pi$ <br> B) $12 \pi$ <br> C) $600 \pi$ <br> D) $900 \pi$ |  |
| 8. | Write down the bearing of P from B . |  |
| 9. | Write down the positive value of $x$ which satisfies the equation $(x+2)^{2}=25$. |  |
| 10. | Michael's gross annual salary is Lm6000. He pays Lm600 as national insurance contributions and another Lm600 as income tax. What percentage of his gross salary does he pay in all? |  |


| FORM 5 MATHEMATICS (Main Paper) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | TIME: 1 h 45 min |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | Total <br> Main | Mental | Global <br> Mark |
| Mark |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

DO NOT WRITE ABOVE THIS LINE
Name $\qquad$
Class $\qquad$

## CALCULATORS ARE ALLOWED BUT ALL NECESSARY WORKING MUST BE SHOWN

## ANSWER ALL QUESTIONS.

1. 

$$
\mathrm{P}=\frac{(2.721+2.342)^{3} \times 102.4}{47.324}
$$

To work out an estimate for P , Alexia first rounds every number to one significant figure. She then works out the calculation mentally.
a) What should Alexia's estimate for the answer be?
b) In this part of the question give both answers correct to five decimal places.
i) Use your calculator to work out the value of P .
ii) Hence work out the difference between the estimate in a) and the answer given by the calculator.
c) Express, to one decimal place, this difference as a percentage of the answer given by the calculator.
2. Solve the equations

$$
\begin{aligned}
& 3 x+4 y=2 \\
& 4 x-3 y=11
\end{aligned}
$$

3. 



In the diagram name a triangle which is congruent to triangle ABG , giving reasons for your answer.
4. On the diagram:
a) Draw the image of P when it is reflected in the $x$-axis and label it Q .
b) Draw the image of Q when it is reflected in the $y$-axis and label it R .
c) Describe the single transformation which maps $P$ to $R$.

5. The picture on the left shows a crane with a load on it. Use the diagram on the right to work out the height of the vertex, V , of the crane above the ground. Give your answer correct to three significant figures.

6. A hang-glider pilot, at a height of $h$ metres above the sea, can see up to a distance $s$ kilometres. It is known that $h$ is directly proportional to the square of $s$.
a) Write down a formula connecting $h$ and $s$.
 (Use $k$ for the constant of proportion).
b) Given that $h=140$ when $s=16$, find the value of $k$ as a fraction in its lowest terms.
c) Work out the height of the hang-glider when the pilot can just see a lighthouse which is 24 kilometres away.
d) What is the greatest distance the pilot can see when the hang-glider is 100 metres above the sea? Give your answer correct to three significant figures.
7. The diagram shows two sectors AOB and COD with the same centre $O$. The area of sector AOB is three times the area of sector COD. Work out, giving your answers correct to one decimal place:
a) the area of the sector AOB ,

b) the area of the sector COD,
c) the radius, $r \mathrm{~cm}$ of the sector COD.

$$
\text { (Area of circle }=\pi \mathrm{r}^{2} \text { ) }
$$

8. a) Some pupils were making number puzzles. Simon's number puzzle was written like this:

$$
2(3 x-1)<18
$$

What is the largest integer Simon could have thought of?


The shaded region A is formed by the lines $y=2, y=3 x$ and $x+y=6$. Write down the three inequalities which define the shaded region A .
9. (To answer this question no knowledge of spreadsheets is required).

|  |  | COLUMN |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ |  |
| ROW | 1 | 1 | 1 | 1 |  |
|  | 2 | 2 | 3 | 4 |  |
|  | 3 | 3 | 6 | 9 |  |
|  | 4 | 4 | 10 | 16 |  |
|  | 5 | 5 | 15 |  |  |
| 6 | 6 |  |  |  |  |
| 7 | 7 |  |  |  |  |

This table shows part of a computer spreadsheet. The number 15 in column $\mathbf{B}$ is labelled $\mathbf{B} 5$. The number 9 in column $\mathbf{C}$ is labelled $\mathbf{C} 3$.
The other numbers in the table are labelled in a similar manner.
The rule for finding the numbers in column $\mathbf{B}$ is:
B1 $=1$
$\mathbf{B} 2=\mathbf{A} 2+\mathbf{B} 1=2+1=3$
$\mathbf{B} 3=\mathbf{A} 3+\mathbf{B} 2=3+3=6$
$\mathbf{B} 4=\mathbf{A} 4+\mathbf{B} 3=4+6=10$
and so on ...
a) Complete the rule for $\mathbf{B} 6$ and $\mathbf{B} 7$.

b) The numbers $\mathbf{C} 2, \mathbf{C} 3, \mathbf{C} 4, \ldots, \mathbf{C} n, \ldots$ can be found from the numbers in column $\mathbf{B}$. Use column $\mathbf{B}$ to complete the rule for $\mathbf{C} 3$ and $\mathbf{C} 4$. ( $\mathbf{C} 2$ is done for you).
$\mathbf{C} 1=1$

$\mathbf{C} 3=\square+\square=\square=$

c) The pattern in column $\mathbf{C}$ is continued.
i) What number will there be in C20?
ii) Write down a formula for $\mathbf{C} n$, in terms of $n$, where $n$ stands for any positive integer.
10. A circle of radius $r$ fits exactly inside the square ABCD of side 2 metres. Work out each of the following and state whether each is rational or irrational.

a) the diameter of the circle,
b) the shaded area of the diagram,
c) the diagonal of the square.
11. Use ruler and compasses only. All construction lines and arcs must be clearly shown.
a) Construct the triangle LMN in which $\mathrm{MN}=7.2 \mathrm{~cm}, \mathrm{LM}=6 \mathrm{~cm}$ and $\mathrm{LN}=6.5 \mathrm{~cm}$.
b) Construct the locus of points:
i) equidistant from M and $\mathrm{N} \quad$ ii) equidistant from $L$ and $M$.

Let these two loci intersect at P. Measure and write down the length of LP.
c) i) What can you say about the lengths of LP, MP and NP?
ii) Hence construct the circle to pass through L, M, and N.

12. a) The wheel shown in the diagram has twelve sections of equal size labelled with letters as shown. In a game a player spins the arrow which is equally likely to stop in any of the sections.
The probability that the arrow stops on $B$ is shown on the probability scale and labelled as $\mathrm{P}(\mathrm{B})$. Show, in a similar way, on this probability scale, the probability that the arrow will stop on:
i) D , ii) a letter of the alphabet.

b) The diagram shows a series of road junctions. At junction A the probability that a car turns left is 0.4 . At junction $B$ the probability that a car turns left is 0.7 . At junction $C$ the probability that a car turns left is 0.2 . Work out the probability that a car passing A will arrive at:
i) point X by the shortest route,
ii) point $Y$ by the shortest route,
iii) junction $D$.

13. In the diagram the circle has centre $\mathrm{O} . \mathrm{QT}$ is a chord, PQR is a tangent and $\angle \mathrm{QST}=x^{\circ}$.
Give reasons for your answers.
a) i) The size of $\angle \mathrm{TOQ}$ in terms of $x$ is $\qquad$
ii) The size of $\angle \mathrm{OQP}$ is $\qquad$

b) i) Using triangle TOQ work out, in terms of $x$, the size of $\angle \mathrm{OQT}$.
ii) Hence work out the size of $\angle \mathrm{TQP}$ in terms of $x$.
c) In this question you have proved a theorem. Fill in the blanks in the general statement of the theorem.
The angle between a chord and a $\qquad$ at the point of contact, is equal to the angle subtended at the circumference in the $\qquad$ .
14. In triangle OAB , the mid-point of $A B$ is P .
a) Given that $\overrightarrow{\mathrm{OA}}=\mathbf{a}$ and $\overrightarrow{\mathrm{OB}}=\mathbf{b}$, express in terms of $\mathbf{a}$ and b the vectors:
i) $\overrightarrow{\mathrm{AB}}$
ii) $\overrightarrow{A P}$
iii) $\overrightarrow{\mathrm{OP}}$

b) The point Q is taken on OP such that $\mathrm{OQ}=\frac{2}{3} \mathrm{OP}$. Express in terms of $\mathbf{a}$ and $\mathbf{b}$ the vectors:
i) $\overrightarrow{O Q}$
ii) $\overrightarrow{A Q}$
c) Given that $X$ is the mid-point of $O B$, express $\overrightarrow{\mathrm{AX}}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Hence show that $\frac{\overrightarrow{\mathrm{AX}}}{\overrightarrow{\mathrm{AQ}}}=k$, where $k$ is a scalar, and find the value of $k$.
15. The diagram shows the graph of $y=x^{2}-3 x-3$.
a) Use the graph to give solutions, correct to one decimal place, of $x^{2}-3 x-3=0$.
b) i) Find the equation of the straight line which should be drawn on the same diagram to solve the equation $x^{2}-4 x-4=0$.
ii) Draw the appropriate straight line graph on the diagram given.
iii) Hence use the graphs of the straight line and the curve to solve $x^{2}-4 x-4=0$. Give solutions correct to one decimal place.

