



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

**LEAVING CERTIFICATE 2010**

**MARKING SCHEME**

**MATHEMATICS**

**ORDINARY LEVEL**



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## GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

## APPLYING THE GUIDELINES

Examples (**not exhaustive**) of the different types of error:

### **Blunders** (i.e. mathematical errors) (-3)

- Algebraic errors :  $8x + 9x = 17x^2$  or  $5p \times 4p = 20p$  or  $(-3)^2 = 6$
- Sign error  $-3(-4) = -12$
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect, (e.g. multiplication instead of division)
- Transposition error, e.g.  $-2x - k + 3 \Rightarrow -2x = 3 + k$  or  $-3x = 6 \Rightarrow x = 2$  or
- $4x = 12 \Rightarrow x = 8$ ; each time.
- Distribution error (once per term, unless directed otherwise) e.g.  $3(2x + 4) = 6x + 4$  or  $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly, e.g.  $(2x - 3)(x + 4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once, e.g.  $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formula
- Error(s) in transcribing formulae from tables (assuming it generates mathematical acceptable answer(s)) Serious errors or over simplifications will merit Attempt marks at most ( check relevant section of scheme)
- Central sign error in  $uv$  or  $u/v$  formulae
- Omission of  $\div v^2$  or division not done in  $u/v$  formula (apply once)
- Vice-versa substitution in  $uv$  or  $u/v$  formulae (apply once)
- Quadratic formula (acceptable) and its application apply a maximum of two blunders

### **Slips** (-1)

- Numerical slips:  $4 + 7 = 10$  or  $3 \times 6 = 24$ , but  $5 + 3 = 15$  is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off, is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per part (a), (b) and (c) of each question. Only applies where a candidate would otherwise have achieved full marks

### **Misreadings** (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)  
However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

## QUESTION 1

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>25 (15, 5, 5) marks</b>	<b>Att (5, 2, 2)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **10 marks** **Att 3**

Express 40 metres as a fraction of 1 kilometre. Give your answer in its simplest form.

**(a)** **10 marks** **Att 3**

1 km = 1000 m [3]	or	
$\frac{40}{1000}$ [7] = $\frac{2}{50}$ [9] = $\frac{1}{25}$ [10]		0.04 [7] = $\frac{4}{100}$ [9] = $\frac{1}{25}$ [10]

\* Accept correct answer without work for full marks. Accept 1:25

\* Accept without work 0.04, 4%, 25:1 or  $\frac{25}{1}$  for [7] marks

\* Accept without work  $\frac{40}{100}$ ,  $\frac{20}{50}$ ,  $\frac{4}{10}$ ,  $\frac{2}{5}$  or 0.4 for [4] marks – **these only**

*Blunders (-3)*

B1 Mathematical error e.g. conversion/decimal error

B2 Fraction error

B3 No simplification

*Slips (-1)*

S1 Simplification not completed to simplest form, between  $\frac{40}{1000}$  and  $\frac{1}{25}$

*Attempts (3 marks)*

A1  $\frac{1}{40}$  or  $\frac{40}{1}$

A2 Some effort at conversion

A3 Mentions 25 without supporting work

*Worthless (0)*

W1 Incorrect answer with no work

Part (b)

25 (15, 5, 5) marks

Att (5, 2, 2)

(i) Calculate the value of  
$$\frac{57.6 + 80.44}{1.3 \times 10^4}$$

and write your answer correct to three decimal places.

- (ii) An importer buys an item for £221 sterling when the rate of exchange is €1 = £0.85 sterling.  
He sells it at a profit of 14% of the cost price.  
Calculate, in euro, the price for which he sells the item.

(i)

15 marks

Att 5

$$\frac{57.6 + 80.44}{1.3 \times 10^4} = \frac{138.04}{13\,000} = 0.0106 = 0.011$$

$$57.6 + 80.44 = 138.04 \quad [5]$$

$$\text{or } 1.3 \times 10^4 = 13\,000 \quad [5] \quad \text{Both } [9]$$

$$\frac{138.04}{13\,000} \quad [12] = 0.0106184154 \quad [14]$$

$$= 0.011 \quad [15]$$

\* Accept correct answer without work for 15 marks    0.01.....[ 14 marks] without work

\* Accept without work for 12 marks :

57.606,    94.1756 - 94.176,    618826.8307- 618826.831 ,    **These only**  
[12]            [11]    [12]            [11]            [12]

*Blunders (-3)*

B1 Mathematical error

*Slips (-1)*

S1 Incorrect or no rounding off

S2 Numerical slips which are not mathematical errors

*Misreading (-1)*

M1 Must not make work easier – see guidelines

(ii) Exchange  
Percentage

5 marks  
5 marks

Att 2  
Att 2

$$\frac{\pounds 221 \times 1.14}{0.85} = \frac{251.94}{0.85} = \text{€}296.40$$

[2] + [2]    [5] + [2]

Sterling to Euro exchange

$$\pounds 221 = \text{€} \frac{221}{0.85} = \text{€}260 \quad [5]$$

Percentage

$$14\% \text{ of } \text{€}260 = \text{€}36.40 \quad [4]$$

$$\text{€}260 + \text{€}36.4 = \text{€}296.40 \quad [5]$$

or

$$\text{€}260 \times 1.14 = \text{€}296.40 \quad [5]$$

Percentage

$$\pounds 221 \times 0.14 = \pounds 30.94 \text{ (14\% of } \pounds 221) \quad [4]$$

$$\pounds 221 + \pounds 30.94 = \pounds 251.94 \quad [5]$$

or

$$\pounds 221 \times 1.14 = \pounds 251.94 \quad [5]$$

Sterling to Euro exchange

$$\frac{\pounds 251.94}{0.85} = \text{€}296.40 \quad [5]$$

\* Accept correct answer without work for full marks [5] + [5]

\* No penalty if € not included

*Blunders (-3)*

B1 Error in finding percentage e.g. decimal or inversion

B2 Error in currency conversion e.g. incorrect operation

*Slips (-1)*

S1 Fails to add percentage profit

*Attempts (2 marks)*

A1 Any relevant step, may get both

*Worthless (0)*

W1 Incorrect answer without work



- (i) What sum of money invested at 5% per annum compound interest will amount to €8682 in 3 years?  
Give your answer correct to the nearest euro.
- (ii) A sum of € $P$  was invested at  $r$  % per annum compound interest.  
The interest for the first year was €220  
The interest for the second year was €228.80  
Calculate  $r$  and  $P$ .

(c) (i)

10 marks

Att 3

I

$$F = P(1+i)^t \Rightarrow 8682 = P(1.05)^3 \Rightarrow \frac{8682}{1.157625} = 7499.83 \Rightarrow P = \text{€}7500$$

[4]                      [7]                      [9]                      [10]

II

$$P = \frac{F}{(1+i)^n} = \frac{8682}{(1+0.05)^3} \quad [4] = \frac{8682}{1.157625} \quad [7] = 7499.83 = \text{€}7500$$

III

€8682 at end year 3

$$P \text{ year 3} = \frac{8682}{1.05} = 8268.57$$

$$P \text{ year 2} = \frac{8268.57}{1.05} = 7874.83$$

$$P \text{ year 1} = \frac{7874.83}{1.05} = 7499.83 = \text{€}7500$$

IV

$$P \text{ year 1} = 100\%; \quad P \text{ year 2} = 105\%; \quad P \text{ year 3} = 110.25\%; \quad P \text{ year 4} = 115.7625\%$$

$$115.7625\% = \text{€}8682 \quad [7]$$

$$100\% = \frac{8682}{1.157625} \times 100 = 7499.83 = \text{€}7500$$

\* Candidates may offer other correct versions

\* *Formulae and Tables*, page 30, use  $F$  for  $A$  and  $i$  for  $\frac{r}{100}$ *Blunders (-3)*B1 Mathematical error e.g. percentages or index Note  $8682(1.05)^3 = 10\,050.50 = 10\,051$  [7]

B2 Incorrect number of years

B3 Fails to finish method IV

*Slips (-1)*

S1 Incorrect or no rounding off

*Attempts (3 marks)*

A1 No compounding of interest - offers €8682 – 15% (€7380) Work must be shown

A2 Answer found by trial and error

A3 5% or 15% of 8682 or mentions 1.05 or 1.15

A4 7499.83 or 7500 without work

*Worthless (0)*

W1 Incorrect answer without work

(c) (ii)

5 marks

Att 2

**Finding  $r$**

**I**

$$F = P(1+i)^t \Rightarrow 220(1+i) = 228.80 \Rightarrow (1+i) = 1.04 \Rightarrow r = 4$$

**II**

$$\text{Interest on } \text{€}220 = 228.80 - \text{€}220 = \text{€}8.80$$

$$\frac{8.80}{220} \times 100 = 4$$

**Finding  $P$**

$$P(0.04) = 220 \Rightarrow P = 5500$$

$$4\% = 220$$

$$1\% = 55$$

$$100\% = 5500$$

\* Candidates may offer other correct versions

\* *Formulae and Tables*, page 30, use  $F$  for  $A$  and  $i$  for  $\frac{r}{100}$

*Blunders (-3)*

B1 Mathematical error

B2 Error in finding % from 1.04, method I

*Attempts (2 marks)*

A1 Finds €8.80

A2 Finds by “trial and error” or  $r = 4\%$  verified

A3 Correct answer without work

*Worthless (0)*

W1 Incorrect answer without work

Note

Award **5 marks** for fully correct with work

Award **2 marks** for some relevant work

Otherwise **0 marks**

## QUESTION 2

<b>Part (a)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (b)</b>	<b>25 (10, 5, 5, 5)marks</b>	<b>Att (3, 2, 2, 2)</b>
<b>Part (c)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>

**Part (a)** **15 marks** **Att 5**

Find the values of  $x$  which satisfy  $2(3 + 4x) \leq 22$ , where  $x \in \mathbb{N}$ .

**(a)** **15 marks** **Att 5**

$$\begin{array}{ll}
 2(3 + 4x) \leq 22 \Rightarrow 6 + 8x \leq 22 & [9] \quad \text{or} \quad 2(3 + 4x) \leq 22 \text{ or } 3 + 4x \leq 11 & [9] \\
 \Rightarrow 8x \leq 16 \Rightarrow x \leq 2 & [12] \quad \quad \quad 4x \leq 11 - 3 \Rightarrow x \leq 2 & [12] \\
 & x \in \{1, 2\} &
 \end{array}$$

- \* Correct answer without work, full marks
- \* No penalty for including 0
- \* Accept marked correctly on a number-line

*Blunders (-3)*

- B1 Mathematical error e.g. distribution error, transposing - once if consistent
- B2  $x$  not a natural number, e.g.  $x \leq -1\frac{1}{2}$  gives negative value
- B3 Only identifies one element of the solution set, 1 or 2
- B4 Verifies one correct value in the inequality, 1 or 2
- B5 Stops at  $x \leq 2$ ,  $x = 2$  or  $x < 2$

*Attempts (5 marks)*

- A1 Any correct relevant multiplication or division
- A2 Tests a non solution in the inequality e.g 3
- A3 0 on its own verified or not

**Part (b)**

**25 (10, 5, 5, 5) marks**

**Att (3, 2, 2, 2)**

Solve for  $x$  and  $y$

$$2x - y = 1$$

$$x^2 - xy = -6.$$

**(b)**

**25 (10, 5, 5, 5) marks**

**Att (3, 2, 2, 2)**

$$2x - y = 1 \Rightarrow y = 2x - 1$$

Step 1 Isolates  $x$  or  $y$

[10]

$$x^2 - xy = -6$$

Step 2 Forms quadratic equation

[5]

$$\Rightarrow x^2 - x(2x - 1) = -6$$

(Penalise error in simplification at Step 3)

$$x^2 - 2x^2 + x + 6 = 0$$

Step 3 Roots of quadratic

[5]

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = -2$$

$$y = 5 \quad \text{or} \quad y = -5$$

Step 4 Values of other coordinate

[5]

\* Error(s) in simplification of quadratic equation apply at the Step 3

\* If equation at Step 2 becomes linear award at most Att 2 + Att 2 for Steps 3 and 4

\* Apply similar scheme if candidate isolates  $x$  at Step 1

\* Random value(s) of  $x$  award attempt marks at most (Step 4) if no work of merit in previous steps

*Blunders (-3)*

B1 Mathematical error – apply at relevant step – see note

B2 Incorrect factors – Step 3

B3 Incorrect roots from factor – Step 3

B4 Only finds one value of  $x$  – Step 3 Note B5 will also apply at Step 4

B5 Only finds one value of  $y$

*Attempts (3 or 2 marks)*

A1 Some relevant work

Note: Don't award multiple Attempts to the same piece of work

Part (c)

10 (5, 5) marks

Att (2, 2)

- (i) Show, by division, that  $3x+1$  is a factor of  $3x^3 + 4x^2 - 89x - 30$ .  
(ii) Hence, or otherwise, solve the equation  $3x^3 + 4x^2 - 89x - 30 = 0$ .

(c) (i)

5 marks

Att2

$$\begin{array}{r} x^2 + x - 30 \\ 3x+1 \overline{) 3x^3 + 4x^2 - 89x - 30} \\ \underline{3x^3 + x^2} \phantom{- 30} \\ 3x^2 - 89x \phantom{- 30} \\ \underline{3x^2 + x} \phantom{- 30} \\ -90x - 30 \\ \underline{-90x - 30} \\ 0 \phantom{0} \end{array}$$

*Blunders (-3)*

B1 Each error in division

B2 Shows clearly  $f(-\frac{1}{3}) = 0$

*Attempts (2 marks)*

A1 Some correct division and stops

A2 Substitutes  $-\frac{1}{3}$  into expression or mentions  $f(-\frac{1}{3})$

A3 Sets up division correctly

(c) (ii)

5 marks

Att 2

$$\begin{aligned}3x^3 + 4x^2 - 89x - 30 &= 0 \\ \Rightarrow (3x+1)(x^2 + x - 30) &= 0 \\ \Rightarrow (3x+1)(x-5)(x+6) &= 0 \\ \Rightarrow x = -\frac{1}{3}, x = 5, x = -6\end{aligned}$$

\* Accept candidates answer from part (i) provided it does not over simplify question

\* Accept  $f(5)$  and  $f(-6)$  fully verified for 4 marks

*Blunders (-3)*

B1 Incorrect factors of quadratic

B2 Incorrect or missing roots from factors, but see S1

*Slips (-1)*

S1 Omits  $x = -\frac{1}{3}$  as a root, if left out [4] at most

*Attempts (2 marks)*

A1 States  $x = -\frac{1}{3}$  is a root and stops in part (ii)

A2 Attempt at factorising quadratic from (i)

A3 Some correct use of “ $-b$ ” formula [Note: Stating formula does not merit attempt mark]

A4 Correct answers without relevant work

A5 Sets up using answer from (i)

A6 Finds  $f(k)$ ,  $k \neq 5$ ,  $k \neq -6$

*Worthless (0 marks)*

W1 Attempts at factorising  $3x^3 + 4x^2 - 89x - 30 = 0$  such as  $x^2(3x + 4) = 89x + 30$

W2 Differentiation

### QUESTION 3

<b>Part (a)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **15 (10, 5) marks** **Att (3, 2)**

Given that  $3(b + a) = t(6 - a)$ , calculate the value of  $a$  when  $t = 3$  and  $b = -4$ .

**(a)** **15 (10, 5) marks** **Att 3, 2**

<b>I</b>	Substitution for $t$ and $b$ :	10 marks	
	Evaluation of $a$ :	5 marks	
	$3(b + a) = t(6 - a)$		
	$3(-4 + a) = 3(6 - a) \Rightarrow -12 + 3a = 18 - 3a \Rightarrow 6a = 30 \Rightarrow a = 5$		
<b>II</b>	$3b + 3a = 6t - at$		substitution merits [10]
	$3a + at = 6t - 3b$		
	$a(3 + t) = 6t - 3b$		
	$a = \frac{6t - 3b}{3 + t} = \frac{6 \times 3 - 3 \times -4}{3 + 3} = \frac{18 + 12}{6} = 5$		rest of work [5]

\* Accept correct answer without work.

\* Once a candidate has substituted correctly for  $t$  and  $b$  he/she is entitled to [10] marks

*Blunders (-3)*

- B1 Mathematical error e.g transposition, distribution, from 5 marks
- B2 Substitution error
- B3 Substitutes one value only
- B4 Interchanges  $t$  and  $b$

*Attempts (2 marks)*

- A1 Some correct effort at isolating /evaluating  $a$ , from 5 marks

*Worthless (0)*

- W1 Incorrect answer without work

Solve for  $x$ 

$$5(x + 1)^2 = 2(x + 1) + 5.$$

Give your answer correct to two decimal places.

(b)

20 (10, 10) marks

Att (3, 3)

Step 1, forming quadratic equation: 10 marks

Step 2, solving quadratic equation: 10 marks

**I**

$$5(x + 1)^2 = 2(x + 1) + 5$$

$$\text{Let } y = x + 1$$

$$5y^2 = 2y + 5 \Rightarrow 5y^2 - 2y = 5 \quad [10]$$

$$5y^2 - 2y - 5 = 0$$

$$\Rightarrow y = \frac{2 \pm \sqrt{4 - 4(5)(-5)}}{2(5)} [4] = \frac{2 \pm \sqrt{104}}{10} = \frac{2 \pm 10.198}{10} [7] = \frac{12.198}{10} \text{ or } \frac{-8.198}{10}$$

$$\Rightarrow y = 1.2198 \text{ or } y = -0.8198 \quad [9]$$

$$\Rightarrow x = 0.22 \text{ or } x = -1.82 \quad [10]$$

**II**

$$5(x + 1)^2 = 2(x + 1) + 5$$

$$\Rightarrow 5x^2 + 10x + 5 = 2x + 7$$

$$\Rightarrow 5x^2 + 8x = 2 \quad [10]$$

$$5x^2 + 8x - 2 = 0$$

$$\Rightarrow x = \frac{-8 \pm \sqrt{64 - 4(5)(-2)}}{2(5)} [4] = \frac{-8 \pm \sqrt{104}}{10} = \frac{-8 \pm 10.198}{10} [7] = \frac{2.198}{10} \text{ or } \frac{-18.198}{10}$$

$$\Rightarrow x = 0.22 \text{ or } x = -1.82 \quad [10]$$

\* Accept candidate's quadratic equation for second 10 marks if not factorisable

\* If quadratic equation reduced to a linear attempt marks at most in Step 2

*Blunders (-3)*

B1 Mathematical error each time

B2 Error in use of quadratic formula to a maximum of 2 (Step 2)

*Slips (-1)*

S1 Fails to round off or rounds off incorrectly – once only

S2 Early rounding off that affects answer

S3 Fails to find  $x$  from  $y$  in method **I***Attempts (3 marks)*A1 Some effort at multiplying out equation - Step 1 Method **11**

A2 If equation becomes linear, maximum possible mark from Step 2 is Attempt

A3 Solves a factorisable quadratic equation even if they use formula

A4 Attempts to factorise the quadratic



**Part (c)****15 (10, 5) marks****Att (3, 2)**

- (i)  $2 + \sqrt{3}$  is a root of the equation  $x^2 - 4x + c = 0$ , where  $c$  is a real number.  
Find the value of  $c$  and write down the other root.
- (ii) The equation  $x^2 + 10x + k = 0$  has equal roots.  
Find the value of the real number  $k$  and write down the value of each root.

**(i)****10 marks****Att 3**

$$x^2 - 4x + c = 0$$

$$\Rightarrow (2 + \sqrt{3})^2 - 4(2 + \sqrt{3}) + c = 0 \quad [4]$$

$$\Rightarrow 4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + c = 0$$

$$\Rightarrow c = 1 \quad [7]$$

$$\text{Other root: } 2 - \sqrt{3} \quad [10]$$

\* Accept any valid method

*Blunders (-3)*

- B1 Mathematical error  
B2 Using decimals  $c \neq 1$

*Attempts (3 marks)*

- A1 Some correct substitution  
A2 Some correct substitution into “ $-b$ ” formula  
A3 States 2<sup>nd</sup> root is  $2 - \sqrt{3}$  and stops must be in surd form  
A4  $c = 1$  without work even if second root found

**(ii)****5 marks****Att 2**

**I**  
Let root =  $p$   
 $(x - p)(x - p) = 0 \Rightarrow x^2 - 2px + p^2 \Rightarrow -2p = 10 \Rightarrow p = -5 \Rightarrow k = (-5)^2 = 25 \quad [4]$

**II**  
 $b^2 - 4ac = 0 \Rightarrow 100 - 4(1)(k) = 0 \Rightarrow k = 25 \quad [4]$   
 $x = -5, \quad [5]$

\* Accept any valid method

*Blunders (-3)*

- B1 Mathematical error

Note:

$x^2 + 10x + 25 \quad [Att 2]$

$(x + 5)(x + 5) \quad [Att 2]$

*Slips (-1)*

- S1 Value of root omitted

$k = 25 \quad [4]$

$x = -5 \quad [5]$

*Attempts (3 marks)*

- A1 Correct answer for  $k$  without work  
A2 Roots found without work  
A3 Correct answer without work

## QUESTION 4

<b>Part (a)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>15 (5, 5, 5) marks</b>	<b>Att (2, 2, 2)</b>

**Part (a)** **15 marks** **Att 5**

Given that  $i^2 = -1$ , simplify  $(4 + 2i)(3 - i)$   
and write your answer in the form  $x + yi$ , where  $x, y \in \mathbb{R}$ .

**(a)** **15 marks** **Att 5**

$$(4 + 2i)(3 - i) = 4(3 - i) + 2i(3 - i) = 12 - 4i + 6i - 2i^2 = 12 + 2i + 2 = 14 + 2i$$

[9]                      [12]                      [14]                      [15]

### *Blunders (-3)*

- B1 Mathematical error
- B2 Error in multiplication – maximum of 2 blunders
- B3  $i^2 \neq -1$ , mis-use of  $i^2$  or avoids use of  $i^2$       B1 and B2 can apply
- B4 Mixes up real and imaginary terms

### *Slips (-1)*

- S1 Numerical slips

### *Attempts (5 marks)*

- A1 Any correct relevant multiplication

### *Worthless (0)*

- W1 Incorrect answer without work

**Part (b)**

**20 (10, 10) marks**

**Att (3, 3)**

Let  $u = 4 + 3i$  and  $w = 6 - 8i$

(i) Find the value of the real number  $k$  such that  $|u| = k|w|$ .

(ii) Express  $\frac{w}{u}$  in the form  $x + yi$ .

**(b) (i)**

**10 marks**

**Att 3**

$$|u| = k|w|$$

$$\Rightarrow |4 + 3i| = k|6 - 8i|$$

$$\Rightarrow \sqrt{16 + 9} = k\sqrt{36 + 64}$$

$$\Rightarrow \sqrt{25} = k\sqrt{100}$$

$$\Rightarrow k = \frac{1}{2} \quad \text{accept } k = \frac{5}{10} = \frac{\sqrt{25}}{\sqrt{100}}$$

Note modulus: One correct  $\sqrt{25}$  or  $\sqrt{100}$  [4]

Two correct  $\sqrt{25}$  and  $\sqrt{100}$  [7]

\* No penalty for using 8 for  $-8$  in formula

\* Accept distance from  $(4, 3)$  to  $(0, 0)$  or  $(6, -8)$  to  $(0, 0)$

*Blunders (-3)*

B1 Incorrect formula e.g.  $\sqrt{\quad}$  omitted

B2 Incorrect substitution e.g. has  $(3i)^2$  and /or  $(8i)^2$  in  $\sqrt{a^2 + b^2}$  - once only

B3 Mathematical error

*Attempts (3 marks)*

A1 Incorrect formula with some correct substitution

A2 Plots  $u$  and/or  $w$

A3 Correct answer without work

A4 Correct modulus formula and stops

A5 Correct substitution for  $u$  and/or  $v$

*Worthless (0)*

W1 Incorrect answer without work

(b) (ii)

10 marks

Att 3

$$\frac{w}{u} = \frac{6-8i}{4+3i} = \frac{6-8i}{4+3i} \times \frac{4-3i}{4-3i} \quad [3]$$

$$= \frac{24-18i-32i+24i^2}{16+9} \quad [7]$$

$$= \frac{0-50i}{25} \quad [9]$$

$$= 0-2i \quad \text{or} \quad = 0-\frac{50i}{25} \quad [10]$$

Note: 0 required in answer

\* Can use multiple of conjugate i.e.  $n(4-3i)$ ,  $n$  a real number,  $n \neq 0$

\* Calculates numerator or denominator, merits 4 marks

\* Calculates numerator and denominator, merits 7 marks

*Blunders (-3)*

B1  $i^2 \neq -1$  or misuse of  $i^2$

B2 Mathematical error in multiplying out numerator – maximum 1 blunder

B3 Mathematical error in multiplying out denominator – maximum 1 blunder

B4 Error in formation of  $\frac{w}{u}$  at final stage e.g. may multiply numerator and denominator

*Attempts (3 marks)*

A1 Substitutes for  $u$  and/or  $w$  and stops

A2 Finds conjugate of  $u$  and stops

A3 Any correct relevant multiplication

Part (c)

15 (5, 5, 5) marks

Att (2, 2, 2)

Let  $z = a + bi$ , where  $a, b \in \mathbb{R}$ .  
Find the value of  $a$  and the value of  $b$  for which  
 $3z - 10i = (2 - 3i)z$ .

(c)

15 (5, 5, 5) marks

Att (2, 2, 2)

**I**

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow 3(a + bi) - 10i = (2 - 3i)(a + bi) \quad [5]$$

$$\Rightarrow 3a + 3bi - 10i = 2a + 2bi - 3ai - 3bi^2$$
$$\Rightarrow 3a + 3bi - 10i = 2a + 2bi - 3ai + 3b \quad [5]$$

Real parts:  $3a = 2a + 3b \Rightarrow a = 3b$   
Imaginary parts:  $3b - 10 = 2b - 3a \Rightarrow 3a + b = 10$   
 $3a + b = 10 \Rightarrow 10b = 10 \Rightarrow b = 1$   
 $\Rightarrow a = 3 \quad [5]$

**II**

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow z = 10i - 3zi$$
$$\Rightarrow a + bi - 10i = -3i(a + bi) \quad [5]$$

$$\Rightarrow a + bi = 10i - 3ai - 3bi^2$$
$$\Rightarrow a + bi = 10i - 3ai + 3b \quad [5]$$

Real parts:  $3a = 2a + 3b \Rightarrow a = 3b$   
Imaginary parts:  $3b - 10 = 2b - 3a \Rightarrow 3a + b = 10$   
 $3a + b = 10 \Rightarrow 10b = 10 \Rightarrow b = 1$   
 $\Rightarrow a = 3 \quad [5]$

**III**

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow z = 10i - 3zi \Rightarrow z + 3zi = 10i \Rightarrow z(1 + 3i) = 10i \quad [5]$$

$$\Rightarrow z = \frac{10i}{1 + 3i} \quad [5]$$

$$\Rightarrow z = 3 + i = a + bi$$
$$\Rightarrow a = 3 \text{ and } b = 1 \quad [5]$$

*Blunders (-3)*

B1 Mathematical error - once per step

*Attempts (2 marks)*

A1 Any relevant work for a given step

## QUESTION 5

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, 2, 2)</b>
<b>Part (c)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, 2, 2)</b>

\* Do not penalise notation

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
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The first term of a geometric sequence is 4 and the common ratio is 0.5.  
Write down the first five terms of the sequence.

<b>(a)</b>	<b>10 marks</b>	<b>Att 3</b>
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**I**

$$T_1 = a = 4,$$

$$T_2 = ar = 4 \times 0.5 = 2$$

$$T_3 = ar^2 = 4 \times 0.5^2 = 1 \text{ or } [2 \times 0.5]$$

$$T_4 = ar^3 = 4 \times 0.5^3 = 0.5 \text{ or } [1 \times 0.5]$$

$$T_5 = ar^4 = 4 \times 0.5^4 = 0.25 \text{ or } [0.5 \times 0.5]$$

**II**

List 4, 2, 1, 0.5, 0.25

\* Accept correct answers with no work

\* Accept in fractional form

*Blunders (-3)*

B1 Decimal error – once if consistent e.g. 0.5 taken as 5 or  $r = 2$

B2 Indices error – each time

B3 Error in formula – see guidelines

*Misreading (-1)*

M1  $r$  taken as 0.05

*Attempts (3 marks)*

A1 Identifies  $a$  as 4 and/or  $r$  as 0.5 and stops

A2 States  $T_1 = 4$

*Worthless (0)*

W1 Treats as an arithmetic sequence but see A1 and A2

W2 Incorrect answer(s) without work

Note: Answers without work

1 term correct 3 marks

2 terms correct 4 marks

3 terms correct 4 marks

4 terms correct 7 marks

5 terms correct 10 marks

**Part (b)****20 (10, 5, 5) marks****Att (3, 2, 2)**

In an arithmetic series, the first term is 6 and the fifth term is 22.

- (i) Find  $d$ , the common difference.  
 (ii) Find  $T_{14}$ , the fourteenth term.  
 (iii) Find  $S_{20}$ , the sum of the first twenty terms .

\* Answers to parts of questions must be clearly identified

**(i)****10 marks****Att 3****I**

$$T_1 = a = 6 \quad [3]$$

$$T_5 = a + 4d = 22 \quad [4]$$

$$\Rightarrow 4d = 22 - 6 \quad [7]$$

$$\Rightarrow d = 4 \quad [10]$$

**II**

$$6, 10, 14, 18, 22 \quad [7]$$

\* Accept correct answer without work

\* Acceptable formula - see guidelines

*Blunders (-3)*

B1 Mathematical error

*Slips (-1)*

S1 Numerical slips

*Attempts (3 marks)*

A1 Correct relevant work

A2  $22 - 4 = 16$  and stops or  $d = 16$

**(ii)****5 marks****Att 2****I**

$$T_{14} = a + 13d = 6 + 13(4) = 6 + 52 = 58$$

**II**

List:  $6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 + 42 + 46 + 50 + 54 + 58$

(Assume final term is answer, otherwise must indicate term 14)

\* Accept candidates  $d$  from (i)

\* Accept correct answer without work

*Blunders (-3)*

B1 Mathematical error

B2 Incorrect term from list

B3 Finds  $S_{14}$  by formula

*Slips (-1)*

S1 Numerical slips

*Attempts (2 marks)*

A1 Identifies  $a$  as 6 for this part of question

*Worthless (0)*

W1 Treats as a geometric series but may have identified  $a$  as 6 as part of this question

(iii)

5 marks

Att 2

I

$$S_{20} = \frac{20}{2}(2a + 19d) = 10(12 + 76) = 10(88) = 880$$

II

$$\text{List: } 6+10+14+18+22+26+30+34+38+42+46+50+54+58+62+66+70+74+78+82 = 880$$

\* Accept candidate's answers from (i) and (ii)

*Blunders (-3)*

- B1 Finds  $T_{20}$  and stops
- B2 Writes complete list but fails to sum
- B3 Finds  $S_{14}$  from (ii)
- B4 Incorrect number of terms in list

*Slips (-1)*

- S1 Numerical slips

*Attempts (2 marks)*

- A1 Identifies  $a$  and/or  $d$
- A2 Correct answer without work.

*Worthless (0)*

- W1 Treats as a geometric series but identification of  $a$  will merit A1



**Part (c)****20 (10, 5, 5) marks****Att (3, 2, 2)**

In a geometric series, the fourth term is 9 and the seventh term is 243.

- (i) Find  $r$ , the common ratio.  
 (ii) Find  $a$ , the first term.  
 (iii) Find  $S_8$ , the sum of the first eight terms.

**(c) (i)****10 marks****Att 3**

- |   |  |
|---|--|
| <p><b>I</b></p> <p><math>T_4 = ar^3 = 9</math> [3]</p> <p><math>T_7 = ar^6 = 243</math> [4]</p> <p><math>\frac{ar^6}{ar^3} = \frac{243}{9} \Rightarrow r^3 = 27</math> [7]</p> <p><math>\Rightarrow r = 3</math> [10]</p> <p><b>II</b></p> <p>List [<math>\frac{1}{3}</math>, 1, 3,] <b>9, 27, 81, 243</b> [7]</p> <p><math>\Rightarrow r = 3</math> [10]</p> | <p>Note: <math>ar^4 = 9</math> for <math>T_4</math><br/> <math>ar^7 = 243</math> for <math>T_7</math><br/> <math>r^3 = 27</math> etc. Accept</p> |
|---|--|

*Blunders (-3)*

- B1 Mathematical error  
 B2 Error in use of formula

*Attempts (3 marks)*

- A1  $T_4$  or  $T_7$  expressed in algebraic form and stops  
 A2 Finds  $243/9 = 27$  and stops  
 A3 Correct answer without work  
 A4 Partial list

**(c) (ii)****5 marks****Att 2**

- |  |  |
|--|--|
| <p><b>I</b></p> <p><math>ar^3 = 9 \Rightarrow a(3^3) = 9 \Rightarrow 27a = 9 \Rightarrow a = \frac{1}{3}</math></p> <p>[2] [5]</p> | <p><b>II</b></p> <p><math>\frac{1}{3}, 1, 3, 9 \Rightarrow a = \frac{1}{3}</math></p> <p>[2] [5]</p> |
|--|--|

\* Accept candidate's  $r$  from (i) as long as it does not oversimplify work

*Blunders (-3)*

- B1 Mathematical error

*Attempts (2 marks)*

- A1 Any relevant step  
 A2 Correct answer without work but allow if full list given in (i)

(c) (iii)

5 marks

Att 2

**I**

$$S_8 = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{1}{3}(6561 - 1)}{3 - 1} = \frac{\frac{1}{3}(6560)}{2} = \frac{3280}{3} = 1093\frac{1}{3}$$

[2] [5]

**II**

List:  $\frac{1}{3} + 1 + 3 + 9 + 27 + 81 + 243 + 729 = 1093.3333$

[2] [5]

\* Accept candidate's  $a$  and  $r$  from (i) and (ii) provided they do not over simplify work

*Blunders (-3)*

- B1 Mathematical error
- B2 Fails to sum list in method II
- B3 Missing or extra terms in list method

*Slips (-1)*

- S1 Numerical slips

*Attempts (2 marks)*

- A1 Finds  $T_8$
- A2 Identifies  $a$  as  $1/3$  in this part
- A3 Correct answer without work

*Worthless (0)*

- W1 Treats as an arithmetic series but identification of  $a$  will merit A2

## QUESTION 6

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 marks** **Att 3**

Let  $h(x) = x^2 + 1$ , where  $x \in \mathbb{R}$ .

Write down a value of  $x$  for which  $h(x) = 50$ .

**Part (a)** **10 marks** **Att 3**

$$h(x) = 50 \Rightarrow x^2 + 1 = 50 \Rightarrow x^2 = 49 \Rightarrow x = \pm 7$$

[3]            [7]            [10]

\* Accept correct answer without work. Accept  $\sqrt{49}$

\* Only one value for  $x$  is required.

*Blunders (-3)*

B1 Mathematical errors

B2 Evaluates  $h(50) = 2501$

*Attempts (3 marks)*

A1 Unsuccessful trial and error, e.g.  $h(5) = 25 + 1$

A2 Any correct relevant step

*Worthless (0)*

W1  $50(x^2 + 1)$  whether continues or not

W2 Incorrect answer with no work

W3 Differentiates

**Part (b)****20 (10, 10) marks****Att (3, 3)**

Let  $g(x) = \frac{1}{x-2}$ , where  $x \in \mathbb{R}$  and  $x \neq 2$ .

(i) Copy and complete the following table:

$x$	0	1	1.5	1.75	2.25	2.5	3	4
$g(x)$		-1		-4		2		

(ii) Draw the graph of the function  $g$  in the domain  $0 \leq x \leq 4$ .

**(b) (i)****10 marks****Att 3**

$x$	0	1	1.5	1.75	2.25	2.5	3	4
$g(x)$	-0.5	-1	-2	-4	4	2	1	0.5

\* Values of  $g(x) = \frac{1}{x-2}$  calculated ( all/some correct ) misreading which oversimplifies, Att 3

\* Accept values as fractions; must be  $\frac{1}{\text{Singleton}}$  or  $\frac{1}{\text{Single number}}$

e.g. for  $x = 1.5$  accept  $\frac{1}{-0.5}$  but not  $\frac{1}{1.5-2}$

**Blunders (-3)**

B1 Treats the function as  $f(x) = \frac{1}{x} - 2$ , even if  $g(x) = \frac{1}{x-2}$  written.

The relevant values for  $f(x) = \frac{1}{x} - 2$  are:

$(0, \text{undefined}), (1.5, -\frac{4}{3}), (2.25, -\frac{14}{9}), (3, -\frac{5}{3}), (4, -\frac{7}{4})$

B2 Treats as  $g(x) = \frac{1}{x+2}$  avoids error with  $-$  sign

**Slips (-1)**

S1 Each un-simplified value to a maximum of 3

**Attempts (3 marks)**

A1 Copies table and stops

A2 Treats  $g(x)$  as  $x-2$

Note: Answers without work

1 value correct 3 marks

2 values correct 4 marks

3 values correct 4 marks

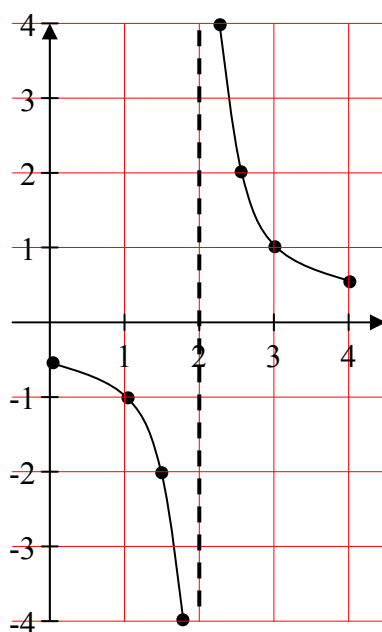
4 values correct 7 marks

5 values correct 10 marks

(b) (ii)

10 marks

Att 3



- \* Consider graph as having 3 features LHS/branch, asymptote (actual or implied) and RHS/branch.
- \* Asymptote  $x = 2$  need not be drawn; an implied vertical asymptote (or visible gap ) will suffice
- \* Has graph of  $x - 2$ : oversimplified Att 3
- \* Accept candidate's values from (i) if not over simplified
- \* Ignore any graph errors outside the given range e.g. graph cutting the horizontal asymptote
- \* Points plotted and not joined and not showing asymptote – [4] marks
- \* Only one branch without a vertical asymptote – [4] marks at most

*Blunders (-3)*

- B1 Left and right branches joined
- B2 Points joined incorrectly
- B3 LHS or RHS branch missing or asymptote missing or not implied
- B4 Serious incorrect scaling of axes e.g. equal distance on  $x$ -axis for given values

*Slips (-1)*

- S1 Each point clearly incorrectly plotted or each point clearly omitted to a maximum of 3 per side

*Attempts (3 marks)*

- A1 Draws axes and stops
- A2 One point correctly plotted
- A3 Any mention of asymptotes
- A4 Table from (i) does not give rise to two branches

Note: If B1 or B2 applied at (i) graph at (ii) will merit attempt mark at most



(c) (ii)

10 marks

Att 3

$$f'(x) = 6 \Rightarrow 1 + \frac{5}{x^2} = 6 \Rightarrow 5 = 5x^2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

[3] [7]

$$f(1) = 1 - \frac{5}{1} = 1 - 5 = -4. \quad \text{Point } (1, -4)$$

$$f(-1) = -1 - \frac{5}{-1} = -1 + 5 = 4. \quad \text{Point } (-1, 4) \quad [10]$$

\* Accept candidates answer from (i) unless it is oversimplified

\* Penalise simplification of  $f'(x)$  errors in this part if necessary

*Blunders (-3)*

B1 Mathematical errors

B2  $f'(x) \neq 6$

B3 Only one solution found for  $x$ , B4 will also apply

B4 Only one value of  $f(x)/y$  found

*Slips (-1)*

S1 Numerical slips

*Attempts (3 marks)*

A1 Mentions slope of  $y = 6x$  is 6

A2 Answer from (i) = 6 and stops

A3 Mentions connection of slope and derivative and stops

## QUESTION 7

<b>Part (a)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (b)</b>	<b>20 (5, 15) marks</b>	<b>Att (2, 5)</b>
<b>Part (c)</b>	<b>15 (5, 5, 5) marks</b>	<b>Att (2, 2, 2)</b>

**Part (a)** **15 marks** **Att 5**

Differentiate  $x^2 - 6x + 1$  with respect to  $x$ .

**(a)** **15 marks** **Att 5**

$$\frac{dy}{dx} = 2x - 6$$

- \* Correct answer without work or notation: full marks.
- \* If done from first principles, ignore errors in procedure – just mark the answer.
- \* Only one non zero term correct, award 12 marks

*Blunders (-3)*

B1 Differentiation error once per term

*Attempts (5 marks)*

A1 A correct step in differentiation from 1<sup>st</sup> principles

A2 A correct coefficient or a correct index of  $x$  in one of the term(s)

A3 Mentions  $\frac{dy}{dx}$  or  $f'(x)$

*Worthless (0)*

W1 No differentiation



- (i) Differentiate  $5 - 3x$  with respect to  $x$  from first principles.
- (ii) Given that  $y = (x^2 - 4)(3x - 1)$ , find the value of  $\frac{dy}{dx}$  when  $x = 2$ .

(b)

5 marks

Att 2

<p><b>I</b> <math>f(x) = 5 - 3x</math>  <math>f(x + h) = 5 - 3(x + h)</math>  <math>= 5 - 3x - 3h</math></p> <hr style="border-top: 1px dotted black;"/> <p><b>II</b> <math>f(x + h) - f(x) = 5 - 3x - 3h - (5 - 3x) = -3h</math></p> <hr style="border-top: 1px dotted black;"/> <p><b>III</b> <math>\frac{f(x + h) - f(x)}{h} = \frac{-3h}{h} = -3</math>  <math>\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = -3</math></p>	<p><b>I</b> <math>y = 5 - 3x</math>  <math>y + \Delta y = 5 - 3(x + \Delta x)</math>  <math>= 5 - 3x - 3\Delta x</math>  <math>y = 5 - 3x</math></p> <hr style="border-top: 1px dotted black;"/> <p><b>II</b> <math>\Delta y = -3\Delta x</math>  <math>\frac{\Delta y}{\Delta x} = -3</math></p> <hr style="border-top: 1px dotted black;"/> <p><b>III</b> <math>\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -3</math></p>
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\* Accept use of  $(x - h)$ *Blunders (-3)*

B1 Any error once per step I, II or III

Note: Must have correct LHS and RHS

*Attempts (2 marks)*A1  $f(x \pm h)$  on LHS or some substitution of  $x \pm h$  for  $x$  on RHS, or equivalent; these onlyA2 Mentions  $\Delta x$  or  $\Delta y$  or similar*Worthless (0)*W1 Answer  $-3$  without work; no attempt at first principles

(b) (ii)

15 marks

Att 5

I	or	II	
$y = (x^2 - 4)(3x - 1)$		$y = (x^2 - 4)(3x - 1)$	
$u = x^2 - 4$	$v = 3x - 1$	$y = 3x^3 - x^2 - 12x + 4$	[9]
$\frac{du}{dx} = 2x$	$\frac{dv}{dx} = 3$		[9]
$\frac{dy}{dx} = (3x - 1)(2x) + (x^2 - 4)(3)$		$\frac{dy}{dx} = 9x^2 - 2x - 12$	[12]
At $x = 2$		At $x = 2$	
$\frac{dy}{dx} = (6 - 1)(4) + (4 - 4)(3) = 20$		$\frac{dy}{dx} = 9(4) - 2(2) - 12 = 36 - 4 - 12 = 20$	[15]

\* Uses  $\frac{u}{v}$  merits 9 marks at most – allow for  $u = \Rightarrow \frac{du}{dx} = \dots$  and  $v = \Rightarrow \frac{dv}{dx} = \dots$  better than A5

*Blunders (-3)*

B1 Differentiation error

B2 Errors in expanding brackets once only unless over simplifies.

B3 Error in substitution, once only

*Slips (-1)*

S1 Numerical slips

*Attempts (5 marks)*

A1  $u$  and/or  $v$  correctly identified and stops (I)

A2 Any correct differentiation

A3 At least one term multiplied correctly

A4 Uses  $3x^3 + 4$  even if completed correctly

A5  $\frac{dy}{dx} = (2x)(3)$

*Worthless (0)*

W1 Substitutes  $x = 2$  into  $y$  and stops

W2  $uv$  formula written and stops

**Part (c)**

**15 (5, 5, 5) marks**

**Att (2, 2, 2)**

The speed,  $v$ , of an object at time  $t$  is given by

$$v = 96 + 40t - 4t^2$$

where  $t$  is in seconds and  $v$  is in metres per second.

- (i) At what times will the speed of the object be 96 metres per second?
- (ii) What will the acceleration of the object be at  $t = 2.5$  seconds?
- (iii) At what value of  $t$  will the acceleration become negative?

\* Units: Penalise as per guidelines.

\* No retrospective marking.

\* No penalty for incorrect notation.

\* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c) (i) to (c) (iii).

**(c) (i)**

**5 marks**

**Att 2**

$$96 = 96 + 40t - 4t^2$$

$$\Rightarrow 4t^2 - 40t = 0$$

$$\Rightarrow t(t - 10) = 0$$

$$\Rightarrow t = 0, t = 10 \text{ s}$$

\* One or both answers correct without work, Att 2

*Blunders (-3)*

B1 Equation  $\neq 96$

B2 Incorrect factors

B3 Incorrect roots from factors but see S2

*Slips (-1)*

S1 No units or incorrect units

S2  $t = 0$  not included

*Attempts (2 marks)*

A1 Attempt at factorising

A2 Trial and error on  $96 + 40t - 4t^2$  even if correct

*Worthless (0)*

W1 Differentiation

(c) (ii)

5 marks

Att 2

$$a = \frac{dv}{dt} = 40 - 8t \quad [4]$$

$$\text{At } t = 2.5 \quad a = 40 - 8(2 \cdot 5) = 20 \text{ m s}^{-2} \quad [5]$$

\* Acceleration as second derivative of  $v$  i.e correct  $d^2v/dt^2$  merits 4

*Blunders (-3)*

B1 Differentiation error

*Slips (-1)*

S1 No units or incorrect units

S2 Substitution error

*Attempts (2 marks)*

A1 Mentions  $dv/dt$  or similar

*Worthless (0)*

W1 Substitutes  $t = 2.5$  into  $v$

(c) (iii)

5 marks

Att 2

**I**

$$\frac{dv}{dt} < 0 \Rightarrow 40 - 8t < 0 \Rightarrow -8t < -40 \Rightarrow t > 5$$

or

**II**

“Acceleration negative (deceleration) after velocity reaches its maximum” or similar

$$\frac{dv}{dt} = 0 \Rightarrow 40 - 8t = 0 \Rightarrow t = 5$$

Acceleration negative after  $t = 5$

\* Correct answer without work, Att 2.

*Blunders (-3)*

B1 Error solving inequality (I) or equation (II)

*Slips (-1)*

S1  $t \leq 5$

*Attempts (2 marks)*

A1 Any correct value offered

A2 Has acceleration  $d^2v/dt^2 = -8$ , therefore acceleration is always negative

*Worthless (0)*

W1  $t = 8$  from  $d^2v/dt^2 = -8$

W2 Attempts to solve  $96 + 40t - 4t^2 < 0$

## QUESTION 8

<b>Part (i)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (ii)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (iii)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (iv)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (v)</b>	<b>5 marks</b>	<b>Att 2</b>

\* Assume answering in order (i) ... (v) No retrospective marking

**Part (i)** **15 marks** **Att 5**

Let  $f(x) = x^3 - 3x + 1$ , where  $x \in \mathbb{R}$ .

(i) Find  $f(-3)$ ,  $f(-2)$ ,  $f(0)$ ,  $f(2)$  and  $f(3)$ .

**(i)** **15 marks** **Att 5**

$$f(x) = x^3 - 3x + 1$$

$$f(-3) = (-3)^3 - 3(-3) + 1 = -27 + 9 + 1 = -17 \quad [5]$$

$$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -1 \quad [6]$$

$$f(0) = (0)^3 - 3(0) + 1 = 0 + 0 + 1 = 1 \quad [9]$$

$$f(2) = (2)^3 - 3(2) + 1 = 8 - 6 + 1 = 3 \quad [12]$$

$$f(3) = (3)^3 - 3(3) + 1 = 27 - 9 + 1 = 19 \quad [15]$$

\* Correct answers without work, full marks.

\* Don't penalise extra values e.g  $f(1)$  and/or  $f(-1)$ .

*Blunders (-3)*

B1 Mathematical errors, each time if different

B2 Use  $x^2$  for  $x^3$

*Slips (-1)*

S1 Arithmetic slips to maximum of 3

*Attempts (5 marks)*

A1 Only finds one value and stops

A2 Some correct substitution into  $f(x)$

A3  $f'(x)$  with some correct substitution

*Worthless (0)*

W1 All incorrect answers without work

Note: Answers without work

1 point/value correct 5 marks

2 points/values correct 6 marks

3 points/values correct 9 marks

4 points/values correct 12 marks

5 points/values correct 15 marks

**Part (ii)**

**10 marks**

**Att 3**

Find  $f'(x)$ , the derivative of  $f(x)$ .

**(ii)**

**10 marks**

**Att 3**

$$f'(x) = 3x^2 - 3$$

- \* Correct answer without work or notation, full marks.
- \* If done from first principles, ignore errors in procedure – just mark the answer.
- \* Only one non zero term correct, award 7 marks

*Blunders (-3)*

B1 Differentiation error once per term.

*Attempts (3 marks)*

- A1 A correct step in differentiation from 1<sup>st</sup> principles
- A2 A correct coefficient or a correct index of  $x$ .

**Part (iii)**

**10 marks**

**Att 3**

Find the co-ordinates of the local maximum point and of the local minimum point of the curve  $y = f(x)$ .

**(iii)**

**10 marks**

**Att 3**

$$f'(x) = 3x^2 - 3 = 0 \quad [3]$$

$$\Rightarrow x^2 - 1 = 0 \Rightarrow (x+1)(x-1) = 0 \Rightarrow x = -1 \text{ or } x = 1. \quad [7]$$

$$f(x) = x^3 - 3x + 1$$

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

$$f(1) = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

Local maximum  $(-1, 3)$ , local minimum  $(1, -1)$ . [10]

\* Accept candidate's  $f'(x)$  from (ii) but see A1

\* Accept implied ' $= 0$ ' if subsequent work supports it.

\* Accept distinguishing max from min by comparing  $y$ -ordinates. Second derivative not required.

\* Correct answers without calculus, Att 3 at most. May be from graph.

*Blunders (-3)*

B1  $f'(x) \neq 0$  (but see 2<sup>nd</sup> asterisk)

B2 Error finding roots

B3 Only finds one root ( B4 will also apply)

B4 Error finding  $f(x)$  value e.g. fails to find  $f(x)$  value or only finds one value or does not use  $f(x)$

*Slips (-1)*

S1 Numerical slips

S2 Does not distinguish between maximum and minimum, or identifies incorrectly

*Attempts (3 marks)*

A1  $f'(x)$  linear and continues

A2  $f''(x)$

*Worthless (0)*

W1  $f(x) = 0$ , whether continues or not

Part (iv)

10 marks

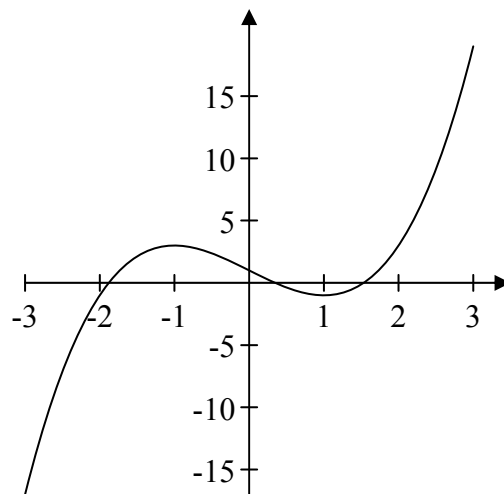
Att 3

Draw the graph of the function  $f$  in the domain  $-3 \leq x \leq 3$ .

(iv)

10 marks

Att 3



- \* Accept candidate's values of  $(x, f(x))$  from previous parts unless oversimplified.
- \* If candidate recalculates points, apply slips and blunders as per guidelines.
- \* Seven (7) points required Only uses 5 points from (i) [8]

*Blunders (-3)*

B1 Scale error, serious

B2 Points not joined or joined incorrectly or joined with a series of straight lines

B3 Axes not in standard form

*Slips (-1)*

S1 Each point incorrectly plotted or omitted

*Attempts (3 marks)*

A1 Plots  $f'(x)$  or graph of a non-cubic function

A2 Answers from part (iii) transferred to this part, carries forward max and min values

A3 Effort at calculation of a point with some substitution e.g.  $f(0)$

A4 Scaled and labelled axes and stops



**Part (v)**

**5 marks**

**Att 2**

Find the range of values of  $k$  for which the equation  $x^3 - 3x + 1 = k$  has three real solutions (roots).

**(v)**

**5 marks**

**Att 2**

$$-1 \leq k \leq 3$$

- \* Accept answer consistent with candidate's graph if cubic
- \* Accept any valid solution
- \* Accept answer clearly indicated on graph
- \* Accept answer using words rather than symbols and  $[3, -1]$  or  $[-1, 3]$
- \* Accept  $-1 < k < 3$

*Blunders (-3)*

B1 Inequalities not as stated

*Attempts (2 marks)*

A1 One correct end-point identified

A2 Solves  $f(x) = 0$  or finds one correct value of  $k$

A3 Mentions local maximum or local minimum or max. and min.



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

**LEAVING CERTIFICATE 2010**

**MARKING SCHEME**

**MATHEMATICS – PAPER 2**

**ORDINARY LEVEL**

## GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

## Application of penalties throughout scheme

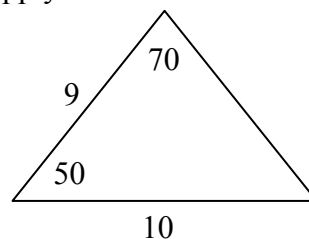
Penalties are applied subject to marks already secured.

**Blunders** - examples of blunders are as follows:

- Algebraic errors:  $8x + 9x = 17x^2$  or  $5p \times 4p = 20p$ .
- Sign error:  $-3(-4) = -12$  or  $(-3)^2 = -9$ .
- Fraction error: Incorrect fraction inversion etc. apply once.
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error:  $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$  or  $-3x = 6 \Rightarrow x = 2$ .  
or  $4x = 12 \Rightarrow x = 8$  each type once per section.
- Distributive law errors (once per pair of brackets)  
 $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$  or  $-(4x + 3) = -4x + 3$  or  $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly:  $(2x - 3)(x + 4) = 8x^2 - 12x$ .
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once per section.  
 $2x^2 - 2x - 3 = (2x - 1)(x + 3)$ .
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

$$\text{e.g. } 2x^2 + 3x + 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(2)}$$

$$\text{or } \frac{10}{\sin 70} = \frac{9}{\sin 50}$$



- Incorrectly treating co-ordinates as  $(x_1, x_2)$  and  $(y_1, y_2)$  when using co-ordinate geometry formula.
- Errors in formula for example:  $\frac{y_2 + y_1}{x_2 + x_1}$  or  $A = P\left(1 + \frac{n}{100}\right)^r$  or  $a^2 = b^2 + c^2 + bc \cos A$   
or  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ , except as indicated in scheme.

**Note:** A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the *Formulae and Tables* booklet.

**Slips** – examples are as follows:

- Numerical slips such as:  $4 + 7 = 10$  or  $3 \times 6 = 24$  but  $5 + 3 = 15$  is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to the incorrect accuracy or an early round-off that affects accuracy are penalised as a slip once in each section.
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once in each section where the candidate would otherwise have obtained full marks in that section. This applies to **Q1** (a) (i), (ii), (b) (i) and (c) (i), (ii) and to **Q5** (a), and (c) (i), (ii).

**Misreadings**

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

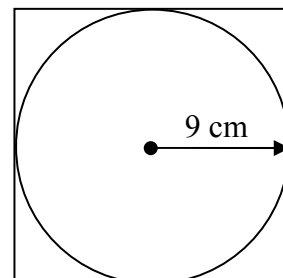
## QUESTION 1

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (15, 5) marks</b>	<b>Att (5, 2)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

A circle is inscribed in a square as shown.  
The radius of the circle is 9 cm.

- (i) Find the perimeter of the square.
- (ii) Calculate the area of the square.



**(a) (i)** **5 marks** **Att 2**

$$l = 9 \times 8 = 72 \text{ cm} \quad \text{or} \quad l = 18 \times 4 = 72 \text{ cm.}$$

**(a) (ii)** **5 marks** **Att 2**

$$A = 18^2 = 324 \text{ cm}^2$$

- \* Accept correct answer without work, including an answer written on a diagram.
- \* Accept in section (ii) an answer consistent with candidate's answer to section (i).

- 5 marks Correct answer.
- 4 marks One slip or misreading.
- 2 marks Work of some merit. e.g.  $2 \times 9$  or 18 or 81 or 2 slips(-1). Otherwise 0 marks.

*Worthless (0 marks)*

W1 Any incorrect answer without work- subject to work of some merit.

**Note: Exception (i) = 324 (4 marks), (ii) = 72 cm<sup>2</sup> (5 marks).**

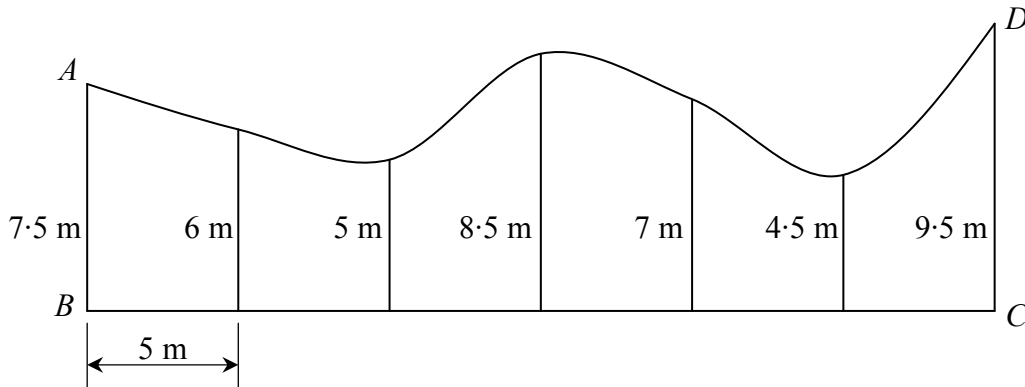
<b>Case 1</b>	<b>Case 2</b>
$l = (18)^2 = 324 \text{ cm}$ (4 marks).	$l = 4 \times 9 = 36$ (2 marks)
$A = (18 \times 4) = 72 \text{ cm}^2$ (5 marks).	$A = 9 \times 9 = 81 \text{ cm}^2$ (5 marks)
<b>Case 3</b>	<b>Not obvious (wrong formula)</b>
$Circle = 2\pi r = 18\pi \text{ cm}$ (4 marks)	(i) = $2\pi r = 18\pi \text{ cm}$ (2 marks)
$Circle = \pi r^2 = 81\pi \text{ cm}^2$ (5 marks)	(ii) = $Area = \pi r^2 = 81\pi$ (2 marks)

**Part (b)**

**20 (15, 5) marks**

**Att (5, 2)**

The diagram shows a sketch of a field  $ABCD$  that has one uneven edge. At equal intervals of 5 m along  $[BC]$ , perpendicular measurements are made to the uneven edge, as shown on the sketch.



- (i) Use Simpson's rule to estimate the area of the field.
- (ii) The actual area of the field is  $200 \text{ m}^2$ . Find the percentage error in the estimate.

**(b) (i) Use of formula  
Calculations**

**10 marks  
5 marks**

**Att 3  
Att 2**

$$\begin{aligned}
 \text{(i) Area} &\approx \frac{h}{3}(F + L + 2\Sigma O + 4\Sigma E) \\
 &= \frac{5}{3}(7.5 + 9.5 + 2(5 + 7) + 4(6 + 8.5 + 4.5)) \quad [10 \text{ marks}] \\
 &= \frac{5}{3}(17 + 2(12) + 4(19)) = \frac{5}{3}(17 + 24 + 76) = \frac{5}{3}(117) = 195 \text{ m}^2. \quad [5 \text{ marks}]
 \end{aligned}$$

- \* Allow  $\frac{h}{3} = (F + L + \text{TOFE})$  and penalise in calculations if formula not used correctly.
- \* Accept correct TOFE *or* TOFE consistent with candidates F and L.

**Substitution:**

- 10 marks Fully correct substitution
- 7 marks One blunder in substitution
- 4 marks Two blunders in substitution
- 3 marks Some merit in candidates work, otherwise 0 marks.

Treat as separate blunders

- Incorrect  $\frac{h}{3}$  (once).
- Incorrect F and/or L or extra terms with F and/or L (once).
- $\Sigma E$  **or**  $\Sigma O$  omitted (once).
- Incorrect TOFE (once), if not consistent with candidates F and L.  
**Note:** e.g. A consistent TOFE =  $\frac{5}{3}(0 + 9.5 + 2(6 + 8.5 + 4.5) + 4(7.5 + 5 + 7))$
- Use of trapezoidal rule. (Answer =  $197.5 \text{ m}^2$ ).

**Calculation :**

- 5 marks Correct or consistent answer.
- 4 marks One slip or misreading.
- 2 marks Work of some merit, otherwise 0 marks

<b>Note: Where calculation work not shown.</b>			
<b>I</b>	<b>II</b>	<b>III</b>	<b>IV</b>
No substitution Ans: 195 (3 marks + 2 marks)	Substitution (mark =*) Ans: 195 m <sup>2</sup> /consistent (* marks + 5 marks)	Substitution (mark =*) Ans: 194/consistent (* marks + 4 marks)	Substitution (mark =*) Ans: #/not consistent (* marks + 0 marks)

**(b) (ii)**

**5 marks**

**Att 2**

(ii) Percentage error: $\frac{5}{200} \times 100 = 2.5\%$
---

\* Accept 2.5 or consistent answer without work. (ii)

**Percentage error:**

- 5 marks Correct or consistent answer.
- 4 marks One slip or misreading.
- 2 marks Work of some merit, otherwise 0 marks.

**Part (c)**

**20 (10, 10) marks**

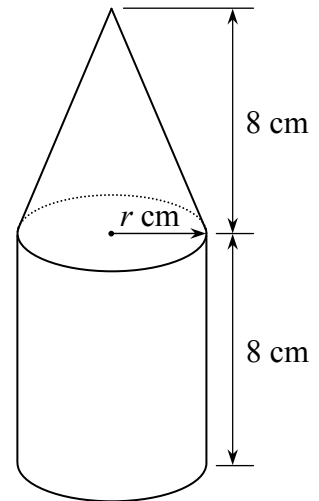
**Att (3, 3)**

A solid metal sphere has diameter 9 cm.

(i) Find the volume of the sphere in terms of  $\pi$ .

The sphere is melted down. All of the metal is used to make a solid shape which consists of a cone on top of a cylinder, as shown in the diagram.

The cone and the cylinder both have height 8 cm. The cylinder and the base of the cone both have radius  $r$  cm.



(ii) Calculate  $r$ , correct to one decimal place.

**(c) (i)**

**10 marks**

**Att 3**

**(c) (ii)**

**10 marks**

**Att 3**

(i) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5)^3 \downarrow_{4\text{marks}} = \frac{243}{2}\pi \text{ cm}^3$ or $121.5\pi \text{ cm}^3$
--

(ii)

$$V = \frac{1}{3}\pi r^2 h + \pi r^2 h \downarrow_{3\text{marks}} = 121.5\pi \downarrow_{4\text{marks}} \Rightarrow \frac{4}{3}r^2(8) = 121.5 \downarrow_{4\text{marks}}$$

$$r^2 = \frac{121.5 \times 3}{4 \times 8} = \frac{364.5}{32} = 11.3906 \downarrow_{7\text{marks}} \Rightarrow r = 3.375 \downarrow_{9\text{marks}} = 3.4 \text{ cm}$$

$$\text{or } r = \sqrt{\frac{121.5 \times 3}{4 \times 8}} = \sqrt{\frac{364.5}{32}} = \sqrt{11.3906} \downarrow_{7\text{marks}} \Rightarrow r = 3.375 \downarrow_{9\text{marks}} = 3.4 \text{ cm}$$

\* Accept an answer in section (ii) consistent with the candidate's answer to section (i).

- 10 marks Fully correct answer
- 9 marks One slip or misreading.
- 7 marks One blunder. e.g.  $381.7$  or  $381.7\pi$  with work.
- 4 marks Two blunders
- 3 marks Some merit in candidates work, otherwise 0 marks.

Treat as separate blunder.

- Incorrect relevant volume of sphere formula i.e.  $k(\pi r^3)$  where  $k \neq \frac{4}{3}$  and continues. (i).

*Attempts (3 marks)*

- A1 Some merit in work, e.g. equation set up or  $h$  substituted into relevant volume formula in (ii).
- A2 Correct formula with any correct substitution.
- A3 Correct answer without work in each section.
- A4  $\pi r^2 h = 121 \cdot 5 \pi$  even if completed.

*Worthless (0 marks)*

- W1 Use of any area formula. e.g.  $4 \pi r^2$ .
- W2 Non sphere formula, e.g.  $\pi r^2 h$  (i). Subject to work of merit.
- W3 Non cone or cylinder used, e.g.  $\frac{4}{3} \pi r^3 = 121 \cdot 5 \pi$  and continues.



## QUESTION 2

<b>Part (a)</b>	<b>5 marks</b>	<b>Att 2</b>
<b>Part (b)</b>	<b>25 (10, 5, 5, 5) marks</b>	<b>Att (3, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, -, -)</b>

*Apply the following to each section of question 2 and question 3.*

If the correct formula is not written, any sign or substitution error is at least a blunder.

### *Blunders (-3)*

- B<sub>a</sub> Two or more incorrect substitutions.
- B<sub>b</sub> Switches  $x$  and  $y$  in substituting or treats as a pair of couples  $(x_1, x_2)$  and  $(y_1, y_2)$ .
- B<sub>c</sub> Error in the central sign in a formula.

### *Slips (-1)*

- S<sub>a</sub> One incorrect non-central sign.
- S<sub>b</sub> One incorrect substitution in the formula.
- S<sub>c</sub> Obvious misreading of one co-ordinate.

### *Attempts*

- A<sub>a</sub> The correct relevant formula written and stops. [If not transcribed from tables.]
- A<sub>b</sub> The co-ordinates of a relevant point written with  $x_1$  and  $y_1$  identified.
- A<sub>c</sub> A correct substitution into relevant formula and stops.

### *Worthless (0 Marks)*

- W<sub>a</sub> Correct formula transcribed from tables and stops.

<b>Part (a)</b>	<b>5 marks</b>	<b>Att 2</b>
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Find the area of the triangle with vertices  $(0, 0)$ ,  $(8, -6)$  and  $(-1, 5)$ .

<b>(a)</b>	<b>5 marks</b>	<b>Att 2</b>
------------	----------------	--------------

$(0, 0)$ ,  $(8, -6)$  and  $(-1, 5)$

$$\text{Area} = \frac{1}{2} |x_1 y_2 - x_2 y_1| = \frac{1}{2} |8 \times 5 - (-1)(-6)| = \frac{1}{2} |40 - 6| = \frac{1}{2} |34| = 17.$$

*or*

$$\begin{aligned} \text{Area} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \downarrow_{2 \text{ marks}} \\ &= \frac{1}{2} |0(-6 - 5) + 8(5 - 0) + (-1)(0 + 6)| = \frac{1}{2} |0 + 40 - 6| = \frac{1}{2} |34| = 17 \end{aligned}$$

*or*

$$\begin{aligned} \text{Area} &= \frac{1}{2} [x_1 y_2 + x_2 y_3 + x_3 y_1 - x_1 y_3 - x_3 y_2 - x_2 y_1] \downarrow_{2 \text{ marks}} \\ &= \frac{1}{2} |0 \times -6 + 8 \times 5 + (-1) \times 0 - 0 \times 5 - (-1)(-6) - 8 \times 0| \\ &= \frac{1}{2} |0 + 40 + 0 - 6 + 0| = 17. \end{aligned}$$

\*  $\frac{1}{2} |-34| = -17$  incurs no penalty.

### *Blunders (-3)*

- B1 Incorrect relevant formula and continues e.g.  $\frac{1}{2} |x_1 y_2 + x_2 y_1|$  or omits the  $\frac{1}{2}$ .
- B2 An incorrect translation or required translation omitted.

### *Attempts (2 marks)*

- A1 Uses the distance formula or the perpendicular distance formula, (at least one substitution).
- A2 Plots one or more points, to the eye.
- A3 A correct answer without work.

### *Worthless (0 marks)*

- W1 Irrelevant formula and stops e.g.  $\frac{1}{2}$  on its own.

**Part (b)****25 (10, 5, 5, 5) marks****Att (3, 2, 2, 2)**

$l$  is the line  $3x - 4y - 15 = 0$ .

(i) Verify that  $(1, -3)$  is a point on  $l$ .

(ii)  $l$  intersects the  $x$ -axis at  $P$ . Find the co-ordinates of  $P$ .

The line  $k$  passes through the point  $(1, -3)$  and is perpendicular to  $l$ .

(iii) Show the lines  $l$  and  $k$  on a co-ordinate diagram.

(iv) Find the equation of  $k$ .

**(b) (i)****10 marks****Att 3**

$$3x - 4y - 15 = 0 \Rightarrow 3(1) - 4(-3) - 15 = 3 + 12 - 15 = 15 - 15 = 0$$

[Hence,  $(1, -3) \in l$ .]

\* Accept consistent answers in this and subsequent sections.

*Blunders (-3)*

B1 Substitution, but work not completed to arrive at LHS = RHS.

*Slips(-1)*

S1 No conclusion if L.H.S.  $\neq$  R.H.S.

*Attempts (3 marks)*

A1 Some substitution attempted or some correct work with the equation. e.g.  $3x - 4y = 15$ .

A2 Plots  $(1, -3)$  for this section.

**(b) (ii)****5 marks****Att 2**

$$3x - 4y - 15 = 0$$

$$y = 0 \Rightarrow 3x - 4(0) - 15 = 0 \Rightarrow 3x = 15 \Rightarrow x = 5.$$

[ $P(5, 0)$ ]

\* Accept a correct answer without work.

*Blunders (-3)*

B1 Finds intercept on the  $y$ -axis. i.e.  $(0, -3\frac{3}{4})$ .

*Attempts (2 marks)*

A1 Some relevant step e.g. writes  $y = 0$  and stops.

A2 Finds a random point on the line.

A3 Writes  $(*, 0)$ .

A4 Attempts to write in form  $y = mx + c$ .

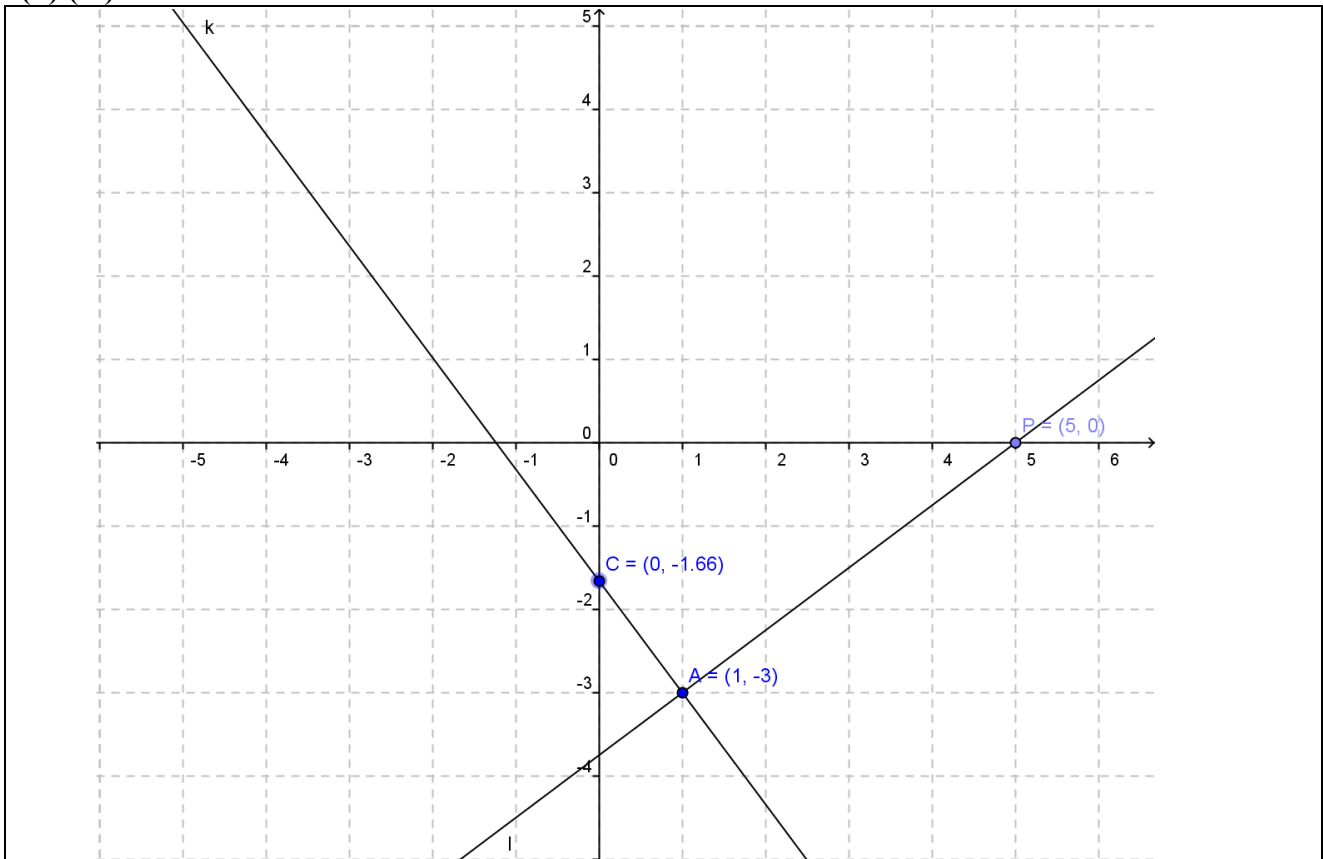
*Worthless (0 marks)*

W1 Writes  $x = 0$  and stops.

(b) (iii)

5 marks

Att 2



- \* Accept candidate's co-ordinates of  $P$  from (ii) and/or candidate's equation of  $k$  from (iv).
- \* Intervals should be indicated or implied.
- \* If (ii) is not answered but  $P(5, 0)$  is labelled on the graph award 5 marks for (ii).
- \* Work must be shown if diagram is not consistent with (ii).
- \* Accept tolerance of  $\pm 10^\circ$  in perpendicular.

*Blunders (-3)*

- B1 Scales unreasonably inconsistent, (to the eye).
- B2 Different scales on  $x$  and  $y$  axes, (to the eye).
- B3 Measure of angle between  $l$  and  $k$  outside tolerance of  $\pm 10^\circ$ .
- B4  $k$  does not pass through  $(1, -3)$ .
- B5 Plots  $P(5, 0)$  on the  $y$ -axis.
- B6 Points plotted but not joined.
- B7 Vertical  $x$ - axis.

*Attempts (2 marks)*

- A1 Draws scaled axes and stops, [for this section b(iii)].
- A2 Plots one correct point or just  $l$  and stops.

*Worthless (0 marks)*

- W1 Draws arbitrary line  $l$  with  $k$  not perpendicular to  $l$ , subject to A1.

(b) (iv)

5 marks

Att 2

$$l: 3x - 4y - 15 = 0 \Rightarrow 4y = 3x - 15 \Rightarrow y = \frac{3}{4}x - \frac{15}{4}.$$

Hence slope of  $l = \frac{3}{4}$ .

$$\frac{3}{4} \times m = -1 \Rightarrow m = -\frac{4}{3}, \text{ the slope of } k.$$

$$y - y_1 = m(x - x_1) \Rightarrow y + 3 = -\frac{4}{3}(x - 1) \downarrow_{5 \text{ marks}} \Rightarrow [3y + 9 = -4x + 4 \Rightarrow 4x + 3y + 5 = 0].$$

or

$$l: 3x - 4y - 15 = 0.$$

$$k \perp l \Rightarrow k: 4x + 3y + c = 0$$

$$(1, -3) \in k \Rightarrow 4(1) + 3(-3) + c = 0 \Rightarrow c = 5 \downarrow_{5 \text{ marks}}$$

$$[k: 4x + 3y + 5 = 0]$$

or

$$y = mx + c$$

$$k: y = -\frac{4}{3}x + d$$

$$(1, -3) \in k \Rightarrow -3 = -\frac{4}{3} + d = 0 \Rightarrow -9 = -4 + 3d \Rightarrow d = -\frac{5}{3} \downarrow_{5 \text{ marks}}$$

$$[y = -\frac{4}{3}x - \frac{5}{3} \Rightarrow 3y = -4x - 5 \text{ or } 4x + 3y + 5 = 0].$$

\* Allow equation of  $k$  work to precede graphing.

*Blunders (-3)*

B1 Incorrect slope of  $k$ , i.e.  $m_1 m_2 \neq -1$ , e.g.  $\frac{4}{3}$  or  $-\frac{3}{4}$ .

B2 Use of  $m_1 m_2 = -1$  omitted or applied incorrectly.

B3 Uses an arbitrary point.

*Misreadings (-1)*

M1  $k$  passes through  $P(5, 0)$ .

*Attempts (2 marks)*

A1 Correct relevant formula not transcribed from tables and stops e.g.  $m_1 m_2 = -1$  or  $m = -\frac{a}{b}$ .

A2  $k$  written as  $4x + 3y$  and stops.

A3 Transposes  $x$  or  $y$  and stops.

A4 Correct answer without work.

**Part (c)****20 (10, 5, 5) marks****Att (3, -, -)**

$A(2, -1)$  and  $B(-4, 7)$  are two points.

(i) Find  $|AB|$ .

(ii) Find  $C$ , the image of  $B$  under the translation  $(2, -1) \rightarrow (-7, 11)$ .

(iii) Show that  $|AB| : |AC| = 2 : 5$ .

**(c) (i)****10 marks****Att 3**

$A(2, -1)$  and  $B(-4, 7)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 2)^2 + (7 + 1)^2} \downarrow_{4 \text{ marks}} = \sqrt{36 + 64} = \sqrt{100} \downarrow_{7 \text{ marks}} = 10$$

**(c) (ii)****5 marks****Hit or miss**

Translation  $(2, -1) \rightarrow (-7, 11) \Rightarrow (-9, 12)$  maps  $B(-4, 7) \rightarrow C(-4 - 9, 7 + 12) = C(-13, 19)$ .

\* Accept correct answer without work.

Award only these marks in section (ii)

5 marks Correct answer.

Otherwise 0 marks.

**(c) (iii)****5 marks****Hit or miss**

$A(2, -1)$  and  $C(-13, 19)$

$$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-13 - 2)^2 + (19 + 1)^2} = \sqrt{225 + 400} = \sqrt{625} = 25$$

$$|AB| : |AC| = 10 : 25 = [2 : 5].$$

Award only these marks in section (iii)

5 marks Correct answer.

Otherwise 0 marks

*Blunders (-3)*

B1 Incorrect formula, e.g.  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$  and continues.

B2 Mathematical error, e.g. Incorrect use of  $\sqrt{\quad}$

B3 Stops at  $\sqrt{100}$  in part c(i).

*Attempts (3 marks)*

A1 Oversimplifies formula, e.g.  $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$  with some correct substitution.

A2 Correct answer without work in c (i).

### QUESTION 3

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (15, 5) marks</b>	<b>Att (5, 2)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

A circle with centre (0, 0) passes through the point (5, -12).

- (i) Find the radius of the circle.
- (ii) Write down the equation of the circle.

**(a) (i)** **5 marks** **Att 2**

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 0)^2 + (-12 - 0)^2} = \sqrt{25 + 144} = \sqrt{169} \downarrow_{2 \text{ marks}} = 13.$$

*or*

$$x^2 + y^2 = r^2 \downarrow_{3 \text{ marks}} \Rightarrow (5)^2 + (-12)^2 = r^2 \Rightarrow 25 + 144 = r^2 \Rightarrow r^2 = 169 \downarrow_{2 \text{ marks}} \Rightarrow r = 13$$

\* Accept correct answer without work.

**(a) (ii)** **5 marks** **Att 2**

$$x^2 + y^2 = 13^2 = 169.$$

\* Accept answer based on candidates work in a (i).

\* Accept correct answer without work.

#### *Blunders (-3)*

- B1 Incorrect relevant formula with substitution, e.g.  $x^2 - y^2 = r^2$  (ii).
- B2 Leaves radius as  $\sqrt{169}$  (i).
- B3 Mathematical error.
- B4 No squares or misuse of Pythagoras.
- B5  $x^2 + y^2 = 13$  or 26 (ii).
- B6 Incorrect or inconsistent centre or radius, e.g. centre = (5, -12).

#### *Attempts (2 marks)*

- A1 A formula with  $(x_2 - x_1)$  or  $(y_2 - y_1)$  and some correct substitution.
- A2 Statement of reference to or some use of Pythagoras, e.g.  $5^2$ .
- A3 Any work of merit, e.g. plots (5, -12) correctly, (x or y-axis scaled).
- A4 Writes  $x^2 + y^2 = r^2$ .
- A5 Statement that radius is the distance from centre to point (5, -12).
- A6 Plots and identifies (0, 0) and (5, -12). (i).

#### *Worthless (0 marks)*

- W1  $5 + 12 = 17$  (i)

Part (b)

20 (15, 5) marks

Att (5, 2)

The circle  $c$  has equation  $x^2 + y^2 = 17$ .

$l$  is the line  $x - 4y - 17 = 0$ .

The line  $l$  is a tangent to  $c$  at the point  $T$ .

- (i) Find the co-ordinates of  $T$ .  
(ii) The point  $T$  is one end-point of a diameter of  $c$ .  
Find the co-ordinates of the other end-point.

(b) (i)

15 marks

Att 5

**I**

$$x - 4y - 17 = 0 \Rightarrow x = 4y + 17 \rightarrow x^2 + y^2 = 17 \Rightarrow (4y + 17)^2 + y^2 = 17$$

$$\Rightarrow 16y^2 + 136y + 289 + y^2 - 17 = 0 \Rightarrow 17y^2 + 136y + 272 \Rightarrow y^2 + 8y + 16$$

$$\Rightarrow (y + 4)^2 = 0 \Rightarrow y = -4 \rightarrow x = 4y + 17 \Rightarrow x = 4(-4) + 17 = -16 + 17 = 1 \quad [\text{Pt}(1, -4)]$$

*or*

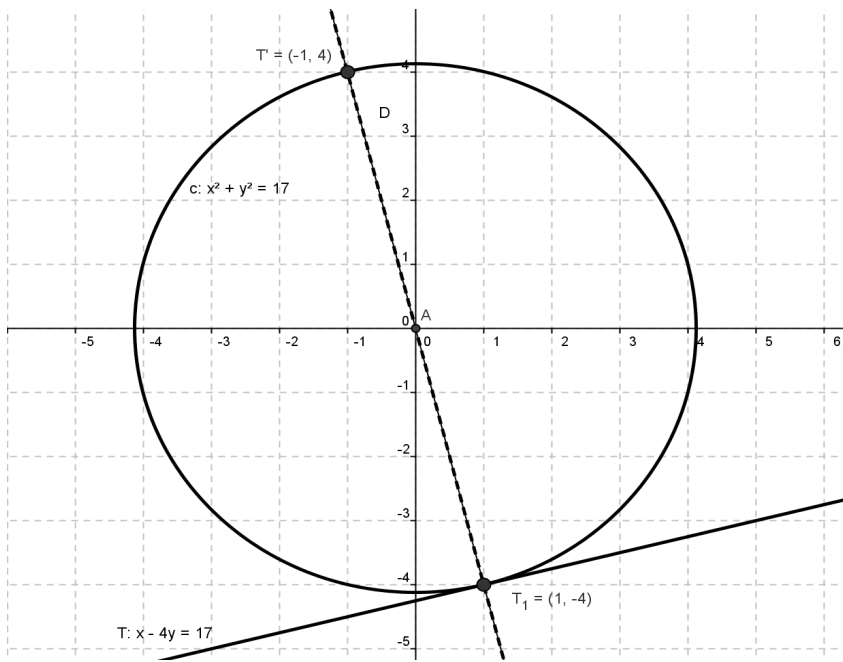
$$y = \frac{-136 \pm \sqrt{18496 - 18496}}{34} = \frac{-136}{34} = -4 \Rightarrow x = 4(-4) + 17 = -16 + 17 = 1 \quad [\text{Pt}(1, -4)]$$

**II**

$$xx_1 + yy_1 = r^2 \Rightarrow (1)x - 4y = 17 \Rightarrow x_1 = 1 : y_1 = -4$$

\* Accept the correct point verified correctly in **both** line and circle equations.

### Case III



$$m_l = \frac{1}{4}$$

$$\Rightarrow m_{\text{radius}} = -4$$

Equation of radius

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 0 = -4(x - 0)$$

$$\Rightarrow 4x + y = 0$$

Solve

$$4x + y = 0 \quad \times 4$$

$$\underline{x - 4y = 17}$$

$$16x + 4y = 0$$

$$\underline{x - 4y = 17}$$

$$17x = 17$$

$$\Rightarrow x = 1$$

$$\Rightarrow 4(1) + y = 0$$

$$\Rightarrow y = -4$$

$$[T = (1, -4)]$$

**Note:**

Correct point without work merits **5 marks**.

Award only these marks in section (b)(i):

15 marks: Correct answer with work.

5 marks: Some work of merit, otherwise 0 marks.

*Attempts (5)*

A1 States centre  $(0, 0)$  or  $r = \sqrt{17}$  or  $r^2 = 17$ .

A2 Correct answer without work.

A3 Correct graphical solution. [Note\*]

A4 Any correct relevant work. e.g.  $x - 4y = 17$ .

A5 Plots  $l$  and/or  $c$  and stops.

A6 Finds only one co-ordinate with work.



**(b) (ii)**

**5 marks**

**Att 2**

$$(1, -4) \rightarrow (0, 0) \rightarrow (-1, 4)$$

*or*

$$\frac{1+x}{2}=0; \frac{-4+y}{2}=0 \Rightarrow x=-1, y=4 \rightarrow (-1, 4)$$

- \* Accept a correct answer without work shown.
- \* Accept answer consistent with work in b(i).

*Blunders (-3)*

- B1 Error in use of translation or central symmetry.
- B2 Find only one co-ordinate with work.
- B3 Incorrect relevant formula and continues.
- B4 Transposing error.

*Slips (-1)*

- S1 Numerical slips to a maximum of 3.

*Attempts (2 marks)*

- A1 Any correct relevant work e.g. (0, 0) mentioned or attempt at translation or central symmetry.

**Part (c)**

**20 (10, 10) marks**

**Att (3, 3)**

A circle has equation  $x^2 + (y-7)^2 = 100$ .

- (i) Write down the co-ordinates of the centre of the circle and the radius of the circle.
- (ii) The point (6,  $h$ ) is on the circle. Find the two possible values of  $h$ .

**c (i)**

**10 marks**

**Att 3**

$$x^2 + (y-7)^2 = 100.$$

Centre (0, 7) or  $h = 0, k = 7$ .

$$\text{Radius } \sqrt{100} = 10$$

*or*

$$x^2 + (y-7)^2 = 100 \Rightarrow x^2 + y^2 - 14y + 49 = 100 \Rightarrow x^2 + y^2 - 14y - 51 = 0.$$

Centre  $(-g, -f) = (0, 7)$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{0 + 49 + 51} = 10.$$

- \* Accept fully correct answer without work shown.

Award only these marks in section (c)(i)

10 marks Correct centre **and** radius.

3 marks Correct centre **or** radius, otherwise 0 marks.

(c) (ii)

10 marks

Att 3

$$x^2 + (y-7)^2 = 100 \Rightarrow 6^2 + (h-7)^2 = 100$$

$$\Rightarrow (h-7)^2 = 100 - 36 = 64 \Rightarrow h-7 = \pm 8$$

$$\Rightarrow h = 7 + 8 = 15 \quad \text{or} \quad h = 7 - 8 = -1.$$

or

$$x^2 + (y-7)^2 = 100 \Rightarrow 6^2 + (h-7)^2 = 100$$

$$\Rightarrow 36 + h^2 - 14h + 49 - 100 = 0 \Rightarrow h^2 - 14h - 15 = 0 \Rightarrow (h-15)(h+1) = 0$$

$$\Rightarrow h = 15 \quad \text{or} \quad h = -1.$$

or

$$\text{Centre} = (0, 7); \text{radius} = 10; \text{point} = (6, h) \Rightarrow \sqrt{(6-0)^2 + (h-7)^2} = 10$$

$$\Rightarrow h^2 - 14h + 85 = 100$$

$$\Rightarrow h^2 - 14h - 15 = 0 \Rightarrow (h-15)(h+1) = 0 \Rightarrow h = 15 \quad \text{or} \quad h = -1$$

\* Accept (6, -1) and (6, 15) verified correctly into the equation of the circle.

Award only these marks in section (c)(i):

10 marks Both values of  $h$  fully correct from correct work.

3 marks Any work of merit, otherwise 0 marks.

*Attempts (3 marks)*

A1 Any work of merit, e.g.  $(6)^2 = 36$ .

A2 States: centre = (0, 0) or radius = 10.

A3 One or both values of  $h$  without work or only one tested.

*Worthless (0)*

W1 Correct formula transcribed from tables and stops.

## QUESTION 4

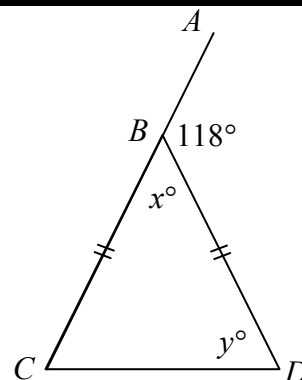
<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
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In the diagram,  
 $|BC| = |BD|$  and  $|\angle ABD| = 118^\circ$ .

(i) Find  $x$ .

(ii) Find  $y$ .



<b>(a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>
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$$x^\circ = 180^\circ - 118^\circ = 62^\circ.$$

<b>(a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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$$|\angle CDB| + |\angle BCD| = 118^\circ$$

$$|\angle CDB| = |\angle BCD| = y$$

$$2y^\circ = 118^\circ \Rightarrow y^\circ = 59^\circ$$

- \* Accept correct answer without work shown.
- \* Accept candidate's value for  $x$  in finding  $y$ .
- \* Accept (a) (ii) answered prior to (a) (i).

### Blunders (-3)

- B1 Incorrect geometrical result e.g. sum of three angles  $\neq 180^\circ$ .
- B2 Transposing error, e.g.  $2y = 118 \Rightarrow y = 116$ .

### Slips (-1)

- S1 Each numerical slip to a maximum of 3.

### Attempts (2 marks each section.)

- A1 Some relevant step or statement, e.g. unmarked angle in triangle =  $y$  and stops.
- A2  $x + y + y = 180$  or similar and stops.
- A3  $y = x$  without finding values and stops, (each section).

### Worthless (0 marks)

- W1 Incorrect answer without work.

Prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal

(b)

20 marks

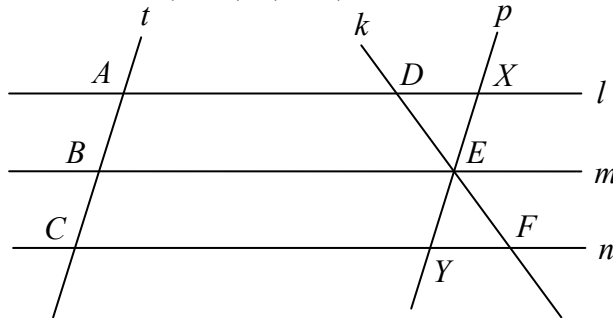
Att 7

$l, m, n$  are three parallel lines.

$t$  is a transversal intersecting the parallel lines such that  $|AB| = |BC|$ .

$k$  is a transversal intersecting  $l, m, n$  at  $D, E$  and  $F$  respectively.

To Prove:  $|DE| = |EF|$



Construction:

Through E, draw the line  $p \parallel t$  intersecting  $l$  at  $X$  and  $n$  at  $Y$  [7marks]

Proof:

$ABEX$  is a parallelogram  $\Rightarrow |AB| = |XE|$  [10 marks]

$BCYE$  is a parallelogram  $\Rightarrow |BC| = |EY|$  [13 marks]

But  $|AB| = |BC|$ , hence  $|XE| = |EY|$

In  $\triangle DEX$  and  $\triangle EYF$

$|\angle DEX| = |\angle FEY|$  ... vertically opposite } [16 marks]

$|\angle DXE| = |\angle EYF|$  ... alternate

$|XE| = |EY|$

Hence,  $\triangle DEX$  and  $\triangle EYF$  congruent [19 marks]

Hence,  $|DE| = |EF|$  [20 marks]

or

**Other valid proofs apply and are acceptable.**

\* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

*Blunders (-3)*

B1 Each step omitted, incorrect or incomplete, except the last.

B2 Steps written in an illogical order. [Penalise once only.]

[Note: Some of the steps above may be interchanged.]

*Attempts (7 marks)*

A1 Any relevant step, stated or indicated, i.e. (minimum 3 parallel lines + 1 transversal).

A2 States or illustrates a special case, e.g. transversals parallel.

*Worthless (0 marks)*

W1 Any irrelevant theorem, subject to the attempt mark.

- (i) Draw a square  $OABC$  with side 4 cm and label the vertices.  
(ii) Draw the image of the square under the enlargement with centre  $O$  and scale factor 2.5.  
(iii) Calculate the ratio  
area of image square : area of original square.  
(iv) Another square,  $PQRS$ , is the image of the square  $OABC$   
under a different enlargement with centre  $O$ .  
The area of  $PQRS$  is  $324 \text{ cm}^2$ .  
Calculate the scale factor of this enlargement.

(c) (i)

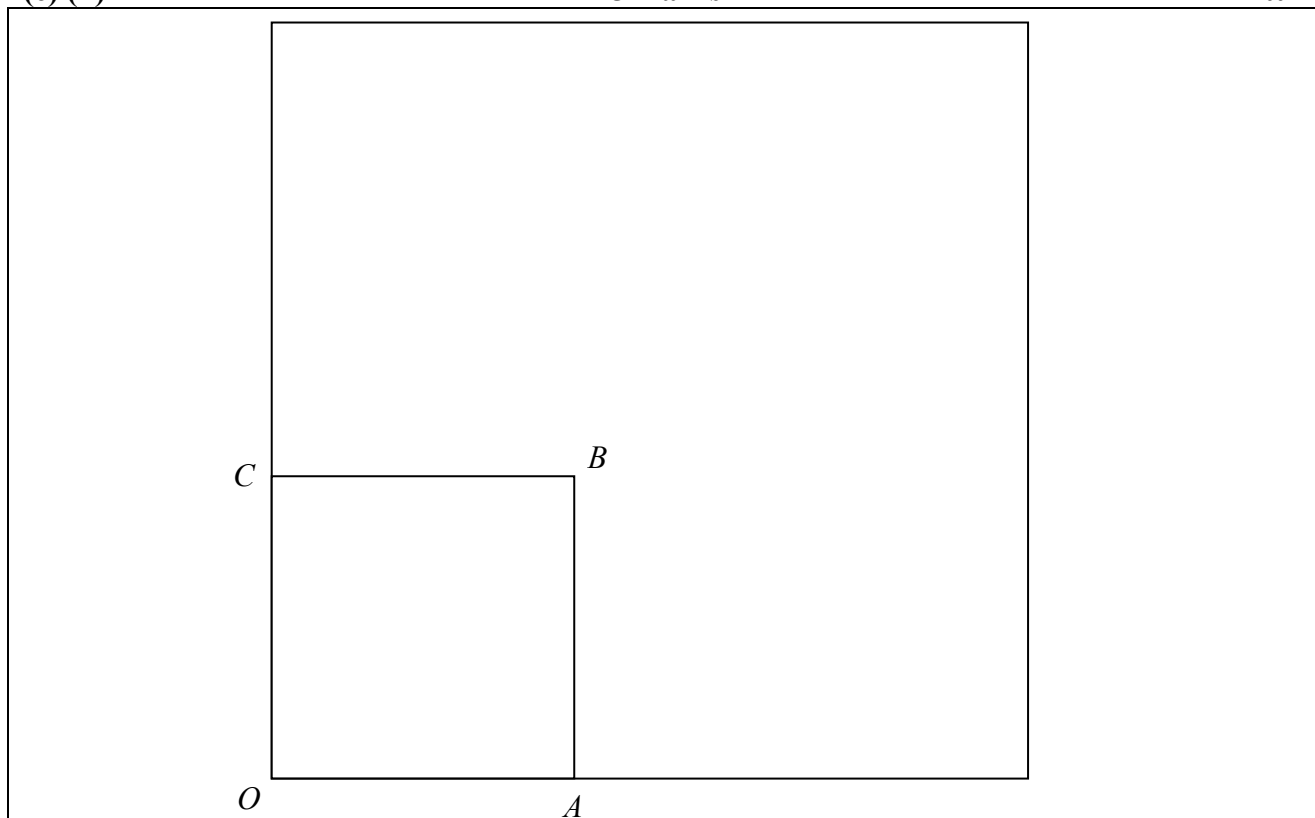
5 marks

Att 2

(c) (ii)

5 marks

Att 2



- \* Accept a quadrilateral with sides within  $\pm 0.5 \text{ cm}$  and angles within  $\pm 5^\circ$ .
- \* Accept image based on candidate's diagram from c (i).
- \* Do not demand labels on the vertices of the image.

*Slips (-1)*

- S1 Sides not straight (no straight edge) but measurements within tolerance.
- S2 Square but with measure of sides outside tolerance.
- S3 Does not label all vertices or labels vertices incorrectly e.g. order not cyclic.
- S4 A rectangle with one pair of sides within tolerance.

Ref:  
Section (i)

*Attempts (2 marks)*

- A1 Some relevant step, e.g. draws a line segment, within tolerance, even if freehand.
- A2 Indicates some knowledge of a square.
- A3 A rectangle or quadrilateral with no side within tolerance.

Award marks for section (ii) as follows:

5 marks: Image is correct size, correct location.

2 marks: Image is correct size, but incorrect location,

or

incorrect size but correct location

or

work of merit worthy of an attempt mark..

**(c) (iii)**

**5 marks**

**Att 2**

Image : Original =  $10^2 : 4^2 = 100 : 16$  or  $25 : 4$ .

\* Accept a correct answer without work shown.

*Blunders (-3)*

B1 Mathematical blunder, e.g.  $(0.25)^2 = 0.5$ .

B2 Ratio as  $4 : 25$  or  $16 : 100$ .

*Attempts (2 marks)*

A1 Correct relevant formula, not transcribed from the tables.

A2 16 or 100 with or without work.

A3 Some work of merit.

**(c) (iv)**

**5 marks**

**Att 2**

Area  $OABC = 16$ .

$$16k^2 = 324 \Rightarrow k^2 = 20.25 \Rightarrow k = 4.5 .$$

*or*

$$\sqrt{324} = 18$$

$$18 \div 4 = 4.5 , \text{ the scale factor.}$$

\* Accept correct answer without work.

*Blunders (-3)*

B1 Misplaced decimal point or leaves as  $\sqrt{20 \cdot 25}$  and stops.

B2 Mathematical error.

*Attempts (2 marks)*

A1 Work of merit, e.g. side of image = 18.

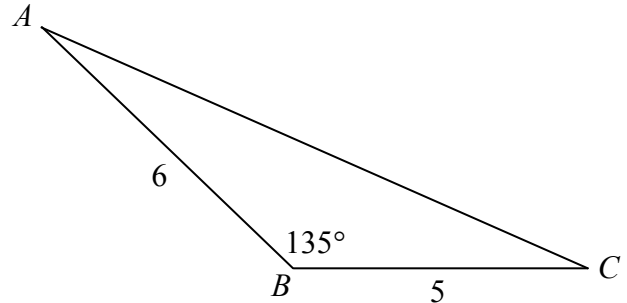
## QUESTION 5

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	25 (5, 5, 5, 10) marks	Att (2, 2, 2, -)
<b>Part (c)</b>	15 (5, 10) marks	Att (-, -)

**Part (a)** **10 marks** **Att 3**

In the triangle  $ABC$ ,  
 $|AB| = 6 \text{ cm}$ ,  $|BC| = 5 \text{ cm}$   
 and  $|\angle ABC| = 135^\circ$ .

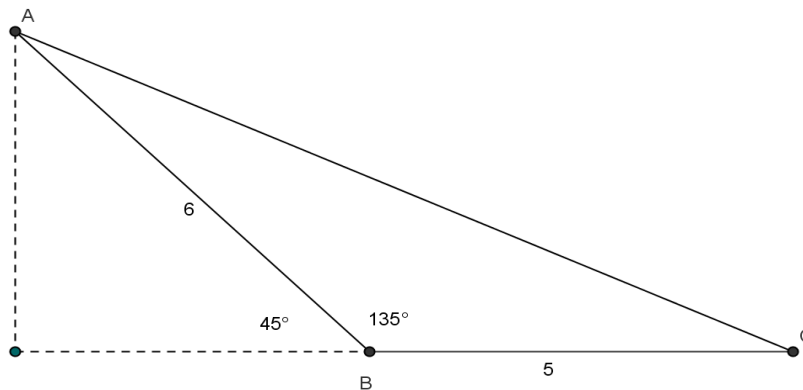
Calculate the area of the triangle,  
 correct to the nearest square centimetre.



**(a)** **10 marks** **Att 3**

Area =  $\frac{1}{2}ac \sin B = \frac{1}{2}(5)(6) \sin 135^\circ = 15(0.7071) = 10.6 = 11 \text{ cm}^2$

or



Area =  $\frac{1}{2} \text{base} \times \text{perp. height} = \frac{1}{2}(5)(6) \sin 45^\circ = 15(0.7071) = 10.6 = 11 \text{ cm}^2$

### Blunders (-3)

- B1  $\frac{1}{2}a \sin 135^\circ$  fully worked.
- B2 Uses radians (or gradient) mode incorrectly – apply once in each part in which it occurs.
- B3 Incorrect area formula.
- B4 Incorrect ratio and continues.
- B5 Incorrect trigonometric function and continues.
- B6 Incorrect function read, e.g. reads  $\cos \theta$  instead of  $\sin \theta$ .
- B7 Misplaced decimal point.
- B8 Incorrect substitution into correct formula and continues.

### Slips (-1)

- S1 Each numerical slip to a maximum of 3.
- S2 Early round off that affects the accuracy of the answer.

Attempts (3 marks)

- A1 Some correct substitution into incorrect relevant formula e.g.  $\frac{1}{2}a \sin C$ .  
 A2 Some correct use of sine rule or cosine rule. e.g.  $|AC| = 5^2 + 6^2 - 2(5)(6)\cos 135^\circ$   
 A3 Answer of  $10 \cdot 6$  or  $11$  without work shown.  
 A4 Area =  $\frac{1}{2} \times 5 \times 6$ , even if finished.

Worthless (0 marks)

- W1 Incorrect answer without work. e.g. "15 without work". Otherwise A4 may apply.  
 W2  $\tan C = \frac{6}{5}$  or  $|AC|^2 = 6^2 + 5^2$  or attempt at Pythagoras.

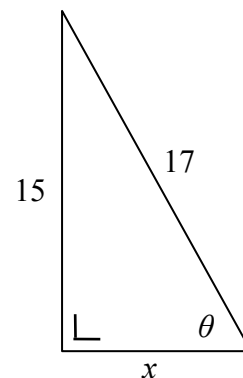
Part (b)

25 (5, 5, 5, 10) marks

Att (2, 2, 2, -)

Consider the right-angled triangle shown in the diagram.

- (i) Find the value of  $x$ .  
 (ii) Write down, as a fraction, the value of  $\sin \theta$ .  
 (iii) Write down, as a fraction, the value of  $\cos \theta$ .  
 (iv) Find the value of  $\sin 2\theta$ .



(b) (i)

5 marks

Att 2

$$x^2 + 15^2 = 17^2$$

$$\Rightarrow x^2 + 225 = 289 \Rightarrow x^2 = 289 - 225 \Rightarrow x^2 = 64 \Rightarrow x = \sqrt{64} \downarrow_{2 \text{ marks}} \Rightarrow x = 8$$

(b) (ii)

5 marks

Att 2

$$\sin \theta = \frac{15}{17}$$

(b) (iii)

5 marks

Att 2

$$\cos \theta = \frac{8}{17}$$

(b) (iv)

10 marks

Hit or miss

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{15}{17} \times \frac{8}{17} = \frac{240}{289}$$

or

$$\sin \theta = \frac{15}{17} \Rightarrow \theta = 61 \cdot 9275 \Rightarrow \sin 2(61 \cdot 9275) = \sin(123 \cdot 855) = 0 \cdot 83$$

or

$$\sin(\theta + \theta) = \sin \theta \cos \theta + \cos \theta \sin \theta = \left(\frac{15}{17} \times \frac{8}{17}\right) + \left(\frac{8}{17} \times \frac{15}{17}\right) = \left(\frac{120}{289}\right) + \left(\frac{120}{289}\right) = \frac{240}{289}$$

- \* Accept answers consistent with candidate's work in previous section, (ii) & (iii).
- \* Accept answer in decimal form for part (iv).
- \* Accept correct or consistent answers without work.



Award only these marks in section (b) (iv):

10 marks: Correct or consistent answer, otherwise 0 marks.

*Blunders (-3) (Apply to sections (i), (ii), (iii))*

- B1 Error in the use of Pythagoras, e.g.  $15^2 = 17^2 + x^2$ .
- B2 Incorrect use of Pythagoras.
- B3 Error in squaring, e.g.  $17^2 = 34$ .
- B4 Incorrect ratio, (each section (i), (ii), (iii)).
- B5 Incorrect trigonometric function and continues.
- B6 Incorrect function read, e.g. reads  $\cos \theta$  instead of  $\sin \theta$ .
- B7 Error in the manipulation of fractions.
- B8 Error in the use of inverse function.

*Misreadings (-1)*

- M1  $\tan \theta = \frac{15}{8}$  (ii) or (iii).

*Slips (-1) (Apply to sections (i), (ii), (iii))*

- S1 Each numerical slip to a maximum of 3.
- S2 Answers in decimal form. ( $\theta = 61.92^\circ$ ), most likely (ii), (iii), (each section).

*Attempts (2 marks) (Apply to sections (i), (ii), (iii))*

- A1 States the theorem of Pythagoras. e.g.  $h^2 = o^2 + a^2$ .
- A2 Some correct use of sine rule or cosine rule, e.g.  $\theta = 61.92^\circ$  and stops (i).
- A3 An exact scaled diagram giving correct answer
- A4 Numerator correct or denominator correct or correct ratio inverted (each section), e.g.  $\tan \theta = \frac{15}{x}$
- A5 Any correct relevant step, e.g.  $17^2 = 289$ ,  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

*Worthless (0 marks)*

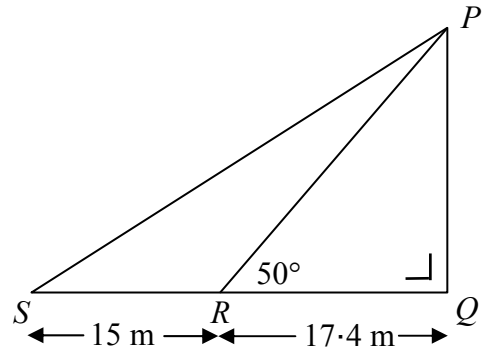
- W1 Incorrect answer without work.
- W2  $\sin 2\theta = 2 \sin \theta$  and stops.
- W3  $17 + 15 = 32$  or  $x + 15 = 17$ .
- W4 Measurement from the diagram. [2.5 cm]

## Part (c)

15 (5, 10) marks

Att (-, -)

A vertical mast  $[PQ]$  is supported by two straight cables  $[PS]$  and  $[PR]$  as shown. The cables are joined to level ground at  $S$  and  $R$  where  $|SR| = 15$  m,  $|RQ| = 17.4$  m and  $|\angle PRQ| = 50^\circ$ .



- (i) Find  $|PR|$ , correct to the nearest metre.  
 (ii) Find  $|PS|$ , correct to the nearest metre.

(c) (i)

5 marks

Att (-, -)

$$\cos 50^\circ = \frac{17.4}{|PR|} \Rightarrow |PR| = \frac{17.4}{\cos 50^\circ} = \frac{17.4}{0.6428} = 27.06 = 27 \text{ m.}$$

or

$$\sin 40^\circ = \frac{17.4}{|PR|} \Rightarrow |PR| = \frac{17.4}{\sin 40^\circ} = \frac{17.4}{0.6428} = 27.06 = 27 \text{ m}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{|PR|}{\sin 90^\circ} = \frac{17.4}{\sin 40^\circ} \Rightarrow |PR| = \frac{17.4}{0.6428} = 27.06 = 27 \text{ m}$$

Award only these marks in section (c) (i)

5 marks: Answer = 27 m.

4 marks: One slip or failure to round off to the required level of accuracy, otherwise 0 marks.

(c) (ii)

10 marks

Att (-, -)

$$a^2 = b^2 + c^2 - 2bc \cos A \Rightarrow |PS|^2 = 15^2 + 27^2 - 2(15)(27) \cos 130^\circ$$

$$\Rightarrow |PS|^2 = 225 + 729 - 810(-0.64279) = 954 + 520.66 = 1474.66 \Rightarrow |PS| = 38.4 = 38 \text{ m}$$

or

$$\tan 50^\circ = \frac{|PQ|}{17.4} \Rightarrow |PQ| = 17.4 \tan 50^\circ = 17.4(1.1918) = 20.73 = 20.7$$

$$|PS|^2 = |SQ|^2 + |PQ|^2 = 32.4^2 + 20.7^2 = 1049.76 + 428.49 = 1478.25$$

$$\Rightarrow |PS| = 38.44 = 38 \text{ m}$$

or

$$|PQ|^2 = (27)^2 - (17.4)^2 \Rightarrow |PQ|^2 = 729 - 302.76 \Rightarrow |PQ|^2 = 426.24 \Rightarrow |PQ| = \sqrt{426.24} = 20.65$$

$$|PS|^2 = (32.4)^2 + (20.65)^2 \Rightarrow |PS|^2 = 1049.76 + 426.42 \Rightarrow |PS|^2 = 1476.18$$

$$\Rightarrow |PS| = \sqrt{1476.18} = 38.42 = 38 \text{ m}$$

Award only these marks in section (c) (ii):

10 marks: Correct or consistent answer.

9 marks: One slip or failure to round off to the required level of accuracy, otherwise 0 marks.

Worthless (0 marks)

W1 Measurement from the diagram. [4.5 cm] / [6.5 cm]

W2 Treats triangle  $SPR$  as right angled triangle or uses sine rule inappropriately in (ii) Note: A2. (Subject to marks already secured).

## QUESTION 6

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

- (i) In how many different ways can a committee of four people be selected from ten people?  
 (ii) If one particular person must be on the committee, in how many different ways can the committee be selected?

**(a) Each section** **5 marks** **Att 2**

<b>(i)</b>	$\binom{10}{4} = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} \downarrow_{2 \text{ marks}} = 210.$
<b>(ii)</b>	$\binom{9}{3} = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} \downarrow_{2 \text{ marks}} = 84$

\* Accept a correct answer without work shown in each section.

### Blunders (-3)

- B1 Treats combination as a permutation. – once in (a).  
 B2 Blunder in evaluating or expanding term.  
 B3 Incorrect  $\binom{9}{3}$  in (ii).

### Attempts (2 marks)

- A1 Attempt at expanding term.  
 A2  $\binom{n}{r}$  with either  $n$  or  $r$  correct or  $\frac{n}{r}$  with either  $n$  or  $r$  correct, (each section).  
 A3 Leaves answer as  $\frac{10!}{4! \times 6!}$  or  $\frac{9!}{3! \times 6!}$ .  
 A4 10!, 4!, 6! in (i).  
 A5 9!, 3!, 6!, 9 or 3 (ii).

### Worthless (0 marks)

- W1 Incorrect answer without work shown e.g. writes 10 – 4 and stops.

Tickets for a raffle are placed in a box. The box contains 15 blue tickets and 10 red tickets. Tickets are drawn at random from the box and they are not replaced.

What is the probability that

- (i) the first ticket drawn is red
- (ii) the first ticket drawn and the second ticket drawn are both red
- (iii) the first ticket drawn is red and the second ticket drawn is blue
- (iv) the first two tickets drawn are different in colour?

**(b) Each section****5 marks****Att 2**

<b>(i)</b>	$P(\text{first ticked red}) = \frac{10}{25} \text{ or } \frac{2}{5} \text{ or } (0.4) \text{ or } 40\%.$ <p><i>or</i></p> $\frac{\binom{10}{1} \downarrow_{2 \text{ marks}}}{\binom{25}{1} \downarrow_{2 \text{ marks}}} = \frac{2}{5} \text{ or } (0.4) \text{ or } 40\%$
<b>(ii)</b>	$P(\text{first, second ticket red}) = \frac{10}{25} \times \frac{9}{24} \downarrow_{2 \text{ marks}} = \frac{90}{600} \text{ or } \frac{3}{20} \text{ or } (0.15) \text{ or } 15\%.$ <p><i>or</i></p> $\frac{\binom{10}{2} \downarrow_{2 \text{ marks}}}{\binom{25}{2} \downarrow_{2 \text{ marks}}} = \frac{3}{20} \text{ or } (0.15) \text{ or } 15\%$
<b>(iii)</b>	$P(\text{first red, second blue}) = \frac{10}{25} \times \frac{15}{24} \downarrow_{2 \text{ marks}} = \frac{150}{600} \text{ or } \frac{1}{4} \text{ or } (0.25) \text{ or } 25\%.$ <p><i>or</i></p> $\frac{\binom{10}{1} \downarrow_{2 \text{ marks}} \times \binom{15}{1}}{\binom{25}{1} \downarrow_{2 \text{ marks}} \times \binom{24}{1}} = \frac{150}{600} \text{ or } \frac{1}{4} \text{ or } (0.25) \text{ or } 25\%$
<b>(iv)</b>	<p>P(two different colours)</p> $= 2 \times \frac{10}{25} \times \frac{15}{24} \downarrow_{2 \text{ marks}} = 2 \times \frac{150}{600} \downarrow_{2 \text{ marks}} = \frac{300}{600} \text{ or } \frac{1}{2} \text{ or } (0.5) \text{ or } 50\% \text{ or } 2 \text{ [Ans(iii)]}$ <p><i>or</i></p> $\frac{\binom{10}{1} \downarrow_{2 \text{ marks}} \times \binom{15}{1}}{\binom{25}{2} \downarrow_{2 \text{ marks}}} = \frac{1}{2} \text{ or } (0.5) \text{ or } 50\%$

\* If the sections of (b) or of (c) are not identified, and it is not obvious which section is being attempted treat each section in order.

\* Accept answers consistent with previous work, e.g #S (accept decimal and percentage form).

\* Award 5 marks for each correct answer without work shown.

*Blunders (-3)*

B1 Addition used instead of multiplication, (each section).

B2 Answers in format  $\binom{n}{r}$ , e.g.  $\binom{10}{25}$  (Apply once only in part b).

*Attempts (2 marks)*

A1 #E correctly identified or given as the numerator or on its own.

#S correctly identified or given as the denominator or on its own.

A2 The correct answer inverted each time or partial correct answer e.g.  $\frac{1}{24}$  in (ii).

A3 Statement of probability theorem awarded once unless specifically adapted to each section.

<i>Attempts (2 marks)</i>			
<b>Section (i)</b>	<b>Section (ii)</b>	<b>Section (iii)</b>	<b>Section (iv)</b>
#E:10	#E: 10,9,90, $\binom{10}{2}$ or 45,	#E: 10,15,150, $\binom{10}{1}$ , $\binom{15}{1}$	#E: 2,10,15, 300, 150, $\binom{10}{1}$ , $\binom{15}{1}$
#S: 25	#S: 25, 24, 600, $\binom{25}{2}$ or 300,	#S: 25, 24, 600, $\binom{25}{1}$ , $\binom{24}{1}$	#S: 25, 24, 600, 300, $\binom{25}{2}$

*Worthless (0 marks)*

W1 Incorrect answer without work shown.(subject to attempts).

A code consists of a four-digit number which is formed from the digits 3 to 9 inclusive.  
No digit can occur more than once in the code.

- (i) Write down the smallest possible four-digit code.  
 (ii) How many different codes are possible?  
 (iii) How many of the four-digit codes are greater than 6000?  
 (iv) How many of the four-digit codes are divisible by 2?

(c) Each section

5 marks

Att 2

(i)	3456	
(ii)	${}^7P_4 = 7 \times 6 \times 5 \times 4 \downarrow_{2 \text{ marks}} = 840$	or $\frac{7!}{3!} \downarrow_{2 \text{ marks}} = 840$
(iii)	$4 \times 6 \times 5 \times 4 \downarrow_{2 \text{ marks}} = 480$	or $4({}^6P_3) \downarrow_{2 \text{ marks}} = 4 \left( \frac{6!}{3!} \right) \downarrow_{2 \text{ marks}} = 480$
(iv)	$6 \times 5 \times 4 \times 3 \downarrow_{2 \text{ marks}} = 360.$	or $3({}^6P_3) \downarrow_{2 \text{ marks}} = 3 \left( \frac{6!}{3!} \right) \downarrow_{2 \text{ marks}} = 360.$

- \* Accept correct answer without work.  
 \* Ans (i) as  $3 \times 4 \times 5 \times 6$  merits 2 marks.

Award marks as follows, in each section:

5 marks: Fully correct answer with or without work.

4 marks: Special Case. Where the codes divisible by 2 refer to the codes greater than 6000.

2 marks: Correct answer given as a list with multiplication clearly indicated but not worked or addition used instead of multiplication  
 or an incorrect answer with a list of digits with at least three of them correct and in consecutive order.(i)  
 or a partial list with three correct digits but not more than five. (ii), (iii), (iv).  
 or  ${}^7P_4$  (ii),  $({}^6P_3)$  or similar (iii),  $({}^6P_3)$  or similar (iv).

0 marks: Incorrect answer without work shown or worthless work  
 or a list of codes or answers from those codes.

<b>Special Case</b>	Reads c(iv) as referring to four digit codes greater than 6000 that are divisible by 2
(iv)	$(4 \times 5 \times 4 \times 1) + (3 \times 5 \times 4) \downarrow_{2 \text{ marks}} + (3 \times 5 \times 4) \downarrow_{2 \text{ marks}} = 80 + 60 + 60 = 200 \downarrow_{4 \text{ marks}}$

## QUESTION 7

**Part (a)** 30 (10, 5, 15) marks Att (3, 2, 5)  
**Part (b)** 20 (10, 5, 5) marks Att (3, 2, 2)

**Part (a)** 30 (10, 5, 15) marks Att (3, 2, 5)

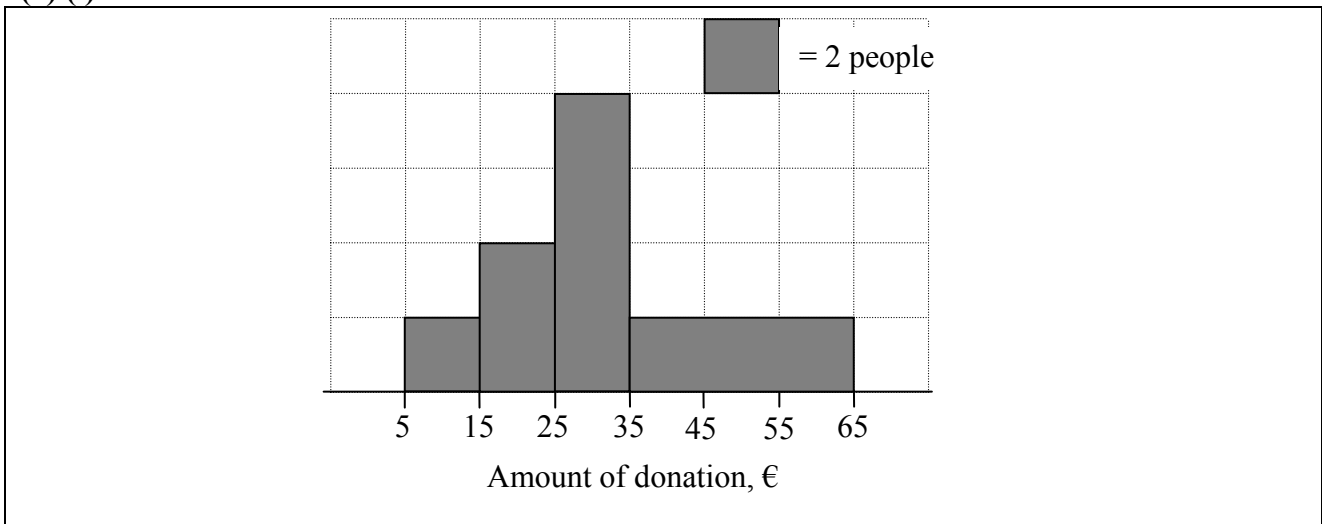
The following table gives the distribution of donations, in euro, made by 20 people to an appeal fund:

Amount of donation, €	5 - 15	15 - 25	25 - 35	35 - 65
Number of people	2	4	8	6

[Note: 5 - 15 means 5 or over but less than 15 etc.]

- (i) Draw a histogram to represent the data.
- (ii) Taking mid-interval values, calculate the mean amount donated.
- (iii) Taking mid-interval values, calculate the standard deviation, correct to one decimal place.

**(a) (i)** 10 marks Att 3



- \* Each rectangle may be blundered only once.
- \* Accept areas of rectangles proportional to frequencies, provided bases are correct.
- \* Do not penalise a space between 0 - 5 on the horizontal axis.
- \* Accept "amount" on vertical axis and "frequency" on horizontal axis.

*Award marks as follows:*

10 marks Correct histogram

7 marks Scale(s) incorrect, rectangles subsequently correct  
 or scales correct, one rectangle incorrect or omitted  
 or scales correct, rectangles correct but spaces put between rectangles.

4 marks Scale(s) incorrect, one rectangle incorrect or omitted  
 or scales correct, two rectangles incorrect or omitted  
 or scales correct, one rectangle incorrect or omitted and spaces between rectangles.  
 or scales incorrect, rectangle subsequently correct but spaces between the rectangles.

3 marks Attempt at answer as below.

Misreadings(-1)

M1 Obvious misreading.

Attempts (3 marks)

A1 Draws scaled horizontal axis and stops, even without labels, or draws a trend graph.

A2 Treats 0-2, 2-4, etc. as intervals and 15, 25, etc. as frequencies.

(a) (ii)

5 marks

Att 2

Mid-interval values: 10, 20, 30, 50

$$\text{Mean } \mu = \frac{10 \times 2 + 20 \times 4 + 30 \times 8 + 50 \times 6}{2 + 4 + 8 + 6} = \frac{20 + 80 + 240 + 300}{20} = \frac{640}{20} = 32$$

or

Interval	Mid-interval (x)	f	fx
5 – 15	10	2	20
15 – 25	20	4	80
25 – 35	30	8	240
35 – 65	50	6	300
		20	640

$$\text{Mean } \mu = \frac{\sum fx}{\sum f} = \frac{640}{20} = 32$$

\* Accept correct answer without work i.e. uses calculator.

\* One table may be used for both sections (a) (ii) and (a) (iii).

\* All answers must be consistent with written mid-interval values and frequency values, otherwise incorrect answer without work merits zero.

Award only these marks in section (a)(ii)

5 marks: Answer of 32.

4 marks: Answer of  $\frac{640}{20}$  or one slip or misreading.e.g. One incorrect mid-interval.

2 marks: Some work of merit.

0 marks Worthless work.

Attempts (2 marks)

A1 Some relevant step e.g. finds the median or modal class or draws a cumulative frequency curve.

A2 A correct multiplication and stops.

A3  $2 + 4 + 8 + 6$  and stops.

A4 One or more correct mid-interval values and stops.

A5 Accept a reasonable estimate of  $30 \leq \mu \leq 34$  without work.

Worthless(0)

W1 Incorrect answer no work, subject to marks already secured.



(a) (iii)

15 marks

Att 5

Deviations  $x - \mu$ :  $10 - 32 = -22$ ,  $20 - 32 = -12$ ,  $30 - 32 = -2$ ,  $50 - 32 = 18$ ,

$$\sigma = \sqrt{\frac{2(-22)^2 + 4(-12)^2 + 8(-2)^2 + 6(18)^2}{2 + 4 + 8 + 6}}$$

$$= \sqrt{\frac{968 + 576 + 32 + 1944}{20}} = \sqrt{\frac{3520}{20}} = \sqrt{176} = 13.26 = 13.3.$$

or

$x$	$f$	$fx$	$ x - \mu $	$(x - \mu)^2$	$f(x - \mu)^2$
10	2	20	22	484	968
20	4	80	12	144	576
30	8	240	2	4	32
50	6	300	18	324	1944
	20	640			3520

$$\sigma = \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}} = \sqrt{\frac{3520}{20}} = \sqrt{176} = 13.26 = 13.3.$$

- \* Accept correct or consistent answer without work i.e. uses calculator.
- \* If one table is used for both sections, see table in section (ii).
- \* Accept either positive or negative deviations.
- \* Accept candidates values from (ii) for (iii).
- \* The above table should be awarded 2 marks for section (ii) and 5 marks for section (iii) where no marks have been awarded in section (ii). If totals omitted award 2 +5 marks.

Award only these marks in section (a)(iii):

15 marks: Correct or consistent answer. Answer of 13.3

14 marks: Answer of  $\sqrt{176}$  or 13.26 or one slip or misreading, e.g. one incorrect deviation or candidate uses sample standard deviation on calculator (Answer 13.139).

5 marks: Some work of merit, e.g. 20 written without work in this section.

0 marks Worthless work, e.g. formula from Tables without further work.

Attempts (5 marks)

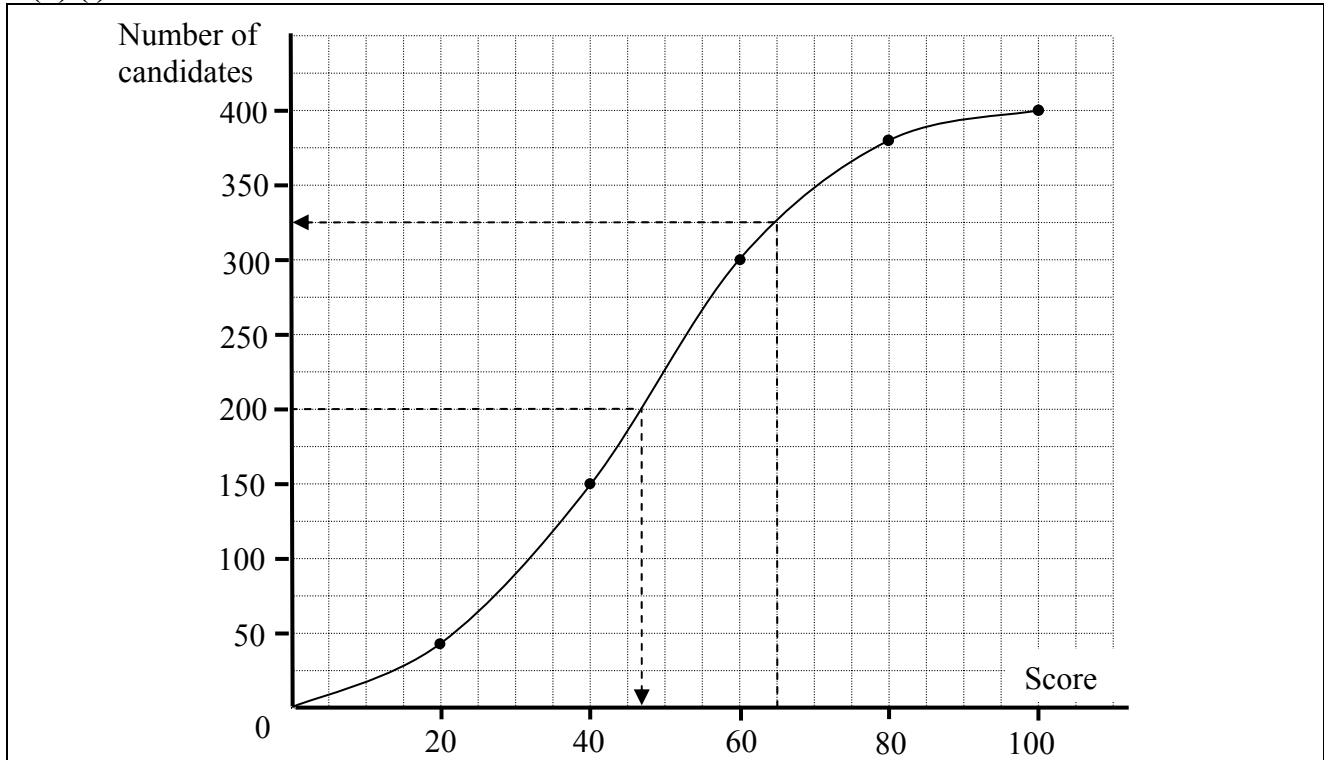
- A1 Any work of merit.
- A2 A correct multiplication and stops.
- A3 Any correct deviation.
- A4 Works with  $\sum f(x - \mu)$ .
- A5  $2 + 4 + 8 + 6$  and stops. (if done in this section).
- A6  $\sum f = 20$ , (if done in this section).
- A7 Accept a reasonable estimate,  $12 \leq \sigma \leq 14$  without work.

**Part (b)****20 (10, 5, 5) marks****Att (3, 2, 2)**

The cumulative frequency table below refers to the scores, in an aptitude test, of 400 candidates who applied for places on a particular course:

Score	$\leq 20$	$\leq 40$	$\leq 60$	$\leq 80$	$\leq 100$
Cumulative frequency	40	150	300	380	400

- (i) Draw the cumulative frequency curve.  
(ii) Use your curve to estimate the median score.  
(iii) Candidates who scored above 65 in the test were called for interview.  
Use your curve to estimate the number of candidates who were called for interview.

**(b) (i)****10 marks****Att 3**

\* Accept frequency on the horizontal.

*Blunders (-3)*

- B1 Scale irregular (apply once).  
B2 Draws a cumulative frequency polygon – apply slips also. [B1 may also apply]  
B3 Draws a cumulative cumulative curve or frequency curve – apply slips also. [B1 may also apply].  
B4 Plots the points above the mid-interval values.

*Slips (-1)*

- S1 Each point omitted or incorrectly plotted (to the eye). [B1 may also apply]  
S2 Each pair of points not joined – including (0, 0) to (20, 40).  
[Note: a point omitted may incur two penalties, S1 and S2, e.g. last point on curve]

*Attempts (3 marks)*

- A1 One correct step e.g. draws axes and stops.  
A2 Draws histogram correctly instead of ogive.  
A3 Makes a cumulative cumulative or frequency table.

**(b) (ii)**

**5 marks**

**Att 2**

Median score 47.

- \* If the candidate draws the correct lines on the graph obtaining the correct answer but does not write the value or writes as  $< 47$ , apply penalty of (-1).
- \* Accept answer based on candidates' graph, allowing tolerance of  $\pm 5$  in the candidate's score with or without work.

*Blunders (-3)*

B1 Starts on the incorrect axis – score 50 which equates to 225 candidates.

*Attempts (2 marks)*

A1 Divides 400 by 2 or candidate's total by 2 and stops.

A2 A line of merit drawn on the graph, allowing a tolerance of  $\pm 10$  candidates for the starting point, otherwise award 0.

A3 Some relevant statement about median.

**(b) (iii)**

**5 marks**

**Att 2**

Score  $> 65$ :

$$325 \Rightarrow 400 - 325 = 75$$

- \* Accept answer based on candidates' graph, allowing tolerance of  $\pm 10$  candidates.

*Blunders(-3)*

B1 Starts from incorrect axis (Answer =26).

B2 Fails to finish. (Answer =325).

*Slips(-1)*

S1 Leaves as  $400 - 325$ .

*Attempts (2 marks)*

A1 A line of merit drawn on the graph allowing a tolerance of  $\pm 5$  in the score for the starting point, otherwise award 0.

## QUESTION 8

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (-, -, -, -)</b>

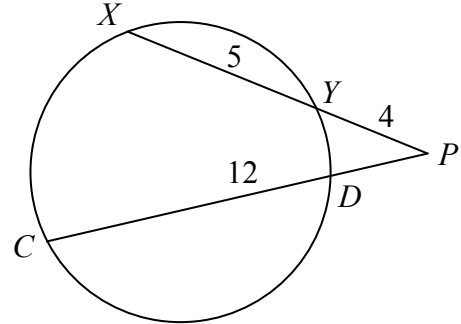
<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
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[XY] and [CD] are chords of a circle which, when produced, intersect at a point  $P$  outside the circle.

$|XY| = 5$ ,  $|YP| = 4$  and  $|CP| = 12$ .

(i) Find  $|XP|$ .

(ii) Find  $|PD|$ .



<b>(a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>
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$$|XP| = |XY| + |YP| = 5 + 4 = 9.$$

<b>(a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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$$|PC| \times |PD| = |PX| \times |PY| \Rightarrow 12 \times |PD| = 4 \times 9 = 36 \Rightarrow |PD| = 3.$$

- \* Accept correct answers without work or an answer clearly indicated on a diagram.
- \* In section (ii) accept candidate's answer from section (i).

### Blunders (-3)

B1 Incorrect operation in a (i), a (ii).

### Misreadings (-1)

M1  $(12+x)(x) = 9(4)$  and continues.

### Attempts (2 marks)

A1 Indicates  $|PD|$  on diagram or states  $|PD| = 12 - x$ .

A2 Geometrical result indicated on diagram or stated without numerical values.

A3 Work of some merit, e.g. begins a correct substitution into result, correct or otherwise.

A4 Addition used instead of multiplication in (ii).

### Worthless (0 marks)

W1 Incorrect answer without work shown.

Prove that a line is a tangent to a circle at a point  $T$  of the circle if and only if it passes through  $T$  and is perpendicular to the line through  $T$  and the centre.

(b)

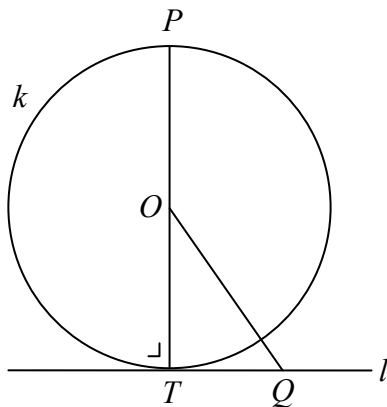
20 marks

Att 7

$[PT]$  is a diameter of circle  $k$ .

$l$  is a line such that  $PT \perp l$  at  $T$ .

To prove:  $l$  is a tangent to circle  $k$ .



Construction: Let  $Q$  be any other point of  $l$ .

Join  $OQ$ .

[7 marks]

Proof:

In  $\triangle OTQ$ ,  $|\angle OTQ| = 90^\circ$ .

Thus,  $[OQ]$  is the hypotenuse of  $\triangle OTQ$ .

[10 marks]

Hence,  $|OQ| > |OT|$

or  $|OQ| > \text{radius}$

[13 marks]

Thus  $Q$  is outside the circle.

[16 marks]

Thus, any other point of  $l$ , except  $t$  is outside the circle.

Thus, the line  $l$  meets the circle at only one point,  $T$ . [19 marks]

Hence, the line  $l$  is a tangent.

When the "only if" case presented as well.

[20

marks]

\* If candidates work is not worthless, Att 7 at least must be awarded.

*Blunders (-3)*

B1 Each step omitted, incorrect or incomplete (except the last two).

B2 Steps written in an illogical order. [Penalise once only.]

[Note: Some of the steps above may be interchanged.]

*Attempts (7 marks)*

A1 Outline diagram with/without  $OQ$  drawn. (Minimum required circle and tangent).

A2 Attempt at proof using special case.

A3 Memorised proof, without a diagram, if attempt can be reconciled with a diagram.

*Worthless (0 marks)*

W1 Any irrelevant theorem, subject to the attempt mark.

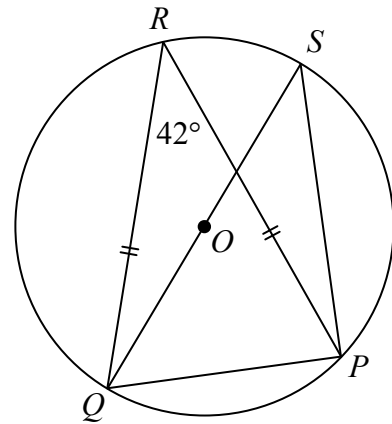
W2 Circle only.

**Part (c)****20 (5, 5, 5, 5) marks****Att (-, -, -, -)**

In the diagram,  $O$  is the centre of the circle  
and  $|\angle PRQ| = 42^\circ$ .

$[QS]$  is a diameter and  $|QR| = |PR|$ .

- (i) Find  $|\angle PSQ|$ .
- (ii) Find  $|\angle SQP|$ .
- (iii) Find  $|\angle QPR|$ .
- (iv) Find  $|\angle RQS|$ .

**(c) (i)****5 marks****Hit or miss**

$|\angle PSQ| = 42^\circ$  .... Angle on same arc.

**(c) (ii)****5 marks****Hit or miss**

$|\angle SQP| = 90^\circ - 42^\circ = 48^\circ$

**(c) (iii)****5 marks****Hit or miss**

$|\angle QPR| = \frac{1}{2}(180^\circ - 42^\circ) = \frac{1}{2}(138^\circ) = 69^\circ$  .... Isosceles triangle & Angles in triangle =  $180^\circ$

**(c) (iv)****5 marks****Hit or miss**

$|\angle RQS| = |\angle RQP| - |\angle SQP| = 69^\circ - 48^\circ = 21^\circ$

\* Accept answer written on a diagram in each section.

\* Accept correct or consistent answer without work in each section.

## QUESTION 9

<b>Part (a)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>
<b>Part (b)</b>	<b>20 (5, 10, 5) marks</b>	<b>Att (2, 3, 2)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **15 (10, 5) marks** **Att (3, 2)**

Let  $\vec{p} = 7\vec{i} + 9\vec{j}$  and  $\vec{q} = -2\vec{i} + 3\vec{j}$ .

(i) Express  $\vec{p} + \vec{q}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

(ii) Hence, calculate  $|\vec{p} + \vec{q}|$ .

**(a) (i)** **10 marks** **Att 3**

$$\vec{p} + \vec{q} = 7\vec{i} + 9\vec{j} - 2\vec{i} + 3\vec{j} \downarrow_{4\text{marks}} = 5\vec{i} \downarrow_{7\text{marks}} + 12\vec{j} \downarrow_{10\text{marks}}.$$

\* Accept correct answer without work.

*Blunders (-3)*

B1 Incorrect combination of components.

*Misreadings (-1)*

M1 Treats as  $\vec{p} - \vec{q}$ .

*Attempts (3 marks)*

A1  $5\vec{i}$  or  $12\vec{j}$  without work shown and stops.

A2 Some effort at combining components.

A3 Work of merit on a diagram e.g. plots one or more of the vectors.

*Worthless (0 marks)*

W1 Incorrect answer without work.

**(a) (ii)** **5 marks** **Att 2**

$$|\vec{p} + \vec{q}| = |5\vec{i} + 12\vec{j}| \downarrow_{2\text{marks}} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} \downarrow_{2\text{marks}} = 13 \downarrow_{5\text{marks}}.$$

\* Accept correct answer without work.

\* Accept candidate's answer from previous part.

*Blunders (-3)*

B1 Blunder in formula e.g. square root omitted or squares omitted or  $-$  instead of  $+$ .

B2  $i^2 \neq 1, j^2 \neq 1$   $i \cdot j \neq 0$  (apply once).

*Attempts (2 marks)*

A1 Writes the square of the coefficients of any of the given components and stops.

A2 Effort at use of relevant square root.e.g.  $\sqrt{p^2 + q^2}$  and stops.

A3 Work of merit on a diagram e.g. plots one or both vectors.

**Part (b)**

**20 (5, 10, 5) marks**

**Att (2, 3, 2)**

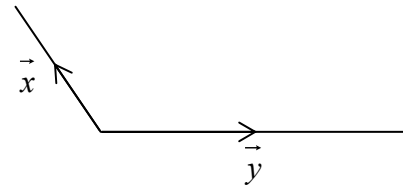
The diagram shows the vectors  $\vec{x}$  and  $\vec{y}$ .

Draw separate diagrams to show each of the following vectors:

(i)  $\vec{y} - \vec{x}$

(ii)  $2\vec{x} + \vec{y}$

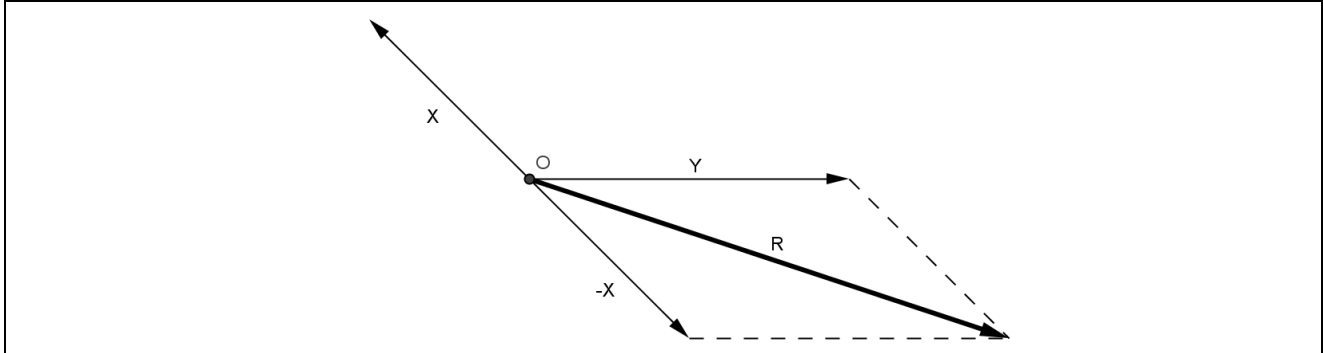
(iii)  $\frac{1}{2}\vec{x} - \vec{y}$



**(b) (i)**

**5 marks**

**Att 2**



- \* Allow candidate to use any starting point, provided they use 3 separate diagrams.
- \* Accept any lengths for first drawing of  $\vec{x}$  and  $\vec{y}$
- \* Accept for direction  $\vec{x}$  approx N.W.,  $\vec{y}$  approx E.
- \* Apply a maximum of one blunder for direction and one blunder for resultant.

Award only these marks in section (b) (i):

5 marks: Complete and correct.

2 marks: Some work of merit, e.g. only one relevant vector drawn.

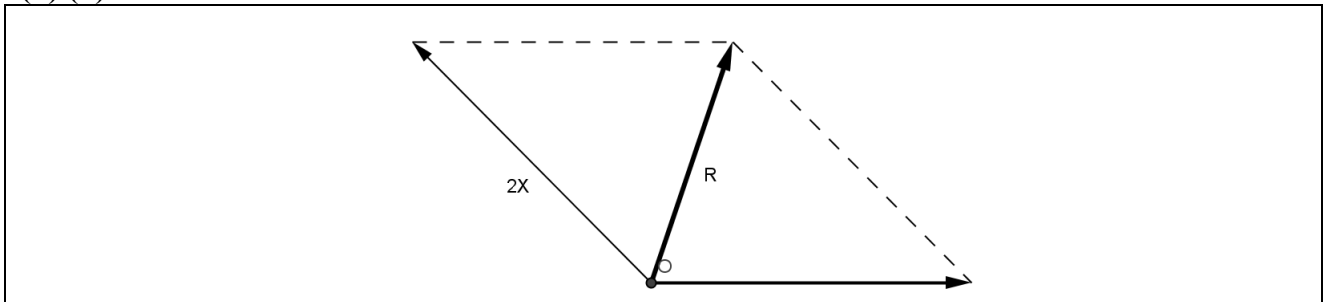
i.e a horizontal line segment or a line segment running N.W. to S.E. (to the eye)

0 marks: No worthwhile work.

**(b) (ii)**

**10 marks**

**Att 3**



- \* Accept lengths where  $|2\vec{x}| > |\vec{x}|$  and  $|\vec{y}| \pm 0.5 \text{ cm}$  from initial diagram of  $\vec{x}$  and  $\vec{y}$ .
- \* Accept for direction  $2\vec{x}$  approx N.W.,  $\vec{y}$  approx E.
- \* Apply a maximum of one blunder for direction, one blunder for length and one blunder for resultant.



Award only these marks in section (b) (ii):

10 marks: Complete and correct.

9 marks: Uses same origin as in previous section.

[Penalise same origin only if answer is otherwise fully correct]

7 marks: One blunder, e.g. length or direction outside tolerance or no resultant.

4 marks: Two blunders, e.g. any 2 of the 3 elements required incorrect.

3 marks: Some work of merit, e.g. only one relevant vector drawn.

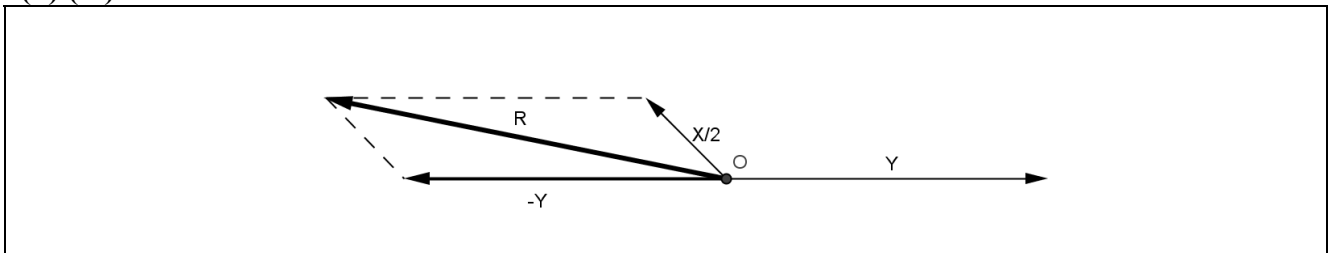
i.e a horizontal line segment or a line segment running N.W. to S.E. (to the eye).

0 marks: No worthwhile work.

(b) (iii)

5 marks

Att 2



\* Accept lengths where  $\left|\frac{1}{2}x\right|, < \left|x\right|$ , and  $\left|y\right| \pm 0.5$  cm from initial diagram of  $\vec{x}$  and  $\vec{y}$ .

\* Accept for direction  $\frac{1}{2}x$  approx N.W.,  $-y$  approx W.

\* Apply a maximum of one blunder for direction, one blunder for length and one blunder for resultant

Award only these marks in section (b) (iii):

5 marks: Complete and correct.

4 marks: Uses same origin as in previous section.

[Penalise same origin only if answer is otherwise fully correct].

2 marks: Some work of merit, e.g. only one relevant vector drawn.

i.e a horizontal line segment or a line segment running N.W. to S.E. (to the eye).

0 marks: No worthwhile work.

Special Case: Resultant drawn as single vector.			
Accepted tolerance	$l = \pm 0.2$ cm	$\theta = \pm 2^\circ$	
	Length	Direction	Marks
Section (i)	5.4 cm	$\theta = -18^\circ$	5/2
Section (ii)	3.7 cm	$\theta = 63^\circ$	10/0
Section (iii)	4.6 cm	$\theta = 170^\circ$	5/2

Part (c)

15 (10, 5) marks

Att (3, 2)

Let  $\vec{r} = 12\vec{i} - 9\vec{j}$  and  $\vec{s} = 9\vec{i} + 12\vec{j}$ .

(i) Show that  $\vec{r} \perp \vec{s}$ .

(ii) If  $\vec{OR} + 3\vec{RU} = \vec{OS}$ , express  $\vec{u}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

(c) (i)

10 marks

Att 3

$$\vec{r} \cdot \vec{s} = (12\vec{i} - 9\vec{j}) \cdot (9\vec{i} + 12\vec{j}) = (12)(9) + (-9)(12) = 108 - 108 = 0. [\Rightarrow \vec{r} \perp \vec{s}]$$

or

$$m_r \times m_s = \frac{-9}{12} \times \frac{12}{9} = -1 \Rightarrow [\vec{r} \perp \vec{s}]$$

or

$$|\vec{r}|^2 + |\vec{s}|^2 = [(12)^2 + (-9)^2] + [(9)^2 + (12)^2] = 144 + 81 + 81 + 144 = 450$$

$$|\vec{rs}|^2 = |\vec{s} - \vec{r}|^2 = |-3\vec{i} + 21\vec{j}|^2 = (-3)^2 + (21)^2 = 9 + 441 = 450 \Rightarrow [\vec{r} \perp \vec{s}]$$

or

$$(\vec{r})^\perp = 9\vec{i} + 12\vec{j} = \vec{s} \Rightarrow [\vec{r} \perp \vec{s}]$$

*Blunders (-3)*

B1 Error in multiplication.

B2  $i^2 \neq 1, j^2 \neq 1$   $i \cdot j \neq 0$  (apply once).

B3 Mathematical error.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

S2 No conclusion or incorrect conclusion if required.

*Attempts (3)*

A1 Any correct multiplication.

A2 Any use of the dot product, e.g.  $\vec{r} \cdot \vec{s} = |\vec{r}| |\vec{s}| \cos \theta$ .

A3 Finds  $|\vec{r}|$  or similar and stops.

A4 Any correct relevant step, e.g. plots one or both vectors.

(c) (ii)

5 marks

Att 2

$$\begin{aligned}\vec{r} + 3\vec{RU} &= \vec{s} \\ \Rightarrow \vec{r} + 3(\vec{u} - \vec{r}) &= \vec{s} \\ \Rightarrow 3\vec{u} &= \vec{s} + 2\vec{r} \\ &= 9\vec{i} + 12\vec{j} + 2(12\vec{i} - 9\vec{j}) \\ &= 33\vec{i} - 6\vec{j} \quad \Rightarrow \vec{u} = 11\vec{i} - 2\vec{j}\end{aligned}$$

Award only these marks in section (c) (ii):

5 marks: Fully correct answer.

4 marks: One slip or misreading.

2 marks: Some work of merit.

0 marks: Worthless work.

*Blunders (-3)*

B1  $\vec{RU} \neq \vec{u} - \vec{r}$

B2 Error in multiplication.

B3 Transposition error.

B4 Combines  $i$ 's with  $j$ 's.

B5 Mathematical error.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (2)*

A1 Any correct substitution of  $\vec{r}$  in terms of  $i$  and  $j$ .

A2 Multiplication by a scalar and stops.

A3  $\vec{RU} = \vec{u} - \vec{r}$  or similar and stops.

A4 Any work of merit, e.g. plots one or both vectors.

## QUESTION 10

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
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Expand  $(1-x)^6$  fully.

<b>(a)</b>	<b>10 marks</b>	<b>Att 3</b>
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$$(1-x)^6 = \binom{6}{0} + \binom{6}{1}(-x) + \binom{6}{2}(-x)^2 + \binom{6}{3}(-x)^3 + \binom{6}{4}(-x)^4 + \binom{6}{5}(-x)^5 + \binom{6}{6}(-x)^6 \quad \downarrow_{4 \text{ marks}}$$

$$= 1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6.$$

- \* Accept long multiplication or Pascal's triangle.
- \* Accept correct answer without work.

### *Blunders (-3)*

- B1 Number of terms equal to 6 or 8.
- B2 Incorrect index, (once only).
- B3 Incorrect coefficient. (once only).
- B4 Incorrect sign or sign between coefficient and variable, (once only).
- B5 Expands  $(1+x)^6$  or  $(1-x)^5$ .

### *Slips (-1)*

- S1 Numerical slips to a maximum of 3.

### *Attempts (3 marks)*

- A1 Any term, including first term, written down correctly.
- A2 If number of terms are less than 6 or greater than 8.
- A3 Gives part of Pascal's triangle or effort at Pascal's triangle.
- A4 Gives coefficients only.
- A5 Any step towards getting a binomial coefficient e.g.  $\binom{6}{2}$ .
- A6 Any correct step towards long multiplication.

### *Worthless (0 marks)*

- W1 Writes  $6(1-x)^5$  or  $6(1-x)^5(-1)$ .

**Part (b)****20 (10, 10) marks****Att (3, 3)**

The starting salary for a job is €24 000 per annum. At the end of each year of employment the annual salary increases by 3%.

- (i) What will the annual salary be after the first increase?  
 (ii) What will the annual salary be after the fourth increase?  
 Give your answer correct to the nearest euro.

**(b) (i)****10 marks****Att 3**

$$S = 24\,000 \left(1 + \frac{3}{100}\right) \downarrow_{4 \text{ marks}} = 24\,000(1.03) \downarrow_{7 \text{ marks}} = €24\,720$$

**or**

$$S = 3\% \text{ of } 24\,000 = 720 \downarrow_{7 \text{ marks}} \Rightarrow S = 24\,000 + 720 = €24\,720$$

\* Accept a correct answer without work.

\*  $3\% = \frac{1}{3}$  and continues merits attempt marks at most in (i) but allow in (ii).

**(b) (ii)****10 marks****Att 3**

$$S = 24\,000 \left(1 + \frac{3}{100}\right)^4 \downarrow_{4 \text{ marks}} = 24\,000(1.03)^4 \downarrow_{4 \text{ marks}}$$

$$= 24\,000(1.12550881) \downarrow_{7 \text{ marks}} = 27\,012.21 \downarrow_{9 \text{ marks}} = €27\,012$$

**or**

*Calculation on year by year basis:*

Year 1: €24 000 + €720

Year 2: €24 720 + €741.6

Year 3: €25 461.6 + €763.85

Year 4: €26 225.45 + €786.76 = €27 012.21 = €27 012

\* Accept a correct or consistent answer without work.

**Blunders (-3)**

- B1 Incorrect or inconsistent *p*.  
 B2 Incorrect or inconsistent *r/i*.  
 B3 Incorrect or inconsistent *n/t*. (1<sup>st</sup> method)  
 B4 Error in formula.  
 B5 Error in substitution.  
 B6 Mathematical error, e.g.  $(1.03)^4 = 4.12$ .  
 B7 Each year omitted, ( 2<sup>nd</sup> method).

**Slips (-1)**

- S1 Early round off that affects the accuracy of the answer- (to a maximum of 3 in 2<sup>nd</sup> method).  
 S2 Numerical slips to a maximum of 3.

**Attempts (3 marks)**

- A1 Work of merit such as correct *p* or *r/i* or *n/t*, e.g.  $3\% = 0.03$ .  
 A2 Simple interest for the four years. ( €2880)

**Worthless (0 marks)**

- W1 Incorrect answer without work.  
 W2 Incorrect formula and stops.  
 W3  $24\,000/3 = 8000$  and stops in (i).

**Part (c)**

**20 (10, 10) marks**

**Att (3, 3)**

- (i) The first term of a geometric series is 5. The sum to infinity of the series is 10. Find the common ratio.
- (ii) Write the recurring decimal  $0.1333\dots$  as an infinite geometric series and hence in the form  $\frac{a}{b}$ , where  $a, b \in \mathbb{N}$ .

**(c) (i)**

**10 marks**

**Att 3**

$$S_{\infty} = \frac{a}{1-r} = \frac{5}{1-r} \downarrow_{3\text{marks}} = 10 \Rightarrow 10 - 10r = 5 \downarrow_{4\text{marks}} \Rightarrow -10r = -5 \downarrow_{7\text{marks}} \Rightarrow r = 0.5.$$

*Blunders (-3)*

B1 Incorrect  $a$ .

B2 Transposition error.

B3 Incorrect relevant formula, e.g.  $\frac{a}{1+r} \Rightarrow r = -0.5$ .

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*

A1 Work of merit such as correct  $a$  identified.

A2 Correct answer without work.

A3 Fails to form equation merits at most an attempt mark.

*Worthless (0 marks)*

W1 Incorrect answer without work.

W2 Correct formula and stops.

W3 Formula for a GP and stops.

W4 Answer =  $\frac{5}{10}$  or  $\frac{5}{10} = \frac{1}{2}$ .

(c) (ii)

10 marks

Att 3

$$\begin{aligned} 0.1333\dots &= \frac{1}{10} + \frac{3}{100} + \frac{3}{1000} + \dots \downarrow_{3 \text{ marks}} \\ &= \frac{1}{10} + \frac{\frac{3}{100}}{1 - \frac{1}{10}} \downarrow_{4 \text{ marks}} = \frac{1}{10} + \frac{1}{30} \downarrow_{7 \text{ marks}} = \frac{4}{30} \text{ or } \frac{2}{15}. \end{aligned}$$

*Blunders (-3)*

- B1 Incorrect  $a$ .
- B2 Incorrect  $r$ .
- B3 Blunder in fractions.
- B4 Incorrect relevant formula, e.g.  $a/(1+r)$  giving answer of  $\frac{7}{55}$ .
- B5 Ignores  $1/10$

*Misreadings (-1)*

- M1 Treats as  $0.1313\dots$

*Slips (-1)*

- S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*

- A1 Some work of merit e.g. states the value for  $a$  or the value for  $r$ .
- A2 Adds 2 or more of the given terms e.g.  $S_2 = \frac{13}{100}$  or  $S_3 = \frac{133}{1000}$ .
- A3 One correct step in adding relevant fractions.
- A4 Correct answer without work
- A5 Works as  $9x = 1.2 \Rightarrow x = \frac{1.2}{9}$  or  $\frac{2}{15}$ .

*Worthless (0 marks)*

- W1 Formula for arithmetic series and stops.
- W2 Incorrect answer without work.

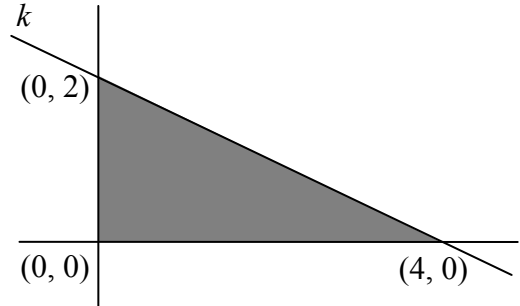
## QUESTION 11

<b>Part (a)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>
<b>Part (b)</b>	<b>35 (20, 5, 10) marks</b>	<b>Att (7, 2, 3)</b>

<b>Part (a)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>
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The line  $k$  passes through the points  $(0, 2)$  and  $(4, 0)$ .

- (i) Find the equation of  $k$ .
- (ii) Write down the three inequalities which define the shaded region in the diagram.



<b>(a) (i)</b>	<b>10 marks</b>	<b>Att 3</b>
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Slope of $k$	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$	[4 marks]
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Equation of $k$ :	$y - y_1 = m(x - x_1)$ $y - 0 = -\frac{1}{2}(x - 4)$ or $y - 2 = -\frac{1}{2}(x - 0)$ or $[x + 2y - 4 = 0]$ .	[10 marks]
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**or**

Equation of $k$ :	$y = mx + c$ $0 = -\frac{1}{2}(4) + c \Rightarrow c = 2.$	[10 marks]
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- \* Accept  $y - 0 = -\frac{1}{2}(x - 4)$  without work.
- \* Apply scheme for Q2, Q3 where relevant.
- \* Error in simplifying  $k$  to be penalised in section a(ii) subject to marks secured.
- \* Treat as TWO blunders when  $m$  is not addressed, i.e  $y - 2 = m(x - 0)$  or  $y - 2 = (x - 0)$ .

### Blunders (-3)

- B1 Incorrect relevant formula and continues.
- B2 Mixes up  $x$ 's and  $y$ 's when substituting.
- B3  $y - 2 = m(x - 0)$  where  $m$  is not equal to  $-\frac{1}{2}$  without work.
- B4  $y - y_1 = -\frac{1}{2}(x - x_1)$  where  $(x_1, y_1)$  is not  $(4, 0)$  or  $(0, 2)$  without work.
- B5 Point substituted incorrectly.

### Slips (-1)

- S1 Numerical slips to a maximum of 3.

### Attempts (3 marks)

- A1 Any work of merit.

### Worthless (0 marks)

- W1 An arbitrary line without work.



**(a) (ii)**

**5 marks**

**Att 2**

$x \geq 0, \quad y \geq 0, \quad x + 2y - 4 \leq 0$ or equivalent.
--

- \* Accept correct inequalities without work.
- \* Accept  $<$  for  $\leq$  and  $>$  for  $\geq$ .
- \* Accept an inequality consistent with candidate's  $k$ . [See 3<sup>rd</sup> \* a (i)]
- \* Award 2 marks for one correct inequality, 4 marks for 2 correct and 5 marks for 3 correct.

*Attempts ( 2 marks )*

- A1 Substitutes any point and stops.
- A2  $x \leq 0$  or  $y \leq 0$  and stops, (without work).
- A3 Incorrect or no conclusion if working on  $K$  e.g.  $x + 2y - 4 = 0 \Rightarrow 0 + 2(0) - 4 = 0$ .
- A4 Mathematical error in testing a point, e.g. sign error.
- A5 Some relevant step e.g.  $x = 0, k$  simplified accept even in a(i).
- A6 Lists all cases, i.e.  $x \geq 0, x \leq 0, y \geq 0, y \leq 0, x + 2y - 4 \leq 0, x + 2y - 4 \geq 0$

*Worthless (0 marks)*

- W1 Writes equation of  $k$  and stops (without simplifying).
- W2 Draws the given diagram or writes (0, 0), (0, 2), (4, 0) only.

**Part (b)**

**35 (15, 5, 5, 10) marks**

**Att (5, 2, 2, 3)**

A contractor has the task of loading containers onto a truck. There are two types of containers: heavy containers which weigh 160 kg each and light containers which weight 40 kg each. The truck can carry, at most, a total weight of 2080 kg.

The time taken to load a heavy container is 3 minutes. The time taken to load a light container is 2 minutes. The total time spent loading a truck cannot be greater than 54 minutes.

- (i) Taking  $x$  as the number of heavy containers and  $y$  as the number of light containers, write down two inequalities in  $x$  and  $y$  and illustrate these on graph paper.
- (ii) The contractor charges €48 to load each heavy container and €36 to load each light container. How many of each should be loaded in order to maximise income?
- (iii) On your graph, show the region where the income is at most €576.

**(b) (i) Inequalities**

**15 (10, 5) marks**

**Att (3, 2)**

Weight:	$160x + 40y \leq 2080$ or $[4x + y \leq 52]$
Time:	$3x + 2y \leq 54$ .

- \* Accept correct multiples or fractions of inequalities or the use of different letters.
- \* Apply (-3), once, if no inequality sign or the incorrect inequality sign is written the first time it occurs.
- \* Accept  $<$  for  $\leq$ .
- \* Better inequality marked out of 10. Other inequality marked out of 5.

	Heavy / $x$	Light / $y$	Maximum
Weight	160	40	2080
Time	3	2	54

**12 Marks**

*Blunders (-3)*

- B1 Mixes up  $x$ 's and  $y$ 's (once if consistent error).
- B2 Confuses rows and columns in table, e.g.  $160x + 3y \leq 2080$  (once if consistent).
- B3 Decimal blunder applies for error with zeros in equation, unless an obvious misreading.

Attempts (3 or 2 marks)

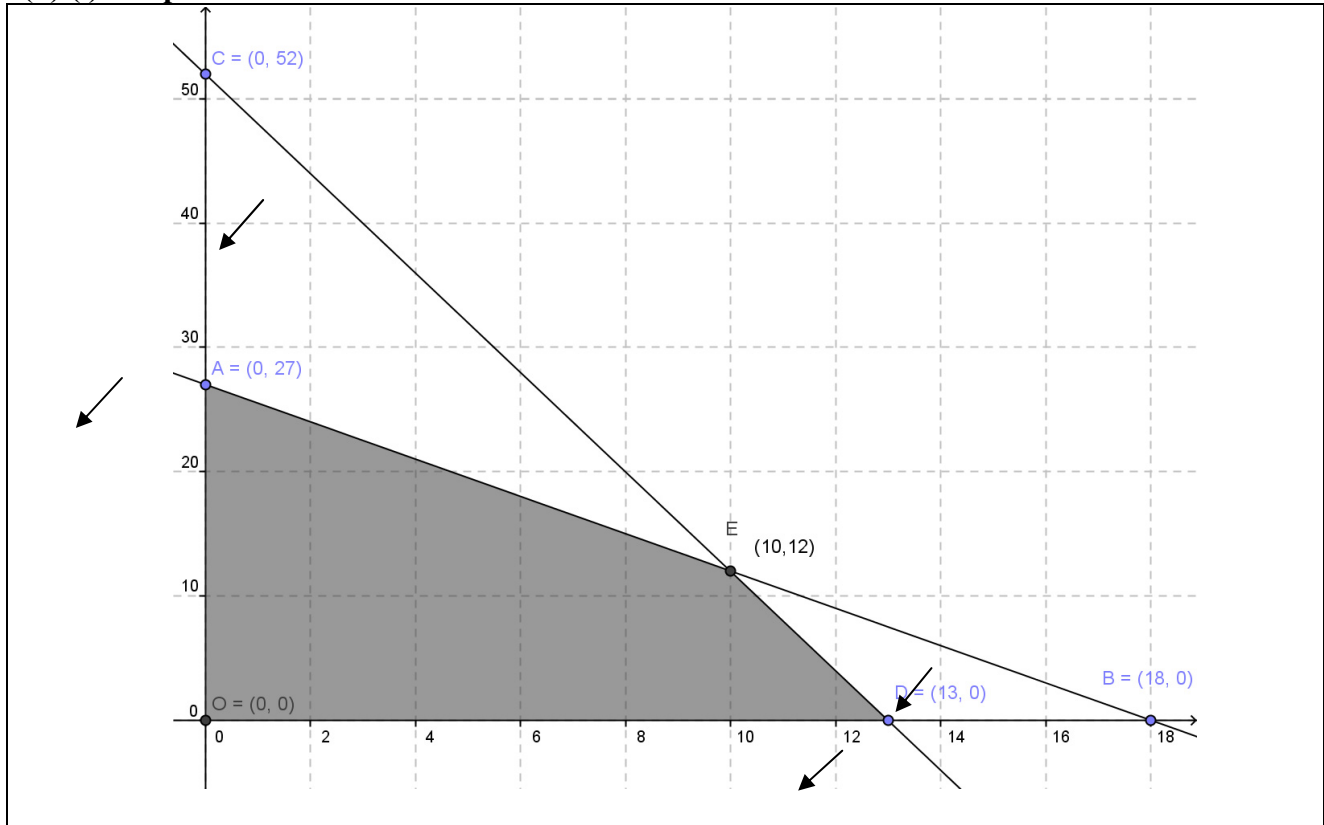
A1 Incomplete relevant data in table and stops, e.g.  $160x$  or  $3x$  or  $40y$  or  $2y$  or  $\leq 2080$  or  $\leq 54$  (each inequality).

A2 Any other correct inequality, e.g.  $x \geq 0$ ,  $y \geq 0$ , (each time).

(b) (i) Graph

5 marks

Att 2



- \* Points or scales required.
- \* Correct shading over-rides arrows or correct arrows overrule shading.
- \* Inequalities not written but correct graph drawn – award 0 + 5 marks.
- \* Two lines drawn and no shading indicated, only one of the following cases applies:
 

Case 1: Two sets of arrows in expected direction	5 marks
Case 2: Two sets of arrows in unexpected direction	5 marks
Case 3: One set of arrows “correct”, the other “incorrect”	2 marks
Case 4: One line with and the other without arrows	2 marks
Case 5: No arrows	2 marks
Case 6: Half-planes consistent with incorrect, penalised inequalities.	5 marks

Blunders (-3)

B1 Blunder in plotting a line or calculations.

B2 Incorrect shading, e.g. one or both of the small triangles shaded, (subject to case 6 above).

B3 Vertical:  $x$ -axis, Horizontal:  $y$ -axis.

Attempts (2 marks)

A1 Some relevant work towards a point on a line.

A2 Draws scaled axes or axes and one line.

**(b) (ii) Intersection/Income****5 marks****Att 2**

$$\begin{array}{rcl}
 4x + y = 52 & & 8x + 2y = 104 \\
 \underline{3x + 2y = 54} & \Rightarrow & \underline{3x + 2y = 54} \\
 & & 5x = 50 \Rightarrow x = 10 \quad \text{and} \quad y = 12
 \end{array}$$

Step 1	Vertices	$48x + 36y$	Income
	(0, 0)	0 + 0	0
Step 2	(13, 0)	624 + 0	624
Step 3	(10, 12)	480 + 432	912
Step 4	(0, 27)	0 + 972	972

Step 5            0 heavy containers and 27 light containers to maximise income.

- \* Accept candidate's own equations from previous sections.
- \* If solving incorrect equations, the point found may be outside the feasible set – award marks for correct work and accept in later sections.
- \* Information does not have to be in table form.
- \* Accept any correct multiple or fraction of  $48x + 36y$  here (may be implied).
- \* Accept work on a feasible set of points formed by axes and one line without further penalty.
- \* Accept only vertices consistent with previously accepted work, not arbitrary ones.
- \* If (18, 0) or (0, 52) is tested and result is used to give maximum income, award attempt 2.
- \* Step 5 must be explicitly written to gain full marks.
- \* Testing only (0, 27) to get 972 merits Att 2 even if the candidate writes 0 heavy containers and 27 light containers to maximise income, i.e. no comparison means the attempt mark at most.

*Blunders (-3)*

- B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.  
 B2  $x$  or  $y$  value only found.

*Attempts (2 marks)*

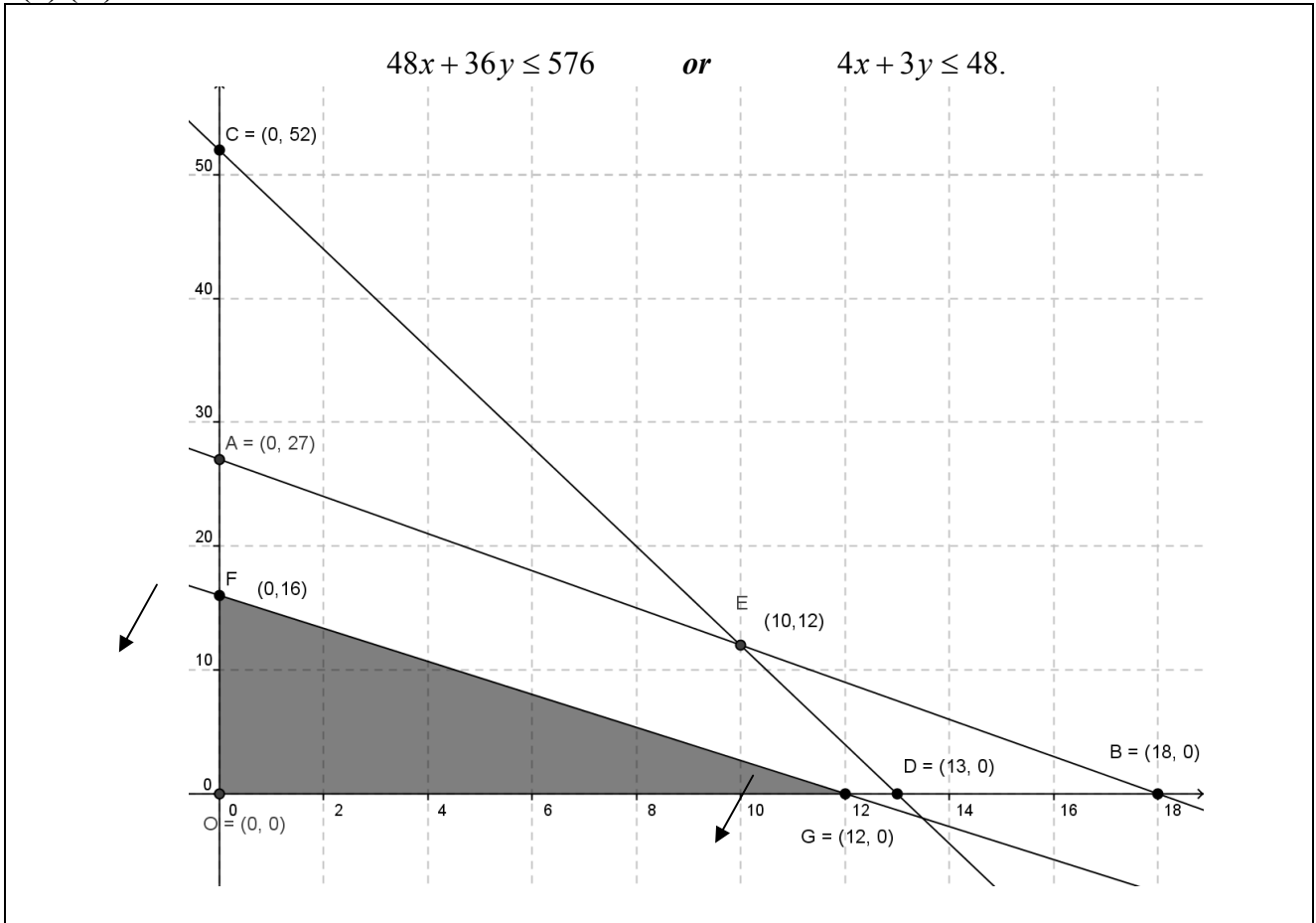
- A1 Correct or consistent point of intersection without work or from a graph.  
 [Should get the *exact same* values from graph as if they had been found algebraically.]  
 A2 Any relevant step towards solving equations.  
 A3 Any relevant work involving  $x$  or  $y$  and/or 48, 36 or similar.  
 A4 Any attempt at substituting co-ordinates into some relevant expression.  
 A5 Any step omitted, subject to the case for awarding 4 marks.

*Worthless (0 marks)*

- W1 Incorrect point of intersection without work and inconsistent with the lines, subject to marks secured.  
 W2 Writing €48 or €36 without further work, subject to marks secured.

*Award marks as follows:*

- 5 marks: Answer is fully correct or consistent.  
 4 marks: The maximum value is identified but step 5 not stated.  
 2 marks: Some relevant work. e.g point of intersection.  
 0 marks: Worthless work.



- \* Accept correct multiples or fractions of the inequality.
- \* No penalty for writing  $48x + 36y = 576$ , provided correct area is indicated.

*Award only marks as follows:*

- 10 marks: Answer is fully correct or consistent, with conclusion with expected shading clearly indicated.
- 3 marks: Some relevant work.
- 0 marks: Worthless work.

*Attempts (3 marks)*

- A1 The point (12, 0) or (0, 16) plotted.
- A2 A subset of the feasible set bounded by the axes shaded.

## MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

### (Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g.  $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$  bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle  $[300 - \text{bunmharc}] \times 15\%$ , agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0





