Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2009

MARKING SCHEME

MATHEMATICS

ORDINARY LEVEL

Contents

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GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES (PAPER 1)

Examples of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error -3(-4) = -12
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$ *or* $4x = 12 \Rightarrow x = 8$; each time.
- Distribution error (once per term, unless directed otherwise) e.g. $3(2x + 4) = 6x + 4$ *or* $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly: e.g. $(2x - 3)(x + 4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formulae: e.g. $T_n = 2a + (n - 1)d$ (only accept use of formulae with one blunder)
- Central sign error in *uv* or *u/v* formulae
- Omission of $\div v^2$ or division not done in *u/v* formula (apply once)
- Vice-versa substitution in *uv* or *u/v* formulae (apply once)
- Quadratic formula (*acceptable*) and its application apply a maximum of two blunders

Slips (-1)

- Numerical slips: $4 + 7 = 10$ or $3 \times 6 = 24$, *but* $5 + 3 = 15$ is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off, is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per part (a), (b) and (c) of each question. Only applies where a candidate would otherwise have achieved full marks in each subpart
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Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

* Incorrect or omitted units: penalise as per guidelines.

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Conor and Alice share 50 apples in the ratio 3 : 7.
(i) How many apples does Conor get?
(ii) How many apples does Alice get?

(a) (i) **5 marks** **Att 2**
(a) (ii) **5 marks** **Att 2**

$3 + 7 = 10$
Conor's share $= \frac{3}{10} \times 50 = 15$
Alice's share $= \frac{7}{10} \times 50 = 35$ or $50 - 15 = 35$.

* Accept correct answer without work for full marks.
* Units not required
* Assume order, Conor: Alice if names not stated

Blunders (-3)
B1 Incorrect numerator -once if consistent
B2 Incorrect denominator - once if consistent
B3 35:15 without work

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 Mentions 10 and stops Att 2 once only
A2 Mentions $\frac{1}{10}$ or 5 or $\frac{1}{10} = 5$ and stops award Att 2
A3 3×50 and/ or 7×50 one or 2 attempts

Worthless (0)
W1 Incorrect answer without work
W2 3×7 =21 and stops
W3 50/3, 50/7, 3/50, 7/50

Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

Barbara works 35 hours a week and she is paid €12.60 per hour.

(i) Find her total weekly pay.

(ii) Barbara pays tax at the rate of 20% on all her income and has weekly tax credits of €53. Calculate her weekly take-home pay.

(iii) In one particular week, Barbara worked 4 additional hours at the same rate of pay. By how much did her take-home pay increase that week?

(b) (i) **5 marks** **Att 2**

Total pay = €12.60 × 35 [2m] = €441 [5m]

* Accept correct answer without work for full marks

Blunders (-3)

B1 Incorrect operation or fails to multiply

B2 Decimal error, but check units

Slips (-1)

S1 Numerical slips to a max of 3

S2 Correct units omitted; see guidelines

Attempts (2 marks)

A1 Finds €12.60 × A; $A \neq 35$, $B \times 35$; $B \neq$ €12.60

Worthless (0)

W1 Incorrect answer without work

(b) (ii) **10 marks** **Att 3**

Gross tax = €441 × 0.2 = €88.20 [4m]

Net tax = €88.20 – €53 = €35.20 [7m]

Take-home pay = €441 – €35.20 = €405.80 [10m]

* Accept candidates figure from (i)

* Accept any mathematically correct method

* Correct answer without work attempt mark only

Blunders (-3)

B1 Incorrect or no use of tax credits

B2 Take-home pay not calculated

B3 Mathematical error

Slips (-1)

S1 Numerical slips

Attempts (3 marks)

A1 20% of incorrect figure and stops

A2 Mentions $20\% = \frac{1}{5}$ and stops

A3 Some work with figures from (i)

Worthless (0)

W1 Incorrect answer without work

(b) (iii) **5 marks** **Att 2**

I Pay increase = €12.60 × 4 = €50.40 [2m]
Tax = €50.40 × 0.2 = €10.08
Increase = €50.40 – €10.08 = €40.32 [5m]

or

II €12.60 × 4[2m] × 0.8 = €40.32 [5m]

or

III 39x€12.60 = €491.40 [2m]
€491.40x0.2= €98.28
€98.28- €53 = €45.28
€491.40-€45.28= €446.12
€446.12- €405.80=€40.32 [5m]

* Accept candidate's answers from (i) and (ii).
* Candidates may offer other correct methods
* No retrospective marking
* Correct answer without work attempt mark only
* Consistent error in b(ii) and b(iii) can generate correct answer in b(iii), blunder error each time

Blunders (-3)
B1 Mathematical error
B2 Fails to find increase in take home pay

Slips (-1)
S1 Numerical slips to a max of 3

Attempts (2 marks)
A1 Mentions 39 and stops

Worthless (0)
W1 Incorrect answer without work

Part (c) **20 (10, 10) marks** **Att (3, 3)**

€7500 was invested for 2 years at r% per annum compound interest.

(i) The amount of the investment at the end of the first year was €7860.
Find the value of r.

(ii) At the start of the second year €X was withdrawn from the account.
The interest earned during the second year was €252.
Find the value of X.

(c) (i) **10 marks** **Att 3**

I $€7500 \times R = 7860$ [3m] $R = \frac{7860}{7500}$[4m] $= 1.048$ [7m] $\Rightarrow r = 4.8$ [10m]

or **II** $7860 - 7500$[3m] $= 360$[4m] $\Rightarrow r = \frac{360 \times 100}{7500}$[7m] $= 4.8$ [10m]

or **III** $A = P(1 + \frac{r}{100})^n$ [3m]

$7860 = 7500(1 + \frac{r}{100})^1$ [4m]

$\frac{7860}{7500} = 1 + \frac{r}{100}$ [7m]

$r = 4.8$ [10m]

* Candidates may offer other correct methods
* Correct answer without work attempt mark only

Blunders (-3)
B1 Mathematical error
B2 Percentage error e.g $\frac{360 \times 100}{7860}$ or similar
B3 Incorrect formula method III
B4 Fails to convert 1.048 to percentage method I

Slips (-1)
S1 Numerical slips to a max of 3

Attempts (3 marks)
A1 Correct relevant formula and stops
A2 Correctly identifies A and/or P in method III

Worthless (0)
W1 Incorrect answer without work

(c) (ii) **10 marks** **Att 3**

I 4.8% = 252 [3m]

P year 2 (100%) [4m]= $\frac{252x100}{4.8} = 5250$ [7m]

X = 7860 – 5250 = 2610. [10m]

or **II**

$\frac{4.8}{100}(7860 - X) = 252$ [4m]

$4.8(7860 - X) = 25200$

$(7860 - X) = 5250$ [7m]

$X = 2610$ [10m]

or **III** 4.8% of €7860 =€377.28 [3m]

$\frac{252}{377.28} \times €7860$[4m]

=€5250 [7m]

$X = 7860 - 5250 = 2610$ [10m]

Lost Interest

€377.28- €252=€125.28 [4m]

4.8%=€125.28 [7m]

100%=€2610 [10m]

* Accept candidate's answer from (i).
* Candidates may offer other correct methods
* Correct answer without work attempt mark only

Blunders (-3)
B1 Mathematical errors (Percentages , fractions, transposing)- **II**

Slips (-1)
S1 Numerical slips to a max of 3

Attempts (3 marks)
A1 4.8% of €7860 = €377.28 and stops
A2 Correct relevant formula
A3 Correct answer by trial and error, even if verified

Worthless (0)
W1 Incorrect answer without work
W2 4.8% of €7500 or €7860 + €252

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

Find the value of $\frac{3x-2y-1}{5}$ when $x=13$ and $y=14$.

(a) **10 marks** **Att 3**

$$\frac{3x-2y-1}{5} = \frac{3(13)-2(14)-1}{5} \text{ [4m]} = \frac{39-28-1}{5}\text{[7m]} = \frac{10}{5}\text{[9m]} = 2 \text{ [10m]}$$

* Correct answer without work; full marks.

Blunders (-3)
B1 Mathematical error once if consistent
B2 Mixes up *x* and *y,* once only
B3 One correct and one incorrect substitution and continues

Misreading (-1)
M1 Uses 31 and /or 41 and continues

Attempts (3 marks)
A1 Any correct relevant multiplication or addition
A2 Correct partial substitution and stops
A3 $\frac{313-214-1}{5}$ and continues or not

Slips (-1)
S1 Numerical slips

Worthless (0)
W1 Incorrect answer without work

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

(i) Find the value of 3^6.

(ii) Write 27 in the form 3^k, where $k \in \mathbf{N}$.

(iii) Find the value of x for which $27 \times 3^x = \frac{1}{729}$.

(b) (i) **5 marks** **Att 2**

I $3^6 = 729$ or **II** 3x3x3x3x3x3 [2m] = 729

* Correct answer without work; full marks.

Blunders (-3)

B1 Indices error with work e.g. $6^3 = 216$

Attempts (2 marks)

A1 Any correct use of indices in base 3 eg $9 = 3^2$ or 3x3x3=27

A2 Attempts to multiply out

Slips (-1)

S1 243 or 2187 with work see **II**

Worthless (0)

W1 Incorrect answer without work

W2 18 ie 3×6

(b) (ii) **5 marks** **Att 2**

I	**II**	**III**
$27 = 3^3$ **or** $27 \div 3 = 9$; $9 \div 3 = 3$; $3 \div 3 = 1$; $\Rightarrow 27 = 3^3$		**or** 3x3=9 9x3=27 $3^3 = 27$

* Correct answer without work; full marks.

Blunders (-3)

B1 Indices error, with work

Attempts (2 marks)

A1 Any correct use of indices in base 3 eg $9 = 3^2$, 27=3x3x3, $\sqrt[3]{27} = 3$

A2 Some correct relevant multiplication or division

Worthless (0)

W1 Incorrect answer without work

W2 Answer 3^9

(b) (iii) **10 marks** **Att 3**

I $27\times3^{x}=\frac{1}{729}\Rightarrow3^{3}\times3^{x}=\frac{1}{3^{6}}$ [3m] $\Rightarrow3^{3+x}=3^{-6}$[7m] $\Rightarrow3+x=-6\Rightarrow x=-9$ [10m]

or **II**

$27\times3^{x}=\frac{1}{729}\Rightarrow3^{x}=\frac{1}{27\times729}$ [3m]=$\frac{1}{19683}$ [3m] $\Rightarrow3^{x}=\frac{1}{3^{9}}$[7m] $\Rightarrow x=$ -9 [10m]

or **III**

$27\times3^{x}=\frac{1}{729}\Rightarrow27\times729\times3^{x}=1$ [$19683\times3^{x}=1$] [3m] $\Rightarrow3^{9}\times3^{x}=3^{0}$[7m] $\Rightarrow x=$ -9 [10m]

* Accept candidates answers from (i) and (ii)

* Correct answer by T +E must be verified using indices, e.g. $3^{3}\times3^{-9}=3^{-6}=\frac{1}{729}$: award 10 marks otherwise Att 3. Unverified: Att 3

* No retrospective/back marking

Blunders (-3)

B1 Error in indices, each time

B2 Transposing errors, each time

Attempts (3 marks)

A1 Substitutes answers from (i) and/or (ii) and stops

A2 Some correct work with indices

A3 27×729 = 19683 and nothing else

Worthless (0 marks)

W1 Incorrect answer without work

W2 Only work $\frac{1}{729}=0.001$ or similar

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $f(x) = x^3 + x^2 - 4x - 4$.

(i) Verify that $f(-2) = 0$.

(ii) Solve the equation $x^3 + x^2 - 4x - 4 = 0$.

(c) (i) **10 marks** **Att 3**

$f(x) = x^3 + x^2 - 4x - 4$

$f(-2) = (-2)^3 + (-2)^2 - 4(-2) - 4$ [4m]

$= -8 + 4 + 8 - 4$ [9m] .

$= 0$ [10m]

Blunders (-3)

B1 Mathematical error each time if different

B2 Error in substitution each time

Attempts (3 marks)

A1 Shows, or attempts to show that $(x+2)$ is a factor

A2 Some correct substitution into $f(x)$

A3 Finds $f(2)$

Worthless (0)

W1 $f(0)$, $f(x+2)$ *or* $f(x-2)$

(c) (ii) **10 marks** **Att 3**

$f(-2)=0 \quad \Rightarrow (x+2)$ is a factor [3m]

I or **II**

$$x+2\overline{)x^3+x^2-4x-4}\quad \begin{matrix} x^2-x-2 \\ x^3+2x^2 \\ -x^2-4x \\ -x^2-2x \\ -2x-4 \\ -2x-4 \\ 0 \quad 0 \end{matrix}$$

[4m]

$x^3+x^2-4x-4=(x+2)(x^2+Ax-2)$

$1=2+A$ x^2 coefficients

$A=-1$

or

$-4=2A-2$ x coefficients

$A=-1$

$x^3+x^2-4x-4=0 \Rightarrow [(x+2)](x^2-x-2)=0$ [4m]

$\Rightarrow [(x+2)](x-2)(x+1)=0$ [7m]

$x=-2 \quad x=2 \quad x=-1$ [10m]

or **III**

$x^3+x^2-4x-4=x^2(x+1)-4(x+1)$ [3m]

$=(x+1)(x^2-4)$ [4m]

$=(x+1)(x-2)(x+2)$ [7m]

$(x+1)(x-2)(x+2)=0$

$x=-1,\ 2,\ -2$ [10m]

* Synthetic division is acceptable

* If roots found by (T +E)/(calculator Function Mode), all to be verified for full marks - otherwise attempt mark only

Blunders (-3)

B1 Incorrect initial divisor/factor: no penalty for $(x-2)$ or $(x+1)$

B2 Error in division/finding quadratic factor, max 2 Blunders

B3 Incorrect linear factors – once only

B4 Incorrect roots from factors or no roots found – once only

Note : If quadratic formula used, apply guidelines

Slips (-1)

S1 $x = -2$ Not given as a root at this part

Attempts (3 marks)

A1 Attempt at, division, comparing coefficients, or some correct factorising

A2 $x = -2$ and stops

A3 Correct quadratic formula and stops

A4 $f(k)$; $k \in R$ with some substitution

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

Simplify $x(2x+7)-3(x-4)$.

(a) **10 marks** **Att 3**

$x(2x+7)-3(x-4)$

$= 2x^2 + 7x - 3x + 12$ [7m]

$= 2x^2 + 4x + 12$ [10m]

* Accept correct answer without work.

Blunders (-3)

B1 Distribution error; once per term/bracket

B2 Mathematical error

Misreadings (-1)

M1 Must not make work easier eg $(2x - 7)$ accept, but $(x + 4)$ a blunder

Attempts (3 marks)

A1 Any correct term by multiplication or correct terms without work unless fully correct

Note:

$2x^2 + 7x = 3x - 12$ [4m] if stops

$2x^2 + 4x + 12 = 0$ [10m]

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

(i) Solve for x and y

$$x + y = 7$$
$$x^2 + y^2 = 29.$$

(ii) Which one of the values of y in **(i)** above satisfies the inequality $6 - 2y < 0$? Justify your answer.

(b) (i) **(5, 5, 5) marks** **Att (2, 2, 2)**

$x + y = 7 \Rightarrow x = 7 - y$ Step 1 Isolates x or y [5m]

$x^2 + y^2 = 29 \Rightarrow (7 - y)^2 + y^2 = 29$
$\Rightarrow 49 - 14y + y^2 + y^2 - 29 = 0$
$\Rightarrow 2y^2 - 14y + 20 = 0$.
or
$y^2 - 7y + 10 = 0$

Step 2 Forms quadratic equation [5m]

$\Rightarrow (y - 2)(y - 5) = 0$
$\Rightarrow y = 2 \text{ or } y = 5$
$x = 7 - y = 7 - 2 = 5 \quad or \quad x = 7 - y = 7 - 5 = 2$

$x = 7 - y = 7 - 2 = 5 \quad or \quad x = 7 - y = 7 - 5 = 2$. (5, 2) and (2, 5)

Step 3 Solutions [5m]

* Apply similar structure if x isolated
* No penalty for excess answers if substitutes into second degree equation

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)
A1 Effort at isolating x or y Step 1
A2 Correct quadratic formula written and stops Step 3
A3 Step 2 *linear*: award at most Att 2 in Step 3
A4 Correct answer(s) by T + E or without work, one or both solutions verified in both equations, award the three attempts (Att 2m), otherwise 0.

Worthless (0)
W1 'Invented' values

(b) (ii) **5 marks** **Att 2**

$6-2y < 0$

I	II	III
$6-2(5)<0$	$6-2(2)<0$	$-2y<-6$
$6-10<0$	$6-4<0$	$y>3$
$-4<0$	$2<0$ False	$\Rightarrow y=5$
	$\Rightarrow y=5$	

* Accept candidate's coordinates from (i).

Blunders (-3)

B1 No conclusion or incorrect conclusion (methods II and III)

B2 Mathematical error

Attempts (2 marks)

A1 Correct answer without work

A2 Uses answer from b(i) and stops

A3 Only one solution at b(i) attempt mark at most

Worthless (0)

W1 Incorrect answer without work

Part (c) **20 (10, 10) marks** **Att (3, 3)**

A rectangle has length $2\sqrt{x}$ cm and width $\sqrt{x}$ cm.

The length of a diagonal of the rectangle is $\sqrt{45}$ cm.

(i) Find the area of the rectangle.

(ii) The area of a square is twice the area of the rectangle.
Find the length of a side of the square.

(c) (i) **10 marks** **Att 3**

$\left(\sqrt{x}\right)^2 + \left(2\sqrt{x}\right)^2 = \left(\sqrt{45}\right)^2$ [4m] or Area Rectangle = lb [3m]

$x + 4x = 45$

$5x = 45$

$x = 9$ cm [7m]

Area Rectangle = lb

Area = $2\sqrt{x}.\sqrt{x}.$

$= 2\sqrt{9}.\sqrt{9}$

$= 2x3x3$

$= 18$ cm^2

Area = $2\sqrt{x}.\sqrt{x}.$

$= 2x$ [4m]

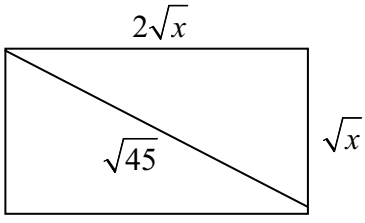
$\left(\sqrt{x}\right)^2 + \left(2\sqrt{x}\right)^2 = \left(\sqrt{45}\right)^2$ [7m]

$x + 4x = 45$

$5x = 45$

$x = 9$

. Area = $2x = 18$ cm^2



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* Incorrect or omitted units: penalises as per guidelines

Blunders (-3)

B1 Incorrect use of Pythagoras

B2 $(\sqrt{x})^2 \neq x$

B3 Mathematical error

Slips(-1)

S1 Incorrect or no units stated for area

Attempts (3 marks)

A1 Correct answer without work

A2 Invents value for x and continues

A3 Correctly states Pythagoras

A4 Correctly labelled diagram

A5 Area rectangle = lb

Worthless (0)

W1 Incorrect answer without work

(c) (ii) **10 marks** **Att 3**

Area of square = $2x18=36\,cm^2$ [3m]	or Area $=2(2x)=4x$ [3m]
$y^2 = 36\ cm^2$. [4m]	Side $= \sqrt{4x}$ [4m]
$y = \sqrt{36}$ [7m]	$= 2\sqrt{x}$ [7m]
$y = 6$ cm [10m]	$= 2\sqrt{9} = 2\times 3 = 6$ cm [10m]

* Accept candidates answer from (i) provided not invented

Blunders (-3)

B1 Incorrect area of square

B2 Mathematical error finding side eg 36/2 offered (index error) Division by any other number merits at most 4m

Attempts (3 marks)

A1 Diagram with some correct, relevant information shown and stops

A2 Area of a square = l^2

Worthless (0)

W1 $2\sqrt{x}$ without work

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 marks** **Att (2, 2)**

Given that $i^2 = -1$, simplify

$$2(3-5i)+7i(2+3i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(a) **10 marks** **Att 3**

$2(3-5i)+7i(2+3i) = 6-10i+14i+21i^2$ [4m]

$= 6-10i+14i+21(-1)$ [7m]

$= -15+4i$ [10m]

Blunders (-3)

B1 Error in multiplication - once per bracket

B2 $i^2 \neq -1$ or mis-use of i^2; B1 and B2 can apply

B3 Sign error

B4 Mixes up real and imaginary terms

B5 Avoids use of i^2

Slip (-1)

S1 Numerical slips

Attempts (3 marks)

A1 Any correct relevant multiplication

Worthless (0)

W1 Incorrect answer without work

Part (b) **20 (10, 10) marks** **Att (3, 3)**

Let $u = 3 + 5i$.

(i) Show that u is a solution of the equation $z^2 - 6z + 34 = 0$.

(ii) Express $\frac{17}{u}$ in the form $x + yi$.

(b) (i) **10 marks** **Att 3**

I

$z^2 - 6z + 34 = (3+5i)^2 - 6(3+5i) + 34$ [3m]

$= 9 + 30i + 25i^2 - 18 - 30i + 34$ [7m]

$= 9 + 30i - 25 - 18 - 30i + 34$ [9m]

$= 43 + 30i - 25 - 18 - 30i$ [9m]

$=0$ [10m]

or **II**

$z^2 - 6z + 34 = 0$

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ [3m]

$z = \frac{6 \pm \sqrt{(-6)^2 - 4(34)}}{2}$ [4m]

$= \frac{6 \pm \sqrt{-100}}{2}$ [7m]

$== \frac{6 \pm 10i}{2}$

$= 3 \pm 5i$ or $3 + 5i$ [10m]

or **III**

$3+5i$ a root $\Rightarrow 3-5i$ other root

$z^2 - (3+5i+3-5i)z = (3+5i)(3-5i) = 0$ [7m]

$z^2 - 6z + 34 = 0$ [10m]

* Note $\sqrt{-100} = 10i$ must appear in method **II**

* $\sqrt{\text{positive number}}$: [4m] at most

Blunders (-3)

B1 Each omitted or incorrect term when multiplying out to a max of 2

B2 $i^2 \neq -1$ or misuse of i^2 (B1 and B2 can apply)

B3 Error in quadratic formula and its application to a max of 2 blunders

Attempts (3 marks)

A1 Any correct substitution

A2 Correct quadratic formula and stops

A3 Identifies 'other' root

Worthless (0)

W1 Treats as a linear equation

(b) (ii) **10 marks** **Att 3**

$$\frac{17}{3+5i}=\frac{17}{3+5i}\times\frac{3-5i}{3-5i} \quad [3m]$$

$51-85i$ or $9-25i^2$ [4m]

$51-85i$ and $9-25i^2$ or $\frac{51-85i}{9-25i^2}$ or $\frac{51-85i}{34}$ [7m]

$= \frac{51}{34}-\frac{85i}{34}$ or $\frac{3}{2}-\frac{5}{2}i$ [10m]

* Can use multiple of conjugate i.e. $n(3-5i)\ n\in R\quad n\neq 0$

Blunders (-3)

B1 $i^2\neq -1$ or misuse of i^2
B2 Mathematical error in multiplying out numerator – max 1 blunder
B3 Mathematical error in multiplying out denominator – max 1 blunder
B4 Inverts at final step

Misreading (-1) Unless work is simplified

Attempts (3 marks)

A1 Substitutes for u and stops
A2 Finds conjugate of u and stops
A3 Any correct relevant multiplication

Worthless (0)

W1 $17 = 3+5i$

Note Incorrect 'conjugate' which does not generate answer in the form $x+iy$, attempt mark at most

Part (c) **20 (5, 15) marks** **Att (2, 5)**

Let $z = 3 - 4i$.

(i) Calculate $|z|$.

(ii) Find the real numbers p and q such that $|z|(p+qi) + (q-pi) = 17 + 7i$.

(c) (i) **5 marks** **Att 2**

I	or **II**
$\|z\| = \sqrt{a^2 + b^2}$ [2m]	$\|z\|^2 = z \times \bar{z}$ [2m]
$= \sqrt{3^2 + (-4)^2}$ [2m]	$\|z\|^2 = (3-4i)(3+4i)$
$= \sqrt{9+16}$	$\|z\|^2 = 9+16 = 25$
$= \sqrt{25}$ [5m]	$\|z\| = \sqrt{25}$ [5m]
$= 5$	$= 5$

* No penalty for using 4 for -4 in formula

* Accept distance from (3,-4) to (0,0) or $\sqrt{a^2 - b^2 i^2}$

Blunders (-3)

B1 Incorrect formula e.g. $\sqrt{\ }$ omitted

B2 Incorrect substitution e.g. has $(-4i)^2$ in $\sqrt{a^2 + b^2}$

Attempts (-2)

A1 Correct formula and stops; modulus or distance

A2 Incorrect formula with some correct substitution

A3 Some correct substitution e.g. $|z| = |3 - 4i|$

A4 Plots $3 - 4i$

A5 Correct answer without work

Worthless

W1 Incorrect answer with out work

(c) (ii) **15 marks** **Att 5**

$|z|(p+qi) + (q-pi) = 17+7i$

$\Rightarrow 5(p+qi)+(q-pi)=17+7i$ [5m]

$\Rightarrow 5p+5qi+q-pi=17+7i$ [8m]

Real parts: $5p+q=17$

Imaginary parts: $5q-p=7$ [12m]

Solving: $\begin{matrix} 5p+q=17 \\ -5p+25q=35 \end{matrix} \Rightarrow 26q=52 \Rightarrow q=2$

$5p+q=17 \Rightarrow 5p+2=17 \Rightarrow 5p=15 \Rightarrow p=3.$ [15m]

* Accept candidates $|z|$ from (i) but if modulus is not real award 8 marks at most

Blunders (-3)

B1 Mathematical error
B2 Error in equating real to real
B3 Error in equating imaginary to imaginary
B4 Only finds one variable

Misreading (-1)

M1 Distributes $|z|$ over $(q - pi)$

Attempts (5 marks)

A1 Substitutes for $|z|$ and stops
A2 Some use of "*like to like*"
A3 Invents $|z|$ and continues with some work of merit

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

* Error in formula: if one error only, then 1×B. Otherwise it is not a valid formula.
* Do not penalise notation

Part (a) **10 marks** **Att 3**

The first term of a geometric sequence is 2 and the common ratio is 3.
Find the second term of the sequence.

(a) (i) **10 marks** **Att 3**

I

$a = 2 \quad r = 3$

$T_n = ar^{n-1} \Rightarrow T_2 = ar^{2-1} = ar = 2 \times 3 = 6$

II Lists:

2, 6, 18, 54, …

(only two terms required)

* Accept correct answer without work

Blunders (-3)

B1 T_3 or T_4 with work otherwise attempt at most

B2 18 with work

Attempts (3 marks)

A1 Correct formula T_n or S_n of a GP and stops

A2 Identifies *a* and stops

Worthless (0)

W1 Incorrect answer without work

W2 Treats as an AP but A2 may apply

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The first term of an arithmetic series is -2 and the second term is 4.

(i) Find d, the common difference.

(ii) Find T_{10}, the tenth term of the series.

(iii) The kth term of the series is 292. Find k.

(iv) Find S_{20}, the sum of the first 20 terms of the series.

(b) (i) **5 marks** **Att 2**

I

$d = T_{n+1} - T_n = T_2 - T_1 = 4-(-2) = 4+2 = 6$ or $d = 4-(-2) = 4+2 = 6$

or

II

$T_n = a+(n-1)d$

$4 = -2+(1)d$

$d = 6$

or

III List

$\overset{+6}{\rightarrow}\ \overset{+6}{\rightarrow}\ \overset{+6}{\rightarrow}\ \overset{+6}{\rightarrow}$

$-2 + 4 + 10 + 16 + 22 + \ldots$

* Accept correct answer without work

Blunders (-3)

B1 Error in formula (see guidelines)

B2 Error in using formula

B3 Sign error e.g. has 2 for 4-(-2)

Attempts (2marks)

A1 Correct formula, T_n or S_n ,of an AP and stops

A2 Identifies a and stops

A3 Lists elements of series but does not identify d at this point

Worthless (0)

W1 Incorrect answer without work

W2 Treats as an GP but A2 may apply

(b) (ii) **5 marks** **Att 2**

I

$T_n = a + (n-1)d$ [2m]

$T_{10} = a + 9d$

$= -2 + 9(6)$

$= -2 + 54$

$= 52$ [5m]

or **II** **List**

-2 + 4+ 10+ 16+ 22+ 28+ 34+ 40+ 46+ 52

III

$S_n = \frac{n}{2}\{2a + (n-1)d\}$ [2m]

$T_{10} = S_{10} - S_9 = 250 - 198 = 52$ [5m]

* Accept candidate's *d* from (i)
* Accept correct answer without work
* If candidates uses the list must indicate/state clearly which part, (ii) to (iv), is being attempted

Blunders (-3)
B1 Error in formula (see guidelines)
B2 Error in using formula
B3 Sign error
B4 Fails to identify 52, in method II, if 52 not final term
B5 46 or 58 as answer method II

Attempts (2marks)
A1 Correct formula, T_n or S_n ,of an AP and stops
A2 Identifies *a* and/or *d* and stops
A3 Partial lists, needs to have at least 3 terms
Must clearly be part(ii); don't back/double mark LIST

Worthless (0)
W1 Incorrect answer without work
W2 Treats as a GP but A2 may apply

(b) (iii) **5 marks** **Att 2**

I

$a+(n-1)d=T_n$

$a+(k-1)d=292$ [2m]

$\Rightarrow -2+(k-1)6=292$

$\Rightarrow (k-1)=294\div 6=49$

$\Rightarrow k=50$ [5m]

II List

-2+4+10+16+22+28+34+40+46+52+58+64+70+76+82+88+94+100+106+112+118+124+130+136+142+148+154+160+166+172+178+184+190+196+202+208+214+220+226+232+238+244+250+256+262+268+274+280+286+ **[292]**

$\mathbf{T_{50}}$ =292

* Accept candidate's *d* from (i)

Blunders (-3)

B1 Error in formula (see guidelines)
B2 Error in using formula
B3 Sign error
B4 Fails to identify 292 as T_{50}, see B5
B5 Offers T_{49} or T_{51} , identifies incorrectly in method II
B6 Mathematical error solving equation - see guidelines

Attempts (2marks)

A1 Correct formula, T_n or S_n ,of an AP and stops
A2 Identifies *a* and/or *d* and stops
A3 Partial lists, needs to have at least 3 terms
Must clearly be part(iii); don't back/double mark LIST

Worthless (0)

W1 Incorrect answer without work
W2 Treats as a GP but A2 may apply

(b) (iv) **5 marks** **Att 2**

I or **II**

$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ [2m] $S_n = \frac{n}{2}\{a+l\} = \frac{n}{2}\{a+T_n\}$ [2m]

$S_{20} = 10\left(-4+19\times 6\right)$ [2m] $= 10\{-2+112\}$ [2m]

$= 10\left(-4+114\right)$ [2m] $=10\{110\}$ [2m]

$= 10\times 110$ [2m] $=1100$ [5m]

$= 1100$ [5m]

or **III** **List**

-2+4+10+ 16+22+28+34+40+ 46+52+58+64+70+76+82+88+94+100+106+112 **[2m]**
=1100 **[5m]**

Blunders (-3)

B1 Error in formula (see guidelines)
B2 Error in using formula
B3 Sign error
B4 Fails to sum terms in method III (List)
B5 S_{19} (988) or S_{21}(1218) in method III
B6 Incorrect total for list method

Attempts (2marks)

A1 Correct formula, T_n or S_n ,of an AP and stops
A2 Identifies *a* and/or *d* and stops
A3 Lists elements of series but does not sum to 20 terms, needs to have at least 3 terms
Must clearly be part(iv); don't back/double mark LIST
A4 Finds T_{20} and stops

Slips (-1)

S1 Numerical slips

Worthless (0)

W1 Incorrect answer without work
W2 Treats as a GP but A2 may apply

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The first two terms of a geometric series are $-6+12+\ldots$.

(i) Find r, the common ratio.

(ii) Find T_7, the seventh term of the series.

(iii) Starting with the first term, how many terms of the series must be added to give a sum of 30.

(c) (i) **5 marks** **Att 2**

$r = \frac{T_2}{T_1} = \frac{ar}{a}$ [2m]

$r = \frac{12}{-6} = -2$ [5m]

* Accept correct answer without work

Blunders (-3)

B1 Sign errors

B2 Index errors

B3 Error in formula

Attempts (2 marks)

A1 Correct GP formula T_n or S_n

A2 Identifies a and stops

A3 -6, 12, -24 and stops Must clearly be part(i); don't back/double mark LIST

A4 $r = \frac{T_{n+1}}{T_n} = \frac{T_2}{T_1}$ and stops

A5 12:-6

Worthless (0)

W1 Incorrect answer without work e.g. 1/2 or -1/2 or 2 or -1:2

W2 Treats as an AP but A2 may apply

Note: If r positive attempt at most in parts (ii) and (iii)

(c) (ii) **5 marks** **Att 2**

I or **II** **List**

$T_n = ar^{n-1}$ [2m]

$T_7 = -6(-2)^{7-1}$

$= -6(-2)^6$

$= -6(64)$

$= -384$ [5m]

-6 +12 - 24 + 48 - 96 + 192 - 384{+768}

T_7 =-384

***** Accept candidate's *r* from (i) but see note page 27

Blunders (-3)

B1 Sign errors
B2 Index errors
B3 Error in formula - see guidelines
B4 Fails to identify T_7 - method II unless final term
B5 Answer 192 or 768 - method II

Attempts (2 marks)

A1 Correct GP formula T_n or S_n
A2 Identifies *a* and/or *r* and stops
A3 -6, 12, -24 and stops -must have at least 3 terms
Must clearly be part(ii); don't back/double mark LIST

Worthless (0)

W1 Incorrect answer without work
W2 Treats as an AP but A2 may apply

(c) (iii) **10 (5, 5) marks** **Att (2, 2)**

I or **II List**

$S_n = \dfrac{a\left(1-r^n\right)}{1-r}$

$\dfrac{-6\left(1-(-2)^n\right)}{1-(-2)} = 30$ [5m]

$-2\left(1-(-2)^n\right) = 30$

$1-(-2)^n = -15$

$(-2)^n = 16$

$n = 4$ [5 m]

-6 +12 - 24 + 48 - 96 + 192 - 384

[5] Step 1

[-6 +12 - 24 + 48]- 96 + 192 - 384

[ ] =30 4 Terms step 2

* Accept candidate's *r* from (i) but see note page 27
* Accept correct answer without work for 5+5 marks
* List method must show at least 4 terms if correct answer not offered

Blunders (-3)
B1 Sign errors
B2 Index errors
B3 Error in formula – see guidelines

Attempts (2 marks)
A1 Correct GP formula T_n or S_n Step 1
A2 Identifies *a* and/or *r* and stops
A3 -6, 12, -24 and stops -must have at least 3 terms
Must clearly be part(iii) don't back/double mark LIST Step 1
A4 Some correct work attempting to solve S_n =30 Step 2

Worthless (0)
W1 Treats as an AP but A2 may apply

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

Let $g(x) = 4 - kx$.

Given that $g(-5) = 34$, find the value of k.

(a) **10 marks** **Att 3**

$g(x) = 4 - kx$

$g(-5) = 4 - k(-5)$ [4m]

$4 + 5k = 34$ [7m]

$5k = 34 - 4$.

$= 30$

$\Rightarrow k = 6$ [10m]

* Accept correct answer without work

Blunders (-3)

B1 Mathematical error

B2 Uses $x = 5$ i.e. Solves $g(5) = 34$

B3 Interchanges coordinates ie has $g(34) = -5$ and continues

Attempts (3 marks)

A1 Uses incorrect value(s) of x but see B3 Attempting to solve by T+E

Worthless (0)

W1 Incorrect answer without work

W2 $-5(4 - kx)$ or $34(4 - kx)$

W3 Differentiates

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

Let $h(x) = x(1 - x^2)$, where $x \in \mathbf{R}$.

(i) Verify that $h(3) + h(-3) = 0$.

(ii) Find the values of x for which $h'(x) = -11$, where $h'(x)$ is the derivative of $h(x)$.

(b) (i) **10 marks** **Att 3**

I	or	**II**
$h(x) = x(1-x^2)$		$h(x) = x(1-x^2) = x - x^3$
$h(3) = 3(1-3^2)$ [3m]		$h(3) = 3 - 3^3$ [3m]
$= 3(1-9)$		$= 3 - 27$
$= 3(-8)$		$= -24$ [4m]
$= -24$ [4m]		
$h(-3) = -3(1-(-3)^2)$		$h(-3) = -3 - (-3)^3$
$= -3(1-9)$		$= -3 - (-27)$
$= +24$ [7m]		$= -3 + 27$
		$= +24$ [7m]

$$h(3) + h(-3) = 24 - 24 = 0 \quad [10\text{m}]$$

Blunders (-3)

B1 Mathematical errors e.g. powers of -3

B2 Incorrect or no conclusion shown, based on work

Attempts (3 marks)

A1 Any partial or full substitution of 3 and/or -3

A2 Uses any value other than 3 and -3

A3 $x(1-x^2) = x - x^3$ fully correct, not partial

Worthless (0)

W1 *3h – 3h = 0*

(b) (ii) **10 (5, 5) marks** **Att (2, 2)**

or

$h(x) = x\left(1-x^2\right)$ $\qquad$ $h(x) = x - x^3$

$u = x \quad \frac{du}{dx} = 1 \quad v = \left(1-x^2\right) \quad \frac{dv}{dx} = -2x$

$h'(x) = \quad (x)(-2x) + \left(1-x^2\right)(1)$ [5m] $\qquad$ $h'(x) = 1 - 3x^2$ [5m]

Step 1 Differentiate 5m

$= -2x^2 + 1 - x^2$ $\qquad$ $h'(x) = 1 - 3x^2$

$h'(x) \quad = -3x^2 + 1$

$h'(x) = -11$

. $\qquad$ $1 - 3x^2 = -11$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$ [5m]

Step 2 Solves *h'(x) =-11* 5m

* Check Guidelines for differentiation blunders
* No penalty for omission of brackets if does not affect answer
* No marks for writing *uv* formula from table

Blunders (-3)
B1 Differentiation Step 1
B2 Error in expanding brackets
B3 Only gets one solution Step 2

Attempts (2 marks)
A1 u and/or v correctly identified and stops Step 1
A2 Any correct differentiation Step 1
A3 Multiplies out *h(x)* and stops Step 1
A4 Substitutes their *h'(x)* = -11 and stops. Step 2
A5 *h'(x)* linear merits Att at most. Step 2

Worthless (0)
W1 Incorrect answer without work
W2 No differentiation unless A3 applies
W3 Writes $\frac{dy}{dx}$ and stops

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Let $f(x) = x^3 - 6x^2 + 9x - 3$, where $x \in$ **R.**

(i) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(ii) Draw the graph of the function f in the domain $0 \le x \le 4$.

(iii) Use your graph to estimate the range of values of x, for which $x < 3$ and $f(x) \ge 0$.

(c) (i) **10 (5, 5) marks** **Att (2, 2)**

$f(x) = x^3 - 6x^2 + 9x - 3 \Rightarrow f'(x) = 3x^2 - 12x + 9$ [5m] Step 1 Differentiation

$f'(x) = 0 \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow (x-1)(x-3) = 0 \Rightarrow x = 1$ or $x = 3$.

$f(1) = 1^3 - 6(1)^2 + 9(1) - 3 = 1 - 6 + 9 - 3 = 1$

$f(3) = 3^3 - 6(3)^2 + 9(3) - 3 = 27 - 54 + 27 - 3 = -3$

(1, 1) is maximum and (3, −3) is minimum.[5 m] Step 2 Finds local max and min

* Accept implied =0 if subsequent work supports it

* Second derivative not necessary to distinguish max and min

Blunders (-3)

B1 Differentiation error

B2 Mathematical error e.g. incorrect factors

B3 Only finds one local turning point but check A3 below

Slip (-1)

S1 Fails to identify local max and min from 2 points or misidentifies

Attempts (2 marks)

A1 Any term differentiated correctly Step 1

A2 States $f'(x)=0$ or $\frac{dy}{dx}=0$ and stops Step 2

A3 $f'(x)$ linear attempt at most in Step 2

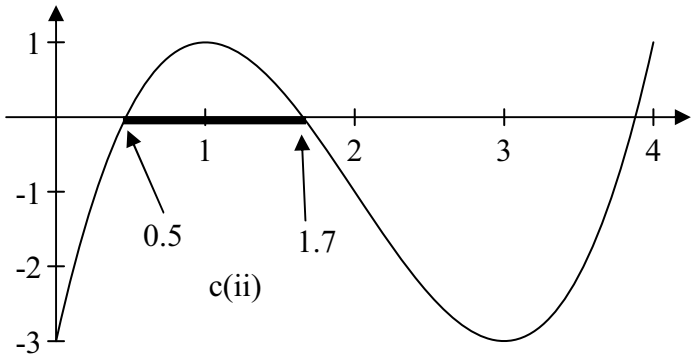
(c) (ii) **5 marks** **Att 2**

$f(x) = x^3 - 6x^2 + 9x - 3$

$f(0) = 0^3 - 6(0)^2 + 9(0) - 3 = -3$

$f(2) = 2^3 - 6(2)^2 + 9(2) - 3 = 8 - 24 + 18 - 3 = -1$

$f(4) = 4^3 - 6(4)^2 + 9(4) - 3 = 64 - 96 + 36 - 3 = 1$



* Accept candidates points from (i)
* If candidates recalculated points apply slips and blunders as per guidelines

Blunders (-3)
B1 Scale error, serious

Slips (-1)
S1 Each of candidate's points incorrectly plotted or omitted
S2 Points not joined, or joined incorrectly, or joins with a series of straight lines

Attempts (2 marks)
A1 Plots $f'(x)$
A2 Answers from part (i) transferred to this part; carries forward max and min values
A3 Effort at calculation of a point with some substitution e.g. $f(0)$
A4 Scaled and labelled axes and stops

(c) (iii) **5 marks** **Att 2**

$f(x) \geq 0$ for $0.5 \leq x \leq 1.7$ tolerance ± 0.2

* Accept answer consistent with candidate graph
* Accept answer clearly indicated on graph with x values identified
* Accept answer using words rather than symbols, and accept [0.5,1.7] [1.7,0.5]
* Accept $0.5 < x < 1.7$

Blunders (-3)
B1 Marked on graph but x values not named
B2 Inequalities not as stated

Attempts (2 marks)
A1 One correct end point identified

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

Differentiate $3x^5 - 7x^2 + 9x$ with respect to x.

(a) **10 marks** **Att 3**

$$\left[\frac{dy}{dx}\right] = 15x^4 - 14x + 9$$

3 Terms of answer (include sign)

* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated: award 4 marks.

Blunders (-3)
B1 Differentiation error once per term.
B2 Term omitted

Attempts (3 marks)
A1 A correct step in differentiation from 1st principles
A2 A correct coefficient or a correct index of x in one of the term(s)

Worthless (0)
W1 No differentiation

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i) Given that $y=(x^2-4x)^5$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(ii) Differentiate $\frac{x^2-1}{x^2+1}$ with respect to x.

Write your answer in the form $\frac{kx}{(x^2+1)^n}$ where $k,\ n \in \mathbf{N}$.

(i) **10 marks** **Att 3**

$y=\left(x^2-4x\right)^5$ Let $u=x^2-4x$ $y=u^5$

$\frac{dy}{dx}=5\left(x^2-4x\right)^4 \vdots (2x-4)$ [7m] $\frac{du}{dx}=2x-4$ $\frac{dy}{du}=5u^4$

term term $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}=5u^4 \vdots (2x-4)$ [7]

Substitution [$x=2$ $u=2^2-4(2)=-4$]

$\left.\frac{dy}{dx}\right|_{x=2}=5\left(2^2-4(2)\right)^4(2(2)-4)=5(4-8)^4(4-4)=5(-4)^4(0)=0$ [10m]

.

* Apply penalties as in guidelines for differentiation

* No penalty for missing brackets if multiplication implied. (Decide by later work.)

* Treat $5\left(x^2-4x\right)^4$ and $(2x-4)$ as separate term/parts – see above

* Part (ii) If differentiation correct accept answer **0** without work for final marks, but answer **0** with <u>no work at all:</u> award attempt 3 only

* Accept $\frac{dy}{dx}=5\left(2^2-4(2)\right)^4(2(2)-4)$ and continues correctly giving answer = 0

Blunder (-3)

B1 Differentiation error once per term -see terms above

B2 Error in substitution - once only

B3 $A\times 0\neq 0$

Attempts (3 marks)

A1 Some correct element of the chain rule e.g. index 4 or coefficient 5

A2 $u=x^2-4x$ and stops

A3 $\frac{dy}{dx}=2x-4$ and continues or not, only attempt

Worthless (0)

W1 Substitutes $x = 2$ into y; no differentiation

(ii) **10 marks** **Att 3**

$$y = \frac{x^2 - 1}{x^2 + 1}$$

Let $u = x^2 - 1$ $\qquad$ $v = x^2 + 1$

$$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{\overset{\text{term}}{\overbrace{(x^2+1)(2x)}} - \overset{\text{term}}{\overbrace{(x^2-1)(2x)}}}{(x^2+1)^2} \quad [7\text{m}]$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2} \quad [10\text{m}]$$

* Apply penalties as in guidelines
* No penalty for missing brackets if multiplication implied. (Decide by later work.)
* No marks for writing *u*/*v* formula from tables and stopping

Blunders (-3)
B1 Differentiation errors, once per term
B2 Errors in expanding to required form – once only, final 3 marks
B3 Error in formula – see guidelines

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 *u* and/or *v* correctly identified and stops
A2 Any correct differentiation

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

A ball is fired straight up in the air.
The height, h metres, of the ball above the ground is given by

$$h = 30t - 5t^2$$

where t is the time in seconds after the ball was fired.

(i) After how many seconds does the ball hit the ground?
(ii) Find the speed of the ball after 2 seconds.
(iii) Find the maximum height reached by the ball.

* Units: penalise as per guidelines.
* No retrospective/back marking.
* No penalty for incorrect notation. e.g has $\frac{dy}{dx}$ at part (ii)
* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii).

(c) (i) **5 marks** **Att 2**

$h = 30t - 5t^2 = 0$ [2m] $\Rightarrow 5t(6-t) = 0 \Rightarrow t = 0$ or $t = 6$ **seconds** [5m]

* Correct answer without work: Att 2
* $t = 0$ not required

Blunders (-3)
B1 Equation $\neq 0$
B2 Incorrect factors
B3 Incorrect roots from factors

Slip (-1)
S1 No units or incorrect units

Attempts (2 marks)
A1 Attempt at factorizing
A2 Any use of Trial and Error, using $h = 30t - 5t^2$ even if correct

Worthless (0)
W1 Differentiation

(c) (ii) **5 marks** **Att 2**

$\frac{dh}{dt} = 30 - 10t$ [2m]
$= 30 - 10(2)$.
$= 10$ m/s [5m]

* Correct answer without work: Att 2.

Blunders (-3)
B1 Differentiation error
B2 Incorrect or no value of t substituted into $\frac{dh}{dt}$
B3 Mathematical error

Attempts (2 marks)

A1 Partial differentiates at part (ii) and stops

A2 $\frac{dh}{dt}$ mentioned or any mention of derivative

Worthless (0)

W1 $t = 2$ substituted into original equation

W2 Effort to use Speed = Distance/Time

(c) (iii) Time **5 marks** **Att 2**

Height **5 marks** **Att 2**

$\frac{dh}{dt} = 30 - 10t = 0$ [2m]

$10t = 30$ Step 1 5 m

$t = 3$ [5m]

$h = 30t - 5t^2 = 30(3) - 5(3)^2$ [2m]

$= 90 - 45$

$= 45$m [5m]

Step 2 5 m

* Correct answer without work Att 2 + Att 2

* Invented value of t [except t=3] e.g. t=10 and subbed into $h = 30t - 5t^2$: award [0m] + Att [2m]

Blunders (-3)

B1 $\frac{dh}{dt} \neq 0$

B2 Incorrect substitution into h

B3 Mathematical error

Slips (-1)

S1 Numerical slips

Attempts (2 marks)

A1 States $\frac{dh}{dt}$ mentioned Step 1

A2 Speed =0 and stops Step 1

Worthless (0)

W1 Substitutes back into $\frac{dh}{dt}$ Step 2

Note: $t = 3$ must be correctly justified to merit [5m] Step 1

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

Let $g(x) = 2(6 - 3x)$, where $x \in \mathbf{R}$.
Find the value of x for which $g(x) = 0$.

(a) **10 marks** **Att 3**

$g(x) = 2(6 - 3x)$ [3m]
$\Rightarrow 12 - 6x = 0$ [4m] $\Rightarrow 6x = 12$ [7m] $\Rightarrow x = 2$ [10 m]

* Correct answers without work; full marks.

Blunders (-3)

B1 Mathematical errors when solving equation - see guidelines
B2 $g(x) \neq 0$
B3 Finds $g(0)$ correctly, answer 12 worth [7m]

Slips (-1)

S1 e.g. Has 2×6 as 11

Attempts (3 marks)

A1 Unsuccessful T +E
A2 Graphical without finding correct answer
A3 Partially multiplies out $2(6 - 3x)$ and stops

Worthless (0)

W1 Incorrect answer without work
W2 Differentiates $g(x)$ but see A3

Part (b) **20 marks** **Att 7**

Differentiate $2x^2 - 5x$ with respect to x from first principles.

(b) **20 marks** **Att 7**

or

$f(x) = 2x^2 - 5x$

I $f(x+h) = 2(x+h)^2 - 5(x+h)$ [8m]

II $= 2x^2 + 4xh + 2h^2 - 5x - 5h$ [11m]

$f(x+h) - f(x) =$

$= 2x^2 + 4xh + 2h^2 - 5x - 5h - 2x^2 + 5x$ [11m]

III $= 4xh + 2h^2 - 5h$ [14m]

IV $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2-5h}{h} = 4x + 2h - 5$ [17m]

V $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = 4x - 5$ [20m]

$y = 2x^2 - 5x$

I $y + \Delta y = 2(x+\Delta x)^2 - 5(x+\Delta x)$ [8m]

II $= 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 5x - 5\Delta x$ [11m]

$y = 2x^2 - 5x$

III $\Delta y = 4x\Delta x + 2(\Delta x)^2 - 5\Delta x$ [14m]

IV $\dfrac{\Delta y}{\Delta x} = 4x + 2\Delta x - 5$ [17m]

V $\lim\limits_{\Delta x\to 0} \dfrac{\Delta y}{\Delta x} = 4x - 5$ [20m]

I $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}$ [8m]

II $= \lim\limits_{h\to 0} \dfrac{2x^2 + 4xh + 2h^2 - 5x - 5h - 2x^2 + 5x}{h}$ [11m]

or III $= \lim\limits_{h\to 0} \dfrac{4xh + 2h^2 - 5h}{h}$ [14m]

IV $= \lim\limits_{h\to 0} (4x + 2h - 5)$ [17m]

V $= 4x + 2(0) - 5$ [20m]

* Accept $h = 0$ or $\Delta x = 0$ in <u>limit</u>.

* In Third method, if first line of LHS is $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ all relevant terms must be present on RHS of subsequent lines

Blunders(-3)

B1 Any error once per step II, III, IV, or V

Note: <u>Forced answer</u>: if earlier error generates derivative other than $4x - 5$, but this is then presented as the derivative, a blunder will apply e.g. $4x + 5$ becomes $4x - 5$

B2 Uses $2x^2 + 5x$ or $2x^2 - 5$

Attempts (7marks)

A1 $f(x \pm h)$ on LHS or some substitution of $x \pm h$ for x on RHS, or equivalent $y + \Delta y$; these only

A2 Treats as a linear

Worthless (0)

W1 Answer $4x - 5$ without work

Note: Must have correct LHS and RHS

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Let $f(x) = \dfrac{1}{x+1}$, $x \in \mathbf{R}$, $x \neq -1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the two values of x at which the slope of the tangent to the curve $y = f(x)$ is -1.

(iii) One of these tangents intersects the positive y-axis.
Find the equation of this tangent.

(c) (i) **5 marks** **Att 2**

I $f(x) = \dfrac{1}{x+1} = (x+1)^{-1}$ [2m] $\Rightarrow f'(x) = -1(x+1)^{-2}$ [5m] $= \dfrac{-1}{(x+1)^2}$

or

II $f(x) = \dfrac{1}{x+1} \Rightarrow f'(x) = \dfrac{(x+1)0 - 1(1)}{(x+1)^2}$ [5m] $= \dfrac{-1}{(x+1)^2}$

* Apply penalties as in guidelines
* No penalty for missing brackets if multiplication implied
* No marks for writing *u*/*v* formula from tables and stopping
* Error in simplification apply later part

Blunders (-3)
B1 Differentiation error
B2 Index error

Attempts (2 marks)
A1 *u* and /or *v* correctly identified and stops
A2 Any correct differentiation

(c) (ii) **5 marks** **Att 2**

$f'(x) = \dfrac{-1}{(x+1)^2} = -1$ [2m] $\Rightarrow 1 = (x+1)^2 \Rightarrow x+1 = \pm 1 \Rightarrow x = 0$ or $x = -2$ [5m]

or

$(x+1)^2 = 1 \Rightarrow x^2 + 2x + 1 = 1 \Rightarrow x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$ [5m]

* Allow candidates answer from (i) unless it oversimplifies question e.g linear
* Apply error in simplification here

Blunders (-3)
B1 Only finds one solution
B2 Mathematical error

Attempt (2 marks)
A1 Finds $f'(-1)$
A2 Mentions $f'(x)$ is slope of tangent; recognising slope

Worthless (0)
W1 Incorrect answer without work
W2 Finds $f(-1)$
W3 Solving $f(x) = -1$

(c) (iii) **10 (5, 5) marks** **Att (2, 2)**

At $x = 0$, $f(x) = 1 > 0$ required point [5m]	Step 1 Identify point
$y - y_1 = m(x - x_1)$ $(x_1, y_1) = (0.1)$ $m = -1$	
$y - 1 = -1(x - 0)$ [5m] $\Rightarrow y - 1 = -x \Rightarrow x + y - 1 = 0$	Step 2 Equation of tangent

* Accept candidates values from part (ii) and see how used. Follow candidate's work for verification

* Invented value award 0 [m] + Att [2m] at most

* Two incorrect values from c(ii), gets corresponding y value(s). Makes unverified judgement and continues to correctly find equation of tangent. Award Att [2m] and [5m]

Blunders (-3)

B1 Mathematical error

B2 Incorrect equation of a line Step 2

B3 Incorrect point Step 1

B4 Incorrect slope Step 2

Attempts (2 marks)

A1 Mentions cuts y-axis at x=0 Step 1

A2 Sketch of $f(x)$ Step 1

A3 Finds $f(-2)$ and stops Step 1

A4 Equation of line formula stated correctly and stops Step 2

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2009

MARKING SCHEME

MATHEMATICS - PAPER 2

ORDINARY LEVEL

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

Application of penalties throughout scheme

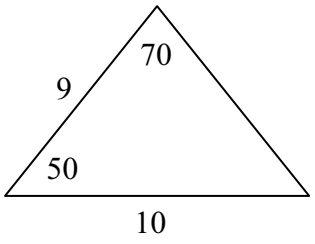
Penalties are applied subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ *or* $5p \times 4p = 20p$.
- Sign error: $-3(-4) = -12$ *or* $(-3)^2 = 6$.
- Fraction error: Incorrect fraction inversion etc. apply once.
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$.
  or $4x = 12 \Rightarrow x = 8$ each type once per section.
- Distributive law errors (once per pair of brackets)
  $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ *or* $-(4x + 3) = -4x + 3$ *or* $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly: $(2x - 3)(x + 4) = 8x^2 - 12x$.
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once
  $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

  e.g. $2x^2 + 3x + 4 = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(3)}$

  or $\dfrac{10}{\sin 70} = \dfrac{9}{\sin 50}$.



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example: $\dfrac{y_2 + y_1}{x_2 + x_1}$ *or* $A = P\left(1 + \dfrac{n}{100}\right)^r$ *or* $a^2 = b^2 + c^2 + bc\cos A$
  or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Note: A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the Tables.

Slips – examples are as follows:

- Numerical slips such as: $4 + 7 = 10$ *or* $3 \times 6 = 24$ *but* $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to the incorrect accuracy or an early round-off that affects accuracy are penalised as a slip once in each section.
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once in each section where the candidate would otherwise have obtained full marks in that section. This applies to **Q1** (a) (i), (ii), (b) (i) and (c) (i), (ii) and to **Q5** (b) (i), (ii) and (c) (i).

Misreadings

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

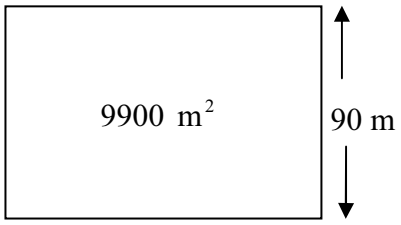
Part (a) **10 (5, 5) marks** **Att (2, 2)**

The area of a rectangular playing pitch is 9900 m^2.

The width of the pitch is 90 m.

(i) Find the length of the playing pitch.

(ii) Find the perimeter of the playing pitch.



(a) (i) **5 marks** **Att 2**

$90 \times L = 9900 \quad \Rightarrow \quad L = 9900 \div 90 = 110$ m.

(a) (ii) **5 marks** **Att 2**

$P = 2 \times L + 2 \times W = 2 \times 110 + 2 \times 90 = 220 + 180 = 400$ m.

* Accept correct answer without work, including an answer written on a diagram.
* Accept in section (ii) an answer consistent with candidate's answer to section (i).
* Any error other than an obvious slip merits the attempt mark at most.

Blunders (-3)

B1 Decimal blunder (i.e. incorrect number of 0s) – once in part (a) if the blunder is the same.

Attempts (2 marks)

A1 Some relevant work, e.g. statement of or correct use of any relevant result.

A2 Wrong operation with the equation set-up.

A3 Perimeter $= L \times L \times W \times W = 110 \times 110 \times 90 \times 90 = 98\,010\,000$ m
or $P = 2(L \times W) = 2(110 \times 90) = 19\,800$ m.

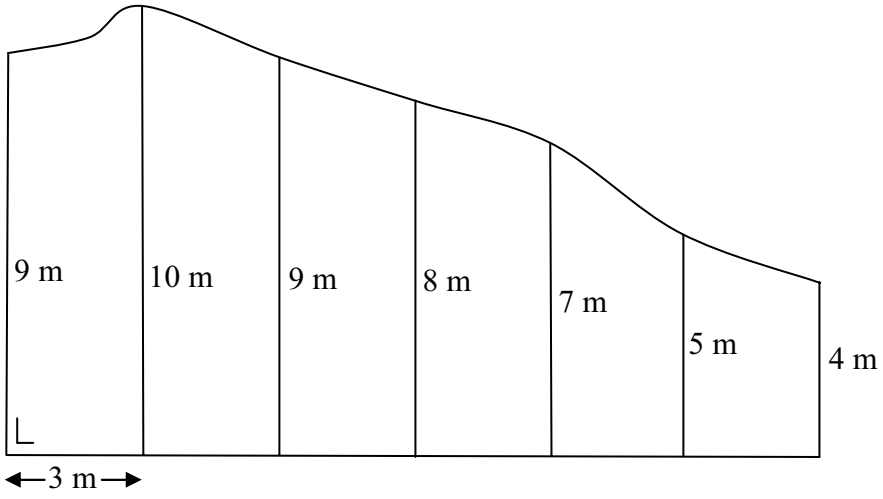
A4 Answers for perimeter of 180, 200, 220, 290, or 310, or equivalent answers consistent with the candidate's answer in (i), without work shown.

Worthless (0)

W1 Incorrect answer without work, subject to A4.

Part (b) **20 (15, 5) marks** **Att (5, 2)**

The sketch shows the garden of a house. At equal intervals of 3 m along one side perpendicular measurements are made to the boundary as shown on the sketch.



(i) Use Simpson's rule to estimate the area of the garden.

(ii) The owner of the house digs an ornamental pond in the garden. The surface area of the pond is $7\,\text{m}^2$.
What percentage of the area of the garden is taken up by the pond?
Give your answer correct to the nearest percent.

(b) (i) Use of formula **10 marks** **Att 3**
Calculations **5 marks** **Att 2**

Area $= \frac{h}{3}\left(F + L + 2\Sigma O + 4\Sigma E\right)$
$= \frac{3}{3}\left(9 + 4 + 2(9 + 7) + 4(10 + 8 + 5)\right)$ [*10 marks*]
$= 1(13 + 32 + 92) = 137\ \text{m}^2.$ [5 *marks*]

(b) (ii) **5 marks** **Att 2**

$$\frac{7}{137} \times 100 = 5.1\% = 5\%.$$

* Allow $\frac{h}{3} = \{\text{F} + \text{L} + \text{TOFE}\}$ and penalise in calculations if formula not used correctly.
* Accept correct TOFE *or* TOFE consistent with candidates F and L.
* Accept correct or consistent answer without work in section (ii).

Blunders (-3)
B1 Incorrect $\frac{h}{3}$ (once).
B2 Incorrect F and/or L or extra terms with F and/or L (once).
B3 Σ E **or** Σ O omitted (once).
B4 Incorrect TOFE (once), if not consistent with candidates F and L.
B5 Mathematical error in dealing with percentage in (ii).

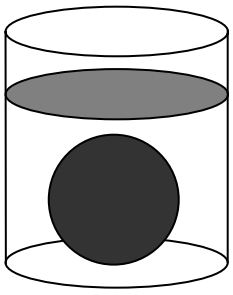
Attempts [3 marks for substituting into formula, 2 marks for calculations in (i), 2 marks in (ii)].
A1 Some relevant step, e.g. identifies F and/or L or odds or evens and stops: 3 marks.
A2 Statement of Simpson's Rule not transcribed from tables: 3 marks.
A3 Σ E **and** Σ O omitted (candidate may be awarded attempt 3 at most and/or attempt 2 marks).
A4 Correct answer without work in (i): 3 marks + 2 marks.
A5 Some correct relevant calculation only: 2 marks.

Worthless (0)
W1 Incorrect answer without work.
W2 Formula transcribed from tables and stops.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

(i) The volume of a sphere is 36π cm^3.
Find the radius of the sphere.

(ii) When the sphere is fully immersed in a cylinder of water, the level of the water rises by 2·25 cm.
Find the radius of the cylinder.



(c) (i) **10 marks** **Att 3**

$$V = \frac{4}{3}\pi r^3 = 36\pi \Rightarrow r^3 = \frac{36\times 3}{4} = 27 \Rightarrow r = \sqrt[3]{27} = 3 \text{ cm.}$$

[*3 marks*] [*10 marks*]

(c) (ii) **10 marks** **Att 3**

$$V = \pi r^2 h = 36\pi \Rightarrow r^2(2.25) = 36 \Rightarrow r^2 = \frac{36}{2.25} = 16 \Rightarrow r = \sqrt{16} = 4 \text{ cm.}$$

* Do not penalise volume of sphere as $\frac{4}{8}\pi r^3$ (Answer $r = 4.16$ cm).
* Accept an answer in section (ii) consistent with the candidate's answer to section (i).

Blunders (-3)
B1 Mathematical blunder e.g. $r^3 = 27 \Rightarrow r = 9$, applied in each section.
B2 Incorrect relevant sphere formula e.g. $\frac{1}{3}\pi r^3$ or πr^3 or $4\pi r^2$ and continues.
B3 Incorrect relevant cylinder formula e.g. $\frac{1}{3}\pi r^2 h$.

Slips (-1)
S1 Each slip to a maximum of 3 in each section.

Attempts (3 marks)
A1 Some relevant step, e.g. equation set up or volume of cylinder in (ii) with *h* substituted.
A2 Correct answer without work in each section.

QUESTION 2

Part (a)	**30 (5, 5, 5, 10, 5) marks**	**Att (2, 2, 2, 3, 2)**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Apply the following to each section of question 2 and question 3.

If the correct formula is not written, any sign or substitution error is at least a blunder.

Blunders (-3)

B_a Two or more incorrect substitutions if the formula is written.

B_b Switches x and y in substituting or treats as a pair of couples (x_1, x_2) and (y_1, y_2).

B_c Error in the central sign in a formula.

Slips (-1)

S_a One incorrect non-central sign in the formula, if the formula is written.

S_b One incorrect substitution in the formula, if the formula is written.

S_c Obvious misreading of one co-ordinate.

Attempts

A_a The correct relevant formula written and stops.

A_b The co-ordinates of a relevant point written with x_1 and y_1 identified.

A_c An incorrect relevant formula, partially substituted.

Part (a) **30 (5, 5, 5, 10, 5) marks** **Att (2, 2, 2, 3, 2)**

$a(-2, 1)$ and $b(4, 5)$ are two points.

(i) Plot the points a and b on a co-ordinate diagram.

(ii) Find the slope of ab.

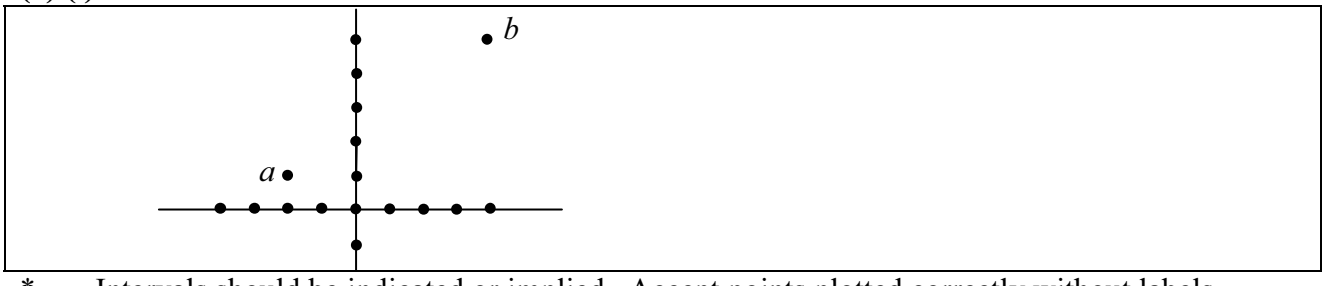
(iii) Find the equation of ab.

K is the line $3x + 2y - 9 = 0$.

(iv) Show that K passes through the midpoint of $[ab]$.

(v) Show that K is perpendicular to ab.

(a) (i) **5 marks** **Att 2**



* Intervals should be indicated or implied. Accept points plotted correctly without labels.

Blunders (-3)

B1 Scales unreasonably inconsistent (to the eye).

B2 Different scales on x and y axes.

B3 Uses a vertical x-axis and a horizontal y-axis.

B4 Point $(-2, 1)$ plotted as $(1, -2)$ and/or (4, 5) plotted as (5, 4).

Slips (-1)

S1 Points plotted but labels reversed.

Attempts (2 marks)

A1 Draws scaled axes and stops.

(a) (ii) **5 marks** **Att 2**

Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-1}{4-(-2)} = \dfrac{4}{6}$ *or* $\dfrac{2}{3}$.

* Accept a correct answer without work shown.

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $\dfrac{y_2 + y_1}{x_2 + x_1}$ or $\dfrac{y_2 - y_1}{x_1 - x_2}$ or $\dfrac{x_2 - x_1}{y_2 - y_1}$.

B2 Answer given is $\frac{6}{4}$, without work.

Attempts (2 marks)

A1 $m = \tan\theta$ or m = vertical/horizontal and stops.

Worthless (0 marks)

W1 Irrelevant formula, even if substituted, but subject to A_b.

(a) (iii) **5 marks** **Att 2**

$y - y_1 = m(x - x_1) \Rightarrow\ y - 1 = \frac{2}{3}(x + 2) \Rightarrow\ 3y - 3 = 2x + 4 \Rightarrow\ 2x - 3y + 7 = 0$.

or

$y - y_1 = m(x - x_1) \Rightarrow\ y - 5 = \frac{2}{3}(x - 4) \Rightarrow\ 3y - 15 = 2x - 8 \Rightarrow\ 2x - 3y + 7 = 0$.

or

$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1} \Rightarrow \dfrac{y-1}{5-1} = \dfrac{x+2}{4+2} \Rightarrow 6y - 6 = 4x + 8 \Rightarrow 2x - 3y + 7 = 0$.

* Accept a correct answer without work shown.
* Do not penalise for errors in simplifying the equation.
* Section (ii) not answered but slope found here for the equation – award 5 + 5 marks.
But if (ii) is answered and slope again found in (iii), marks awarded in (ii) stand.

Blunders (-3)

B1 Uses an arbitrary point for the line.
B2 Uses an incorrect or inconsistent slope.
B3 Incorrect relevant formula and continues e.g. $y + y_1 = m(x + x_1)$. [Both signs incorrect.]

(a) (iv) **10 marks** **Att 3**

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{1+5}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3).$

K: $3x + 2y - 9 = 0.$

$3(1) + 2(3) - 9 = 3 + 6 - 9 = 9 - 9 = 0.$ [Hence, $(1, 3) \in K$.]

* Accept a correct midpoint without work shown but if the midpoint is incorrect, penalise (-3) and if a negative conclusion is omitted after substitution, penalise (-1).

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ or $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$.

B2 Substitution, but work not completed to arrive at LHS = RHS.

Attempts (3 marks)

A1 Some substitution attempted or some work at simplifying or plotting the equation.

Worthless (0 marks)

W1 Irrelevant formula, even if substituted, but subject to A_b.

(a) (v) **5 marks** **Att 2**

K: $3x + 2y - 9 = 0 \Rightarrow 2y = -3x + 9 \Rightarrow y = -\frac{3}{2}x + \frac{9}{2}.$

Hence slope of $K = -\frac{3}{2}$.

Slope of $ab = \frac{2}{3}$.

$-\frac{3}{2} \times \frac{2}{3} = -1.$ [Hence, $K \perp ab$.]

Blunders (-3)

B1 Blunder in slope of K.

B2 Use of $m_1 m_2 = -1$ omitted or applied incorrectly.

Attempts (2 marks)

A1 Correct relevant formula and stops e.g. $m_1 m_2 = -1$ or $m = -\frac{a}{b}$ or $y = mx + c$.

A2 Transposes x or y and stops.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

$p(3, 0)$ is a point.

t and s are two distinct points on the y-axis and $|\,pt\,| = |\,ps\,| = 5$.

(i) Find the co-ordinates of t and the co-ordinates of s.

(ii) Find the area of the triangle tsp.

(iii) $ptus$ is a parallelogram in which $[ts]$ is a diagonal.
Find the co-ordinates of the point u.

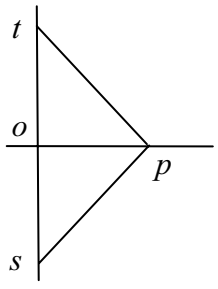
(b) (i) **10 marks** **Att 3**

$p(3, 0)$ and $t(0, y)$

$|\,pt\,| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0-3)^2 + (y-0)^2} = 5$

$\Rightarrow \sqrt{9 + y^2} = 5 \Rightarrow 9 + y^2 = 25 \Rightarrow y^2 = 25 - 9 = 16 \Rightarrow y = \pm 4$.

Thus, $t(0, 4)$ and $s(0, -4)$.



or

$|\,tp\,|^2 = |\,op\,|^2 + |\,ot\,|^2 \Rightarrow 5^2 = 3^2 + y^2 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$.

Thus, $t(0, 4)$ and $s(0, -4)$.

* Accept labels on points s and t interchanged.

* Accept a fully correct answer without work, otherwise apply A2.

Blunders (-3)

B1 Incorrect formula e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ and continues.

B2 Error in use of theorem of Pythagoras.

B3 t and/or s not written in co-ordinate form or written as $(4, 0)$ / $(-4, 0)$.

Attempts (3 marks)

A1 Oversimplifies formula e.g. $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$ with some correct substitution.

A2 Partially correct answer without work: $4, -4, \pm 4$, $(4, 0)$, $(-4, 0)$ or $x = 0$.

A3 Plots t and s on the y-axis without attempt at finding co-ordinates or $(3, 0)$ plotted.

A4 Attempt at use of theorem of Pythagoras.

Worthless (0 marks)

W1 Irrelevant formula and stops.

(b) (ii) **5 marks** **Att 2**

$t(0, 4)$, $s(0, -4)$, $p(3, 0)$

Area $= \frac{1}{2}\,|\,ts\,| \times |\,op\,| = \frac{1}{2} \times 8 \times 3 = 12$.

or

Area $=$ area Δpto + area Δpos $= \frac{1}{2}(4)(3) + \frac{1}{2}(4)(3) = 12$.

or

$t(0, 4) \to (0, 0)$, $s(0, -4) \to (0, -8)$, $p(3, 0) \to (3, -4)$

Area $= \frac{1}{2}\,|\,x_1 y_2 - x_2 y_1\,| = \frac{1}{2}\,|\,0 \times -4 - (3) \times -8\,| = \frac{1}{2}\,|\,0 + 24\,| = 12$.

or

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}\,|\,0(-4 - 0) + 0(0 - 4) + 3(4 + 4)\,| = \frac{1}{2}\,|\,0 + 0 + 24\,| = 12.$

or

Area $= \frac{1}{2}[x_1 y_2 + x_2 y_3 + x_3 y_1 - x_1 y_3 - x_3 y_2 - x_2 y_1]$

$= \frac{1}{2}\,|\,0 \times -4 + 0 \times 0 + 3 \times 4 - 0 \times 0 - 3 \times -4 - 0 \times 4\,| = \frac{1}{2}\,|\,0 + 0 + 12 + 0 + 12 - 0\,| = 12.$

* $\frac{1}{2}|-24| = -12$ incurs no penalty.

* Accept correct or consistent answer without work, including trigonometric answer.

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $\frac{1}{2}|x_1y_2 + x_2y_1|$ or omits the $\frac{1}{2}$.

B2 Error in use of translation.

B3 Arbitrary points taken for this section.

Attempts (2 marks)

A1 Uses the distance formula or the perpendicular distance formula.

A2 Correct substitution of base or height.

Worthless (0 marks)

W1 Irrelevant formula and stops e.g. $\frac{1}{2}$ on its own.

(b) (iii) **5 marks** **Att 2**

Translation $p(3, 0) \to t(0, 4)$ maps $s(0, -4) \to u(-3, 0)$.

or

Translation $p(3, 0) \to s(0, -4)$ maps $t(0, 4) \to u(-3, 0)$.

or

Midpoint of $[ts] = \left(\frac{0+0}{2}, \frac{4-4}{2}\right) = (0, 0)$

Midpoint of $[pu] = \left(\frac{3+x}{2}, \frac{0+y}{2}\right) = (0, 0) \Rightarrow x = -3,\ y = 0$.

* Accept a fully correct or consistent *u* without work.

Blunders (-3)

B1 An incorrect translation used.

B2 Blunder in use of translation, e.g. two incorrect co-ordinates, having used correct translation.

B3 Incorrect relevant midpoint formula and continues.

B4 Parallelogram used is *pust* or similar.

Slips (-1)

S1 One correct and one incorrect ordinate having used correct translation.

S2 *u* not written in co-ordinate form or written as $(0, -3)$.

Attempts (2 marks)

A1 Plots parallelogram *ptus* on a co-ordinate diagram.

A2 States "diagonals of a parallelogram bisect each other" and stops.

A3 Names the translation *pt* or *ps*.

A4 Writes $y = 0$.

QUESTION 3

Part (a)	**30 (5, 5, 10, 5, 5) marks**	**Att (2, 2, 3, 2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **30 (5, 5, 10, 5, 5) marks** **Att (2, 2, 3, 2, 2)**

The circle C has equation $x^2 + y^2 = 25$.

(i) Write down the radius of C.

(ii) Verify that the point $(4, -3)$ is on C.

(iii) The line T is a tangent to C at the point $(4, -3)$. Find the equation of T.

(iv) On a co-ordinate diagram, draw the circle C and the tangent T.

(v) L is a tangent C and L is parallel to the x-axis.
Find the two possible equations of L.

(a) (i) **5 marks** **Att 2**

Radius $= \sqrt{25} = 5$.

* Accept $r = 5$ without work.

* Accept a circle drawn with 5 shown as radius.

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $x^2 + y^2 = r$.

B2 Writes $r^2 = 25$ and writes $r = 12.5$.

Slips (-1)

S1 Writes $x^2 + y^2 = 5^2$ without writing $x^2 + y^2 = r^2$.

Misreadings (-1)

M1 Writes $x^2 - y^2 = r^2$ followed by $x^2 - y^2 = 5^2$.

Attempts (2 marks)

A1 Relevant step e.g. mentions (0, 0) or writes the distance formula or gets a point on the circle.

A2 Correct relevant formula and stops e.g. $x^2 + y^2 = r^2$ or $(x-h)^2 + (y-k)^2 = r^2$.

A3 Writes $r = 25$, with or without work.

Worthless (0 marks)

W1 Writes $r = 12.5$ without work shown.

(a) (ii) **5 marks** **Att 2**

$x^2 + y^2 = 25 \Rightarrow 4^2 + (-3)^2 = 25 \Rightarrow 16 + 9 = 25 \Rightarrow 25 = 25$. [Hence, $(4, -3)$ is on C.]

or

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0-4)^2 + (0+3)^2} = \sqrt{16+9} = \sqrt{25} = r$. [Hence, $(4, -3) \in C$.]

* Any error other than an obvious slip merits the attempt mark at most.

* Accept "distance from $(4, -3)$ to (0, 0) is 5 which is the radius".

* Accept candidate's radius from (i) above and answer must be consistent for full marks.

* Penalise an omitted or incorrect conclusion (-1).

Attempts (2 marks)

A1 Relevant step e.g. 3^2 or refers to (0, 0) or 5.

A2 Accurate diagram drawn with $(4, -3)$ shown on the circle.

A3 States or refers to theorem of Pythagoras.

(a) (iii) **10 marks** **Att 3**

Slope of $op = \dfrac{0+3}{0-4} = -\dfrac{3}{4}$.

Slope of tangent T: $m_1 m_2 = -1 \Rightarrow -\frac{3}{4} m_2 = -1 \Rightarrow m_2 = \frac{4}{3}$.

Equation of T: $y + 3 = \frac{4}{3}(x - 4) \Rightarrow 3y + 9 = 4x - 16 \Rightarrow 4x - 3y - 25 = 0$.

or

Equation of T: $y = mx + c \Rightarrow y = \frac{4}{3}x + c \Rightarrow -3 = \frac{4}{3}(4) + c \Rightarrow c = -\frac{25}{3}$.

$y = \frac{4}{3}x - \frac{25}{3} \Rightarrow 3y = 4x - 25 \Rightarrow 4x - 3y - 25 = 0$.

or

$x_1 x + y_1 y = r^2 \Rightarrow 4x - 3y = 25$.

* In the third method, accept candidate's radius from (i).

* Apply a maximum of one blunder in finding slope of radius, one blunder in finding slope of tangent and one blunder in finding the equation.

Blunders (-3)

B1 Incorrect relevant slope formula e.g. $\dfrac{y_2 + y_1}{x_2 + x_1}$ or $\dfrac{y_2 - y_1}{x_1 - x_2}$ or $\dfrac{x_2 - x_1}{y_2 - y_1}$ and continues.

B2 Uses an arbitrary point for the line.

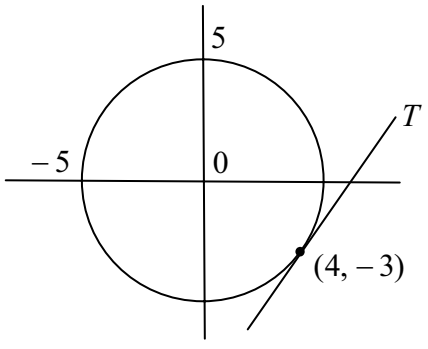
Attempts (3 marks)

A1 Correct or consistent answer without work shown.

A2 States relevant formula.

A3 Shows knowledge of what a tangent is – in this section only.

(a) (iv) **5 marks** **Att 2**



* Accept a free-hand diagram of a circle, reasonably drawn.

* Scales must be indicated or implied for full marks.

Blunders (-3)

B1 Scales unreasonably inconsistent (to the eye).

B2 Different scales on x and y axes.

B3 Uses a vertical x-axis and a horizontal y-axis.

B4 T does not pass through $(4, -3)$.

B5 An incorrect or omitted circle.

B6 An incorrect or omitted tangent.

Attempts (2 marks)

A1 Draws scaled axes and stops.

A2 Any relevant work e.g. (0, 0) or work at finding points on K.

(a) (v) **5 marks** **Att 2**

Equation of L: $y = 5$ and $y = -5$.

Blunders (-3)

B1 Fails to get the equation of the second line.

B2 Gives $x = 5$ and/or $x = -5$ as an answer.

B3 Adds at least one correct line to the diagram above without writing an equation.

Attempts (2 marks)

A1 Some relevant work e.g. refers to $(0, 5)$ or $(0, -5)$ or states lines have same slope.

A2 Gives 5 and/or -5 as an answer.

A3 Gives a correct relevant formula e.g. equation of a line.

A4 Gives slope = 0.

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

The point $c(1, -6)$ is the centre of the circle K, as shown.

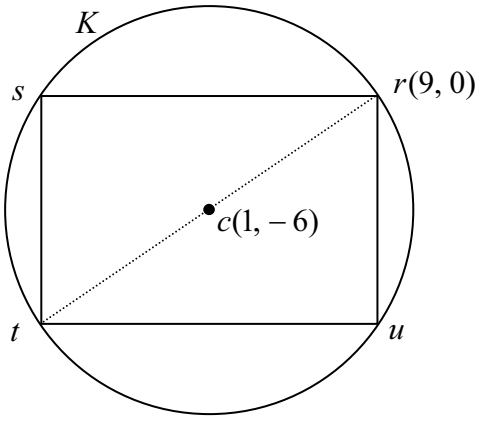
The point $r(9, 0)$ is on the circle.

(i) Find the radius of the circle.

(ii) Write down the equation of the circle.

The vertices of the rectangle *rstu* are on the circle and *sr* is horizontal.

(iii) Find the co-ordinates of *t*, the co-ordinates of *s* and the co-ordinates of *u*.



(b) (i) **5 marks** **Att 2**

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9-1)^2 + (0+6)^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$.

or

$(x-1)^2 + (y+6)^2 = r^2 \Rightarrow (9-1)^2 + (0+6)^2 = r^2 \Rightarrow r^2 = 100 \Rightarrow r = 10$.

(b) (ii) **5 marks** **Att 2**

$(x-h)^2 + (y-k)^2 = r^2 \Rightarrow (x-1)^2 + (y+6)^2 = 100$.

* Accept an answer consistent with candidate's answer to (i) without work shown.

Attempts (2 marks)

A1 Correct answer without work shown in (i).

A2 Find co-ordinates of t or attempts to use the translation $r \to c$.

A3 Correct relevant formula and stops e.g. $(x-h)^2 + (y-k)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$.

A4 Answer of $x^2 + y^2 = 100$.

Worthless (0 marks)

W1 Answer of $x^2 + y^2 = r^2$.

(b) (iii) **10 marks** **Att 3**

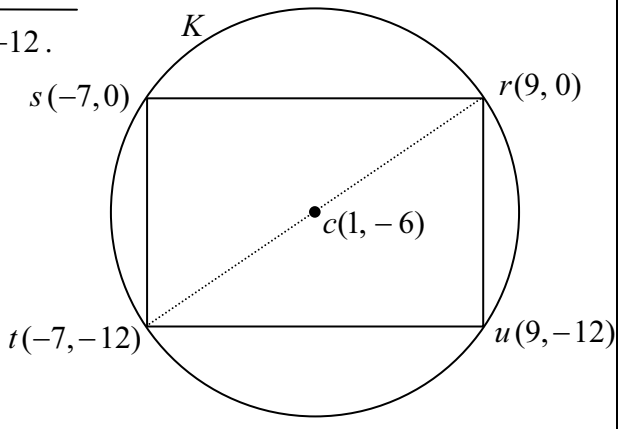
Coordinates of t:

$(9, 0) \rightarrow (1, -6)$ maps $(1, -6) \rightarrow t(-7, -12)$.

or

$\frac{1}{2}(9 + x) = 1 \Rightarrow x = -7$; $\frac{1}{2}(0 + y) = -6 \Rightarrow y = -12$.

$t(-7, -12)$.



Coordinates of s:

sr is $x-$axis $\Rightarrow$ y co-ordinate of s is 0

$sr \perp y-$axis $\Rightarrow$ x co-ordinate of s is -7

$s(-7, 0)$.

Coordinates of u:

$tu \parallel x-$axis $\Rightarrow$ y co-ordinate of u is -12

$tu \perp y-$axis $\Rightarrow$ x co-ordinate of u is 9

$u(9, -12)$.

* Accept a correct answer without work shown.

* Accept the co-ordinates found in any order.

* If a candidate merits at least an attempt mark, deduct a maximum of 3 marks for each point.

Blunders (-3)

B1 Error in use of transformation, unless an obvious slip.

B2 Picks an arbitrary point for s or u provided that the y-co-ordinate of s is 0 and the x-co-ordinate of u is 9.

Misreadings (-1)

M1 Misreads labels e.g. $u(-7, 0)$ and $s(9, -12)$.

Attempts (3 marks)

A1 Plots the points r and c.

A2 Correct relevant formula, e.g. midpoint, and stops.

A3 Relevant statement e.g. $sr \parallel x-$axis or has slope 0.

A4 Attempt to find equation of tr.

QUESTION 4

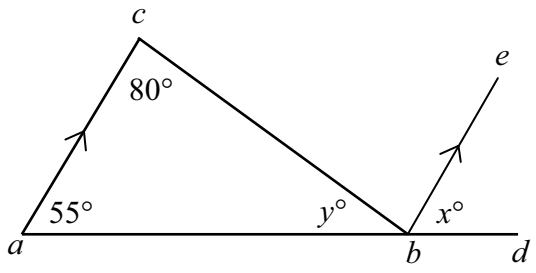
Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

In the diagram, *ac* is parallel to *be,*
$|\angle bca| = 80°$ and $|\angle cab| = 55°$.

(i) Find *x*.

(ii) Find *y*.



(a) (i) **5 marks** **Att 2**

$x° = 55°$, corresponding angles.

(a) (ii) **5 marks** **Att 2**

$y° + 80° + 55° = 180° \Rightarrow y° = 180° - (55° + 80°) = 180° - 135° = 45°$.

* Accept correct answers without work shown.
* Accept use of candidate's *x* value in finding *y*.
* Allow *x* and *y* in any order, based on the work of the candidate.
If work is not shown and $x + y = 100$ given, award 0 + 5 marks.

Blunders (-3)
B1 Incorrect geometrical result, e.g. sum of angles in triangle $\neq 180°$.

Attempts (2 marks)
A1 Statement of or use of any relevant result.
A2 Answer of *x* = 80 without work shown.

Worthless (0)
W1 Incorrect answer without work, except for A2.
W2 Stating "corresponding angles" without application to the question.

Part (b) **20 marks** **Att 7**

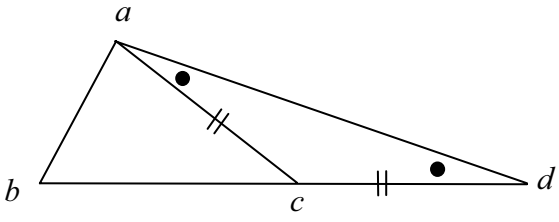
Prove that the sum of the lengths of any two sides of a triangle is greater than that of the third side.

(b) **20 marks** **Att 7**

abc is a triangle.

To prove: $|bc|+|ac|>|ab|$.

Construction: Produce bc to d such that $|cd|=|ac|$. Join a to d. [*7* marks]



Proof:

In Δacd, $|ac|=|cd|$

$\Rightarrow |\angle dac|=|\angle cda|$ …isosceles Δ [*10 marks*]

$|\angle dac|+|\angle cab|>|\angle cda|$ [*13 marks*]

$\Rightarrow |bd|>|ab|$ … side opp. greatest $\angle$ [*16 marks*]

But $|bd|=|bc|+|cd|$

Thus, $|bc|+|cd|>|ab|$ [*19 marks*]

Thus, $|bc|+|ac|>|ab|$ [*20 marks*]

* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

Blunders (-3)

B1 Each step omitted, incorrect or incomplete, except the last.

B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

Attempts (7 marks)

A1 Any relevant step, stated or indicated, e.g. triangle with additional relevant information.

A2 States or illustrates a special case, e.g. measuring the sides of the triangle.

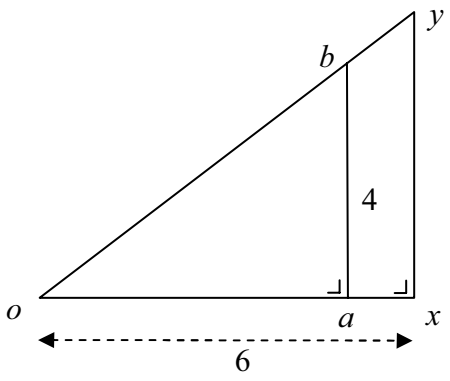
Worthless (0 marks)

W1 Any irrelevant theorem, subject to the attempt mark.

W2 Triangle only.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The right-angled triangle oxy is the image of the triangle oab under the enlargement of centre o and scale factor 1·2.
$|ab| = 4$ and $|ox| = 6$.



(i) Find $|xy|$.
(ii) Find $|oa|$.
(iii) Find the area of the triangle oab.
(iv) Find the area of the figure $axyb$.

(c) (i) **5 marks** **Att 2**

$|xy| = 4 \times 1.2 = 4.8$.

(c) (ii) **5 marks** **Att 2**

$|oa| \times 1.2 = 6 \Rightarrow |oa| = 6 \div 1.2 = 5$.

(c) (iii) **5 marks** **Att 2**

Area $oab = \frac{1}{2}|oa| \times |ab| = \frac{1}{2} \times 5 \times 4 = 10$.

(c) (iv) **5 marks** **Att 2**

Area $oxy = \frac{1}{2}|ox| \times |xy| = \frac{1}{2} \times 6 \times 4.8 = 14.4$.

or

Area oxy = area $oab \times 1.2^2 = 10 \times 1.44 = 14.4$.

Area $axyb$ = area oxy – area oab = $14.4 - 10 = 4.4$.

* Accept a correct or consistent answer without work in each section.
* Accept area of triangle found by trigonometric method.

Blunders (-3)
B1 Incorrect scale factor used.
B2 Incorrect ratio.
B3 $4 \div 1.2$ or 4×0.2 in (i) and stops.
B4 Multiplication used in (ii) i.e. 6×1.2.
B5 Incorrect area formula for triangle.
B6 Does not square scale factor in section (iv).

Attempts (2 marks)
A1 Some relevant step, e.g. indication of a correct multiplication.
A2 Attempt at a ratio.
A3 Some substitution into a correct area formula.

QUESTION 5

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

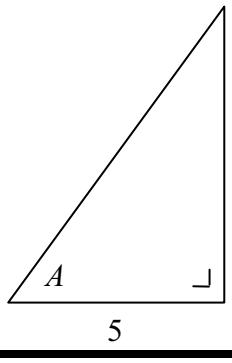
Part (a) **10 (5, 5) marks** **Att (2, 2)**

The length, 5, of a side of the right-angled triangle is shown and A is the angle indicated,

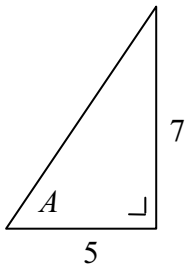
where $\tan A = \frac{7}{5}$.

(i) Copy the diagram into your answer book and on it mark the side of length 7.

(ii) Find the length of the third side.



(a) (i) **5 marks** **Att 2**



* Accept any clear indication of required side.

Attempts (2 marks)

A1 Diagram copied correctly.
A2 Writes tan A = opposite/adjacent.
A3 Incorrect side marked or both sides marked.

(a) (ii) **5 marks** **Att 2**

$x^2 = 5^2 + 7^2 = 25 + 49 = 74 \Rightarrow x = \sqrt{74}$ *or* 8.6.

* Accept a correct trigonometric method.
* Accept an answer consistent with the candidates answer given in (i).

Blunders (-3)

B1 Error in the use of theorem of Pythagoras e.g. $7^2 - 5^2$.

Attempts (2 marks)

A1 Statement or use of any relevant result or any correct step e.g. 5^2.
A2 Correct answer without work shown: $\sqrt{74}$ or 8.6… required.
A3 An exact scaled diagram giving the correct answer [8.60].

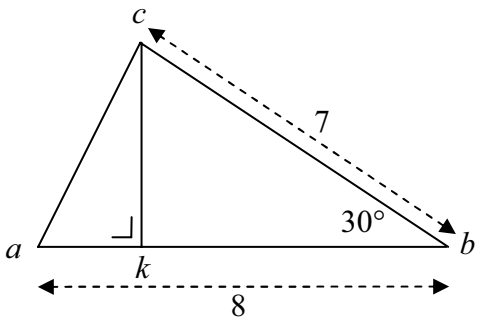
Worthless (0 marks)

W1 Incorrect answer without work.
W2 Work such as 5 + 7 or 5 + 7 = 12.
W3 Side measured from question paper [4.8 cm].

Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

In the triangle abc,
$|ab| = 8$ cm, $|bc| = 7$ cm
and $|\angle abc| = 30°$.

(i) Find the area of the triangle abc.
(ii) Given that $ck \perp ab$, find $|ck|$.
(iii) Given that $|ac| = 4$ cm, find $|\angle kca|$ correct to the nearest degree.



(b) (i) **5 marks** **Att 2**

Area $abc = \frac{1}{2} \times 7 \times 8 \times \sin 30° = 14$ cm^2.

(b) (ii) **10 marks** **Att 3**

$\frac{1}{2}|ab| \times |ck| = 14 \Rightarrow \frac{1}{2}(8)|ck| = 14 \Rightarrow |ck| = \frac{14}{4}$ or 3.5 cm.

or

$\sin 30° = \frac{|ck|}{7} \left[or \ \frac{|ck|}{\sin 30°} = \frac{7}{\sin 90°} \right] \Rightarrow |ck| = 7 \sin 30° = 7 \times 0.5 = 3.5$ cm.

(b) (iii) **5 marks** **Att 2**

$\cos|\angle kca| = \frac{|ck|}{|ac|} = \frac{3.5}{4} = 0.875 \Rightarrow |\angle kca| = 28.9° = 29°$.

* Accept an answer consistent with candidate's work in previous sections.
* If $\cos|\angle kca| > 1$, then award attempt 2 at most.

Blunders (-3)
B1 Uses radians (or gradient) mode incorrectly – apply once in part (b) and once in part (c).
B2 Incorrect area formula.
B3 Incorrect ratio and continues.
B4 Incorrect trigonometric function and continues.
B5 Incorrect function read e.g. cosine instead of sine and continues.
B6 Error in use of sine rule.
B7 Misplaced decimal point.
B8 Error in use of inverse function.
B9 Incorrect substitution into correct formula and continues.

Misreadings (-1)
M1 Finds $|\angle kac|$.

Attempts (3 marks or 2 marks)
A1 Correct answer without work shown.
A2 Trigonometric function correctly defined or found.
A3 Attempt at constructing trigonometric fractions.
A4 Incorrect relevant formula with some correct substitution.

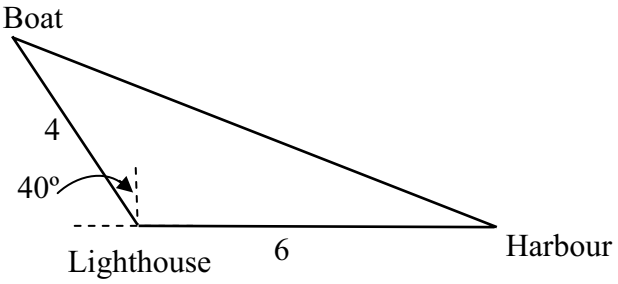
Worthless (0 marks)
W1 Writes formula from Tables and stops.
W2 Measurement from the diagram.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

A harbour is 6 km due East of a lighthouse.
A boat is 4 km from the lighthouse.
The bearing of the boat from the lighthouse is N 40° W .

(i) How far is the boat from the harbour? Give your answer correct to one decimal place.

(ii) Find the bearing of the boat from the harbour?
Give your answer correct to the nearest degree.



(c) (i) **10 marks** **Att 3**

$a^2 = b^2 + c^2 - 2bc\cos A \Rightarrow a^2 = 4^2 + 6^2 - 2(4)(6)\cos 130°$

$\Rightarrow a^2 = 16 + 36 - 48(-0.64279) = 52 + 30.85392 = 82.85392 \Rightarrow a = 9.102 = 9.1$ km.

(c) (ii) **10 marks** **Att 3**

$$\frac{\sin H}{4} = \frac{\sin 130}{9.1} \Rightarrow \sin H = \frac{4\sin 130}{9.1}$$

$$\Rightarrow \sin H = \frac{4 \times 0.7660}{9.1} = \frac{3.064}{9.1} = 0.3367 \Rightarrow H = 19.67 = 20°.$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \cos H = \frac{9.1^2 + 6^2 - 4^2}{2(9.1)(6)}$$

$$\Rightarrow \cos H = \frac{82.81 + 36 - 16}{109.2} = \frac{102.81}{109.2} = 0.94148 \Rightarrow H = 19.69 = 20°.$$

Bearing is W 20° N *or* N 70° W.

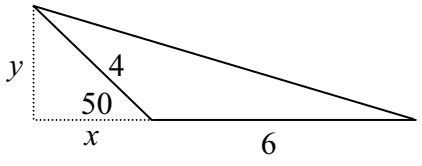
(c) Alternative solution **20 (10, 10) marks** **Att (3, 3)**

(i) $x = 4\cos 50° = 4(0.6428) = 2.57$

$y = 4\sin 50° = 4(0.7660) = 3.06$

$d^2 = 8.57^2 + 3.06^2 = 73.4449 + 9.3636 = 82.8085$

$\Rightarrow d = 9.0999 = 9.1$ km.



(ii) $\sin H = 3.06 \div 9.1 = 0.3363 \Rightarrow H = 19.65° = 20°.$

or $\cos H = 8.57 \div 9.1 = 0.9418 \Rightarrow H = 19.65° = 20°.$

or $\tan H = 3.06 \div 8.57 = 0.3571 \Rightarrow H = 19.65° = 20°.$

Blunders (-3)
As in part (b) where relevant and
B9 Error in use of cosine rule.
B10 Angle $A \neq 130°$ e.g taken as $40°$ or $140°$.

Attempts (3 marks)
As in part (b), where relevant.

Worthless (0 marks)
W1 Measurement from a diagram.
W2 Treats given triangle as right angled or use of sine rule in (i).

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

(i) Evaluate $\binom{7}{2}$.

(ii) Evaluate $\binom{7}{2}+\binom{7}{5}$.

(a) Each section **5 marks** **Att 2**

(i)	$\binom{7}{2}=\frac{7\times 6}{1\times 2}=21.$
(ii)	$\binom{7}{2}+\binom{7}{5}=21+21=42$ ***or*** $2\binom{7}{2}=42$ ***or*** $\binom{7}{2}+\binom{7}{5}=21+\frac{7\times 6\times 5\times 4\times 3}{1\times 2\times 3\times 4\times 5}=21+21=42.$

* Accept a correct answer without work shown in each section.

Blunders (-3)

B1 Treats combination as a permutation. – once in (a).

B2 Blunder in evaluating or expanding term.

Attempts (2 marks)

A1 Attempt at expanding term.

A2 Term expanded without multiplying out.

A3 In (ii) answer given is: answer to (i) + ${}^{7}C_{5}$.

Worthless (0 marks)

W1 Incorrect answer without work shown e.g. ½ or ⅕ or writes 7 – 2 or 7 – 5 and stops.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

There are 210 boys and 240 girls in a school. The school has a junior cycle and a senior cycle. The number of boys and the number of girls in each cycle is shown in the table.

	Boys	Girls
Junior cycle	120	130
Senior cycle	90	110

(i) A student is picked at random.
What is the probability that the student is a boy?

(ii) A student is picked at random.
What is the probability that the student is in the senior cycle?

(iii) A junior cycle student is picked at random.
What is the probability that the student is a girl?

(iv) A boy is picked at random.
What is the probability that he is in the senior cycle?

(b) Each section **5 marks** **Att 2**

(i)	P(student a boy) = $\frac{120+90}{210+240}=\frac{210}{450}$ *or* $\frac{7}{15}$.
(ii)	P(student in senior cycle) = $\frac{90+110}{210+240}=\frac{200}{450}$ *or* $\frac{4}{9}$.
(iii)	P(junior cycle girl) = $\frac{130}{120+130}=\frac{130}{250}$ *or* $\frac{13}{25}$.
(iv)	P(boy in senior cycle) = $\frac{90}{120+90}=\frac{90}{210}$ *or* $\frac{3}{7}$.

* If the sections of (b) or of (c) are not identified, and it is not obvious which section is being attempted treat each section in order.

* Accept answers consistent with previous work (e.g. incorrect addition of *S*), including decimal and percentage form.

* Award 5 marks for each correct answer without work shown.

Slips (-1)

S1 The addition required for final answer omitted or incorrect in each section.

Attempts (2 marks)

A1 #(E) correctly identified or given as the numerator or
#(S) correctly identified or given as the denominator.

A2 The correct answer inverted each time or partial correct answer e.g. $^{90}/_{450}$ in (ii).

A3 Statement of probability theorem awarded once unless specifically adapted to each section.

Worthless (0 marks)

W1 Use of ${}^nP_r$ or ${}^nC_r$.

W2 Incorrect answer without work shown.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Three boys and two girls are seated in a row as a group.
In how many different ways can the group be seated if

(i) there are no restrictions on the order of seating

(ii) there must be a boy at the beginning of the row

(iii) there must be a boy at the beginning of the row and a boy at the end of the row

(iv) the two girls must be seated beside each other?

(c) Each section **5 marks** **Att 2**

(i)	$5! = 120$ *or* $5\times4\times3\times2\times1 = 120$.
(ii)	$3\times4! = 72$ *or* $3\times4\times3\times2\times1 = 72$.
(iii)	$3\times3!\times2 = 36$ *or* $3\times3\times2\times1\times2 = 36$.
(iv)	$4! \times 2! = 48$.

Award marks as follows, in each section:

5 marks: fully correct answer with or without work.

2 marks: correct answer in factorial notation or given as a list with multiplication clearly indicated but not worked or a correct relevant factorial written or addition used instead of multiplication.

0 marks: incorrect answer without work shown or worthless work.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**40 (5, 5, 5, 10, 5, 5, 5) marks**	**Att (2, 2, 2, 3, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

Find the median of the numbers 3, 9, 2, 1, 13, 5, 8.

(a) (i) **10 marks** **Att 3**

Array: 3, 9, 2, 1, 13, 5, 8 Ordered array: 1, 2, 3, 5, 8, 9, 13
Median = 5

Award marks as follows:

10 marks Correct answer of 5 - accept a correct answer without work shown.

9 marks One number omitted from the ordering (obvious misreading) and the median of the remaining six found correctly.

7 marks The seven numbers ordered correctly but an incorrect or no answer given *or* the seven numbers ordered incorrectly but the middle number selected.

4 marks Answer 1 given without any attempt at reordering the array.

3 marks Attempt at reordering the array with either no or an incorrect selection *or* states there is no median (confusing median with mode) *or* defines median *or* gives answer as 2 or 3 or 8 or 9 or 13 without work *or* gives $\frac{1}{2}(3+5)$ or $\frac{1}{2}(5+8)$ without work *or* an attempt at finding the mean (answer of 5.857…).

0 marks Worthless work.

Part (b) **40 (5, 5, 5, 10, 5, 5, 5) marks** **Att (2, 2, 2, 3, 2, 2, 2)**

A car-park opens at 07:30. The number of cars entering the car-park during 15 minute intervals on a particular morning is recorded in the following table:

Time	07:30 – 07:45	07:45 – 08:00	08:00 – 08:15	08:15 – 08:30	08:30 – 08:45	08:45 – 09:00
Number of cars	20	40	100	165	105	50

[Note: 07:30 - 07:45 means 07:30 or later, but not including 07:45 etc.]

(i) How many cars entered the car-park from 07:45 to 08:30?

(ii) What was the maximum number of cars that could have entered the car park by 08:20?

(iii) Copy and complete the following cumulative frequency table:

Time	Before 07:45	Before 08:00	Before 08:15	Before 08:30	Before 08:45	Before 09:00
Number of cars						

(iv) Draw the cumulative frequency curve (ogive).

Use your curve to estimate

(v) the median time

(vi) the number of cars that had entered the car-park by 08:10

(vii) the time by which 75% of the cars had entered the car-park.

(b) (i) **5 marks** **Att 2**

40 + 100 + 165 = 305.

(b) (ii) **5 marks** **Att 2**

20 + 40 + 100 + 165 = 325.

* Accept a correct answer without work shown.

* Candidate may use cumulative frequency table or curve to get the correct answer.

* Candidate must clearly indicate the answer to (ii) – see M1 below.

Slips (-1)
S1 Writes the addition but does not add.
S2 One element incorrect, omitted or in excess with work shown.

Misreadings (-1)
M1 Finds the minimum number of cars in (ii) [Answer 160].

Attempts (2 marks)
A1 Answer of 40 or 165 in (i) without work.
A2 Answer of 165 in (ii) without work.
A3 At least two correct elements of the table, with addition indicated, written in (i) and/or (ii), subject to S2.
A4 Graphical attempt at an answer.

Worthless (0 marks)
W1 Single answers written, without work, other than those listed above.

(b) (iii) **5 marks** **Att 2**

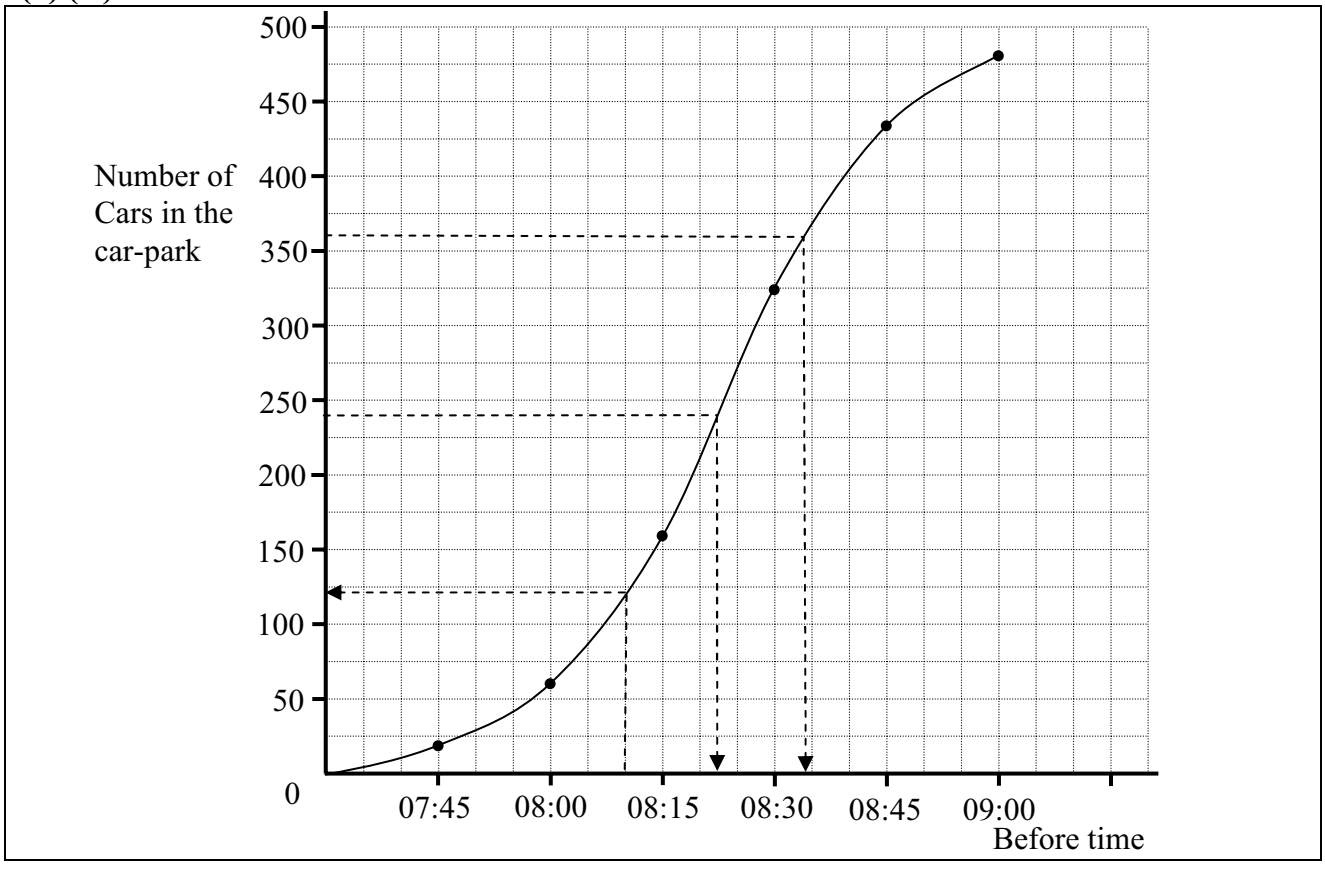
Time	Before 07:45	Before 08:00	Before 08:15	Before 08:30	Before 08:45	Before 09:00
Number of cars	20	60	160	325	430	480

* Deduct 1 mark for each incorrect and inconsistent or omitted entry subject to blunders and attempt mark.

Blunders (-3)
B1 Subtracts instead of adds.

Attempts (2 marks)
A1 One correct frequency and stops.
A2 Copies the given table and stops.

(b) (iv) **10 marks** **Att 3**



* Accept frequency on the horizontal.
* Accept a graph based on the candidates table, subject to A3, provided the candidate's table has some merit e.g. two correct entries or work at combining the cells of the original table.

Blunders (-3)
B1 Scale irregular (apply once).
B2 Draws a cumulative frequency polygon – apply slips also. [B1 may also apply]
B3 Draws a cumulative cumulative curve – apply slips also. [B1 may also apply]

Slips (-1)
S1 Each point omitted or incorrectly plotted (to the eye). [B1 may also apply]
S2 Each pair of points not joined – including (07:30, 0) to (07:45, 20).
[Note: a point omitted may incur two penalties, S1 and S2.]

Attempts (3 marks)
A1 One correct step e.g. draws axes and stops.
A2 Draws histogram correctly instead of ogive.
A3 The original table plotted.

(b) (v) **5 marks** **Att 2**

Median time 08:22.

* If the candidate draws the correct lines on the graph obtaining the answer but does not write the value, apply penalty of (-1). Refers to sections (v), (vi) and (vii).
* Accept answer based on candidates' graph, allowing tolerance of ± 05 minutes.

Blunders (-3)
B1 Starts on the incorrect axis – 08:15 which equates to 160 cars.

Attempts (2 marks)
A1 Divides 480 by 2 and stops.
A2 A relevant line drawn on the graph, allowing a tolerance of ± 10 cars for the starting point, otherwise award 0.
A3 Some relevant statement about median.

(b) (vi) **5 marks** **Att 2**

124 cars.

* Accept answer based on candidates' graph, allowing tolerance of ± 6 cars.

Attempts (2 marks)
A1 Answer of 100 or 160.
A2 A relevant line drawn on the graph allowing a tolerance of ± 5 minutes for the starting point, otherwise award 0.

(b) (vii) **5 marks** **Att 2**

75% of cars = $480 \times 0.75 = 360$ cars.
Time = 08:34.

* Accept answer based on candidates' graph, allowing tolerance of ± 04 minutes.

Attempts (2 marks)
A1 Answer of 360 or $480 \times \frac{3}{4}$.
A2 A relevant line drawn on the graph allowing a tolerance of ± 10 cars for the starting point, otherwise award 0.

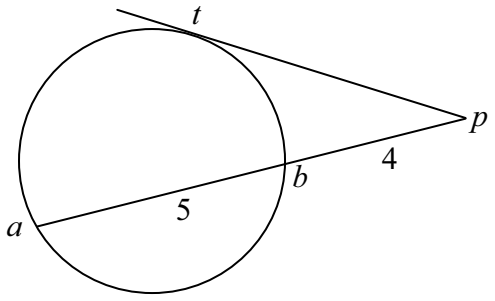
QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (5, 5, 5, 5) marks**	**Hit or miss**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

pt is a tangent to the circle at *t*.
pa intersects the circle at *b*.
$|ab| = 5$ and $|bp| = 4$.

(i) Find $|pa|$.
(ii) Find $|pt|$.



(a) (i) **5 marks** **Att 2**

$|pa| = 4 + 5 = 9$.

(a) (ii) **5 marks** **Att 2**

$|pt|^2 = |pa| \times |pb| = 9 \times 4 = 36 \quad \Rightarrow \quad |pt| = \sqrt{36} = 6$.

* Accept correct answers without work or an answer clearly indicated on a diagram.

Attempts (2 marks)
A1 Geometrical result indicated on a diagram or stated without numerical data.
A2 Some relevant step, e.g. begins a correct substitution into result, correct or otherwise.
A3 Addition used instead of multiplication in (ii).

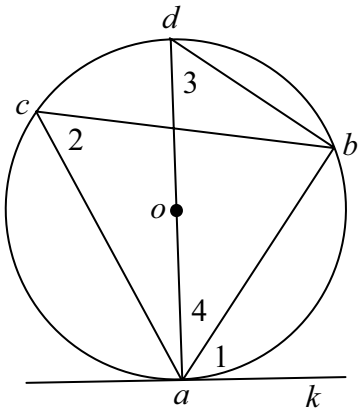
Worthless (*0 marks*)
W1 Incorrect answer without work shown.

Part (b) **20 marks** **Att 7**

Prove that an angle between a tangent *ak* and a chord $[ab]$ of a circle has degree-measure equal to that of any angle in the alternate segment.

(b) **20 marks** **Att 7**

Circle, centre *o,* chord $[ab]$ and tangent *ak*. *c* is a point on the circle in the alternate segment.
To Prove: $|\angle bak| = |\angle bca|$.



Construction: Draw the diameter $[ad]$.
Join *d* to *b*. [*7 marks*]

Proof:
$|\angle abd| = 90°$ … angle in semicircle [*10 marks*]
Thus, $|\angle 4| + |\angle 3| = 90°$ … angles in triangle [*13 marks*]
But, $|\angle 1| + |\angle 4| = 90°$ … $ad \perp ak$
Thus, $|\angle 3| = |\angle 1|$ [*16 marks*]
But $|\angle 3| = |\angle 2|$ … angles on same arc
Thus, $|\angle 1| = |\angle 2|$ [*19 marks*]
i.e. $|\angle bak| = |\angle bca|$. [*20 marks*]

* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

Blunders (-3)

B1 Each step omitted, incorrect or incomplete (except the last).

B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

Attempts (7 marks)

A1 Any relevant step, stated or indicated, e.g. circle with additional relevant information.

A2 States or illustrates a special case, e.g. measuring the angles on a diagram.

A3 Proves an angle on a diameter is a right angle.

Worthless (0 marks)

W1 Any irrelevant theorem, subject to the attempt mark.

W2 Circle only.

Part (c) **20 (5, 5, 5, 5) marks** **Hit or miss**

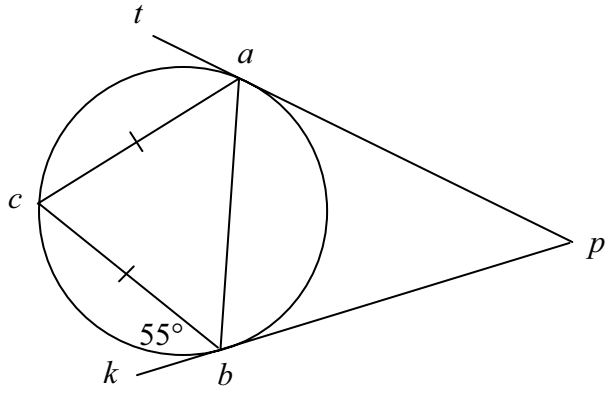
pt and pk are tangents to the circle at a and b, respectively.
c is a point on the circle such that
$|ca| = |cb|$ and $|\angle kbc| = 55°$.

(i) Find $|\angle bac|$.

(ii) Find $|\angle cba|$.

(iii) Find $|\angle acb|$.

(iv) Find $|\angle bpa|$.



(c) (i) **5 marks** **Hit or miss**

$|\angle bac| = 55°$ …. Angle in alternate segment.

(c) (ii) **5 marks** **Hit or miss**

$|\angle cba| = |\angle bac| = 55°$ …. Isosceles triangle.

(c) (iii) **5 marks** **Hit or miss**

$|\angle acb| = 180° - 2(55°) = 70°$ ….Angles in triangle = 180°.

(c) (iv) **5 marks** **Hit or miss**

$|\angle abp| = 180° - (55° + 55°) = 70°$; $|\angle bpa| = 180° - 2(70°) = 40°$.

* Accept answer written on a diagram in each section.

* Accept correct or consistent answer without work in each section.

QUESTION 9

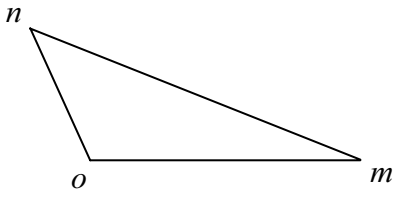
Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

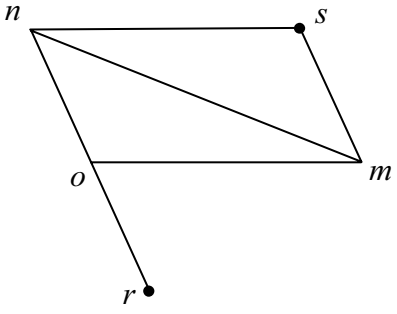
The diagram shows the triangle *omn,* where *o* is the origin. Copy the diagram and on it show

(i) the point *r* such that $\vec{r} = -\vec{n}$

(ii) the point *s* such that $\vec{s} = \vec{m} + \vec{n}$.



(a) (i) **5 marks** **Att 2**
(a) (ii) **5 marks** **Att 2**



* Answers may be on separate diagrams.

Blunders (-3)

B1 Diagram not to scale (to the eye).

B2 $\vec{r} = 2\vec{n}$.

Slips (-1)

S1 $\vec{r} = -\vec{m}$.

S2 $\vec{s} = \vec{m} \quad -\vec{n}$.

Attempts (2 marks)

A1 Copies the diagram – apply only once if no additional work added to diagram.
One diagram drawn with arrows merits one attempt unless the candidate clearly indicated that both sections are being attempted.

A2 The vector $\overrightarrow{no}$ indicated.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Let $\vec{a} = 7\vec{i} + \vec{j}$ and $\vec{b} = 5\vec{i} - 5\vec{j}$.

(i) Express $\vec{a} + \vec{b}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $\overrightarrow{ab}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Hence, or otherwise, calculate $(\vec{a} + \vec{b}) \cdot \overrightarrow{ab}$, the dot product of $\vec{a} + \vec{b}$ and $\overrightarrow{ab}$.

(iv) Is $(\vec{a} + \vec{b}) \perp \overrightarrow{ab}$? Give a reason for your answer.

(b) (i) **5 marks** **Att 2**

$\vec{a} + \vec{b} = 7\vec{i} + \vec{j} + 5\vec{i} - 5\vec{j} \downarrow = 12\vec{i} - 4\vec{j}. \downarrow$

[*2 marks*] [*5 marks*]

(b) (ii) **5 marks** **Att 2**

$\overrightarrow{ab} = \vec{b} - \vec{a} \downarrow = 5\vec{i} - 5\vec{j} - (7\vec{i} + \vec{j}) = -2\vec{i} - 6\vec{j}. \downarrow$

[*2 marks*] [*5 marks*]

(b) (iii) **5 marks** **Att 2**

$(\vec{a} + \vec{b}) \cdot \overrightarrow{ab} = (12\vec{i} - 4\vec{j}) \cdot (-2\vec{i} - 6\vec{j}) \downarrow = (12)(-2) + (-4)(-6) = -24 + 24 = 0. \downarrow$

[*2 marks*] [*5 marks*]

* Accept a correct or consistent answer without work shown in sections (i), (ii), and (iii).

(b) (iv) **5 marks** **Att 2**

Yes. $(\vec{a} + \vec{b}) \cdot \overrightarrow{ab} = 0 \Rightarrow (\vec{a} + \vec{b}) \perp \overrightarrow{ab}$ ***or*** statement "the dot product is zero".

Blunders (-3)

B1 $\overrightarrow{ab} = \vec{a} + \vec{b}$ *or* $\vec{a} - \vec{b}$ *or* $\vec{a} \cdot \vec{b}$ and continues.

B2 $\vec{i}^2 \neq 1$ or $\vec{j}^2 \neq 1$ or $\vec{i} \cdot \vec{j} \neq 0$, applied once.

B3 Incorrect relevant formula e.g. $|\vec{m}||\vec{n}|\sin\theta$ or $|\vec{m}| = \sqrt{a^2 - b^2}$.

Attempts (2 marks)

A1 Relevant work on a diagram e.g. plots one or more of the vectors.
A2 Correct relevant formula and stops.
A3 Finds the length of one vector and stops.
A4 Some correct work in multiplication using $(\vec{a} + \vec{b})$ and/or $\overrightarrow{ab}$.
A5 Writes $\cos\theta$ in terms of dot product.
A6 Writes $\cos 90° = 0$.
A7 One correct component in (i) and/or (ii) merits the attempt mark if no work is shown.

Worthless (0 marks)

W1 Incorrect answer without work, subject to A7.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $\vec{p} = 2\vec{i} + 5\vec{j}$ and $\vec{q} = \vec{i} - \vec{j}$.

(i) Find the scalars *k* and *t* such that $k\vec{p} + t\vec{q} = 14\vec{j}$.

(ii) Show that $|\vec{p} + \vec{q}| < |k\vec{p} + t\vec{q}|$.

(c) (i) **10 marks** **Att 3**

$k\vec{p} + t\vec{q} = 14\vec{j} \Rightarrow k(2\vec{i} + 5\vec{j}) + t(\vec{i} - \vec{j}) = 0\vec{i} + 14\vec{j}$ [*3 marks*]

$\Rightarrow (2k + t)\vec{i} + (5k - t)\vec{j} = 0\vec{i} + 14\vec{j}$.

$\vec{i}$ components: $2k + t = 0$

$\vec{j}$ components: $5k - t = 14$ [*7 marks*]

Solving simultaneously: $7k = 14 \Rightarrow k = 2$

$2(2) + t = 0 \Rightarrow t = -4$. [10 *marks*]

(c) (ii) **10 marks** **Att 3**

$|\vec{p} + \vec{q}| = |3\vec{i} + 4\vec{j}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. [*3 marks*]

$|k\vec{p} + t\vec{q}| = |0\vec{i} + 14\vec{j}| = 14$. [*7 marks*]

and $5 < 14$ [or $\sqrt{25} < \sqrt{196}$]. [Thus, $|\vec{p} + \vec{q}| < |k\vec{p} + t\vec{q}|$]. [10 *marks*]

Blunders (-3)

B1 Mixes up the *i* and the *j* components.

B2 Blunder in formula e.g. square root omitted or squares omitted or − instead of +.

Slips (-1)

S1 Interchanges $\vec{p}$ and $\vec{q}$ to find $k\vec{q} + t\vec{p} = 14\vec{j}$.

Attempts (3 marks)

A1 Correct answer without work shown.

A2 Some effort at scalar multiplication or combining components.

A3 Finds the square of the coefficients of any of the given components and stops.

A4 Effort at use of relevant square root.

A5 Relevant work on a diagram e.g. plots one or both vectors.

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

€6000 is invested at 5% per annum compound interest.
Find the value of the investment at the end of 10 years, correct to the nearest euro.

(a) **10 marks** **Att 3**

$A = 6000(1 + 5/100)^{10} = 6000(1.05)^{10} = 6000(1.62899) = 9773.36 = €9773.$

or

Calculation on year by year basis:

Year 1:	€6000	+	€300
Year 2:	€6300	+	€315
Year 3:	€6615	+	€330.75
Year 4:	€6945.75	+	€347.2875
Year 5:	€7293.04	+	€364.652
Year 6:	€7657.69	+	€382.8845
Year 7:	€8040.57	+	€402.0285
Year 8:	€8442.60	+	€422.13
Year 9:	€8864.73	+	€443.2365
Year 10:	€9307.96	+	€465.398 = €9773.358 = €9773.

Blunders (-3)

B1 Sign error in formula – uses 1 – 0.05.
B2 Subtracts in the long calculation.
B3 Each year omitted in the calculation on a year by year basis.
B4 Writes $(1.05)^{10} = 10.5$.

Slips (-1)

S1 Early round-off that affects the accuracy of the answer – (maximum of 3 in long calculation).
S2 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 Mention of 0.05 or 1.05 or $5/100$ or $105/100$.
A2 Some relevant step e.g. 5% of 6000 = 300 and stops or 105% of 6000 = 6300 and stops.
A3 Correct answer without work.
A4 Simple interest calculated for the ten years (Answer €9000).

Worthless (0 marks)

W1 6000/5 = 1200 or $6000 \times 50\% = 3000$.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

(i) Expand $(1 + x)^5$ fully.

(ii) Simplify $(1 + x)^5 - (1 - x)^5$.

(iii) Hence, find the value of $(1 + \sqrt{2})^5 - (1 - \sqrt{2})^5$.
Give your answer in the form $k\sqrt{2}$ where $k \in \mathbf{N}$.

(b) (i) **10 marks** **Att 3**

$$(1+x)^5 = \binom{5}{0} + \binom{5}{1}(x) + \binom{5}{2}(x)^2 + \binom{5}{3}(x)^3 + \binom{5}{4}(x)^4 + \binom{5}{5}(x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5.$$

* Accept a correct answer without work.
* Accept long multiplication or Pascal's triangle.

Blunders (-3)
B1 The number of terms is 5 or 7.
B2 Incorrect power in a term.
B3 Incorrect coefficients in a term.
B4 Incorrect sign or puts a sign between the coefficient and corresponding variable.

Slips (-1)
S1 Expands $(1-x)^5$.

Attempts (3 marks)
A1 The number of terms is less than 5 or greater than 7.
A2 Any term, including the first term, written correctly.
A3 Effort at Pascal's triangle.
A4 Gives all coefficients only.
A5 Any step towards getting a coefficient, including writing it in combination form.
A6 Any correct step towards long multiplication.

Worthless (0 marks)
W1 Writes $5(1+x)^4$.

(b) (ii) **5 marks** **Att 2**

$$(1+x)^5 - (1-x)^5 = 1+5x+10x^2+10x^3+5x^4+x^5-1+5x-10x^2+10x^3-5x^4+x^5.$$
$$= 10x+20x^3+2x^5.$$

* Accept an answer consistent with candidate's answer to (i), if not oversimplified.

Blunders (-3)
B1 Expands $(1-x)^5$ incorrectly using binomial or long multiplication.
B2 Writes $(1-x)^5$ as $-1-5x-10x^2-10x^3-5x^4-x^5$.

Slips (-1)
S1 Expands $(1+x)^5 + (1-x)^5$ as $2+20x^2+10x^4$, otherwise blunder.
S2 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Any relevant work.

(b) (iii) **5 marks** **Att 2**

$$(1+\sqrt{2})^5 - (1-\sqrt{2})^5 = 10(\sqrt{2})+20(\sqrt{2})^3+2(\sqrt{2})^5 = 10\sqrt{2}+40\sqrt{2}+8\sqrt{2} = 58\sqrt{2}.$$

* Accept an answer consistent with candidates answer from (ii), if not oversimplified.

Award marks as follows:
5 marks: Answer fully correct.
2 marks: Some relevant attempt.
0 marks: Worthless work.

Attempts (2 marks)
A1 Identifies $x=\sqrt{2}$ and stops.
A2 Correct answer without work shown, including decimal answer [82.02].

Worthless (0 marks)
W1 An incorrect answer without work.

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

The first term two terms of a geometric series are $6 + \frac{18}{4} + \ldots$

(i) Find S_{20}, the sum of the first 20 terms of the series, correct to one decimal place.

(ii) Find S_∞, the sum to infinity of the series.

(iii) Find $S_\infty - S_{20}$.

(c) (i) **10 marks** **Att 3**

$a = 6, \quad r = 18/4 \div 6 = 3/4$ [*3 marks*]

$$S_{20} = \frac{6\left(1-(3/4)^{20}\right)}{1-3/4} \downarrow = \frac{6(1-0.00317)}{0.25} = \frac{5.98098}{0.25} = 23.92392 = 23.9. \downarrow$$

[*4 marks*] [*10 marks*]

(c) (ii) **5 marks** **Att 2**

$$S_\infty = \frac{a}{1-r} = \frac{6}{1-3/4} \downarrow = \frac{6}{1/4} = 24. \downarrow$$

[*2 marks*] [*5 marks*]

or

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{6\left(1-(3/4)^n\right)}{1-3/4} \downarrow = \frac{6}{1/4} = 24. \downarrow$$

[*2 marks*] [*5 marks*]

(c) (iii) **5 marks** **Att 2**

$S_\infty - S_{20} = 24 - 23.9 = 0.1$

* Accept an answer consistent with (i) and (ii).

Blunders (-3)

B1 Incorrect *a*.
B2 Incorrect *r*.
B3 Blunder in fractions.
B4 Incorrect relevant formula e.g. + instead of – in formula – answer 3.4.
B5 Finds limit as $n \to 0$ in the second method in (ii).

Slips (-1)

S1 Numerical slips to a maximum of 3.
S2 One incorrect sign in the S_n formula.

Attempts (3 marks or 2 marks)

A1 Correct relevant formula and stops.
A2 Some relevant step e.g. states the value for *a* or the value for *r*.
A3 Adds 2 or more of the given terms e.g $S_2 = 42/4$ or $S_3 = 111/8$.
A4 One correct step in adding relevant fractions.
A5 Treats as arithmetic series with further work, e.g. identifies *a*.
A6 Writes $T_n = ar^{n-1}$ or $6(3/4)^{n-1}$ or gives $T_3 = 27/8$ or $T_4 = 81/32$.
A7 Correct answer without work.

Worthless (0 marks)

W1 Formula for arithmetic series and stops.
W2 Incorrect answer without work.

QUESTION 11

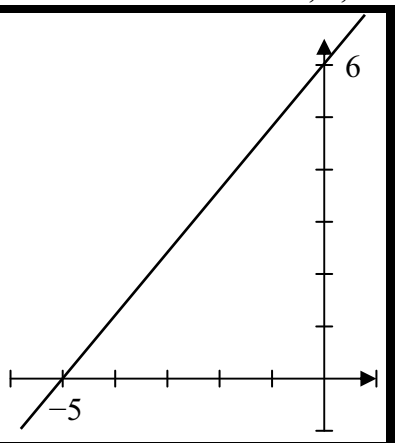
Part (a)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (b)	**35 (20, 10, 5) marks**	**Att (8, 4, 2)**

Part (a) **15 (5, 5, 5) marks** **Att 2, 2, 2**

The diagram shows the line $6x - 5y + 30 = 0$.

(i) Copy the diagram into your answer book and on it show the set of points which satisfy the inequality $6x - 5y + 30 \leq 0$.

(ii) Using the same diagram, illustrate the inequality $y \geq 2$.



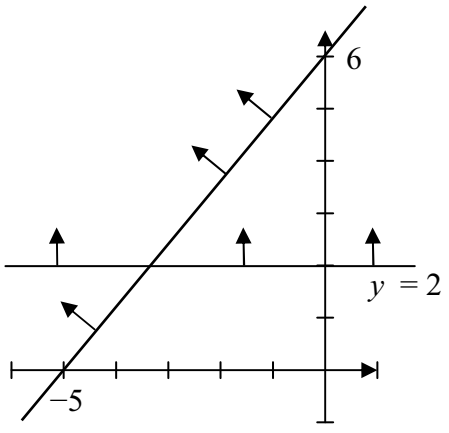
(a) (i)	**5 marks**	**Att 2**
(a) (ii) Draws line	**5 marks**	**Att 2**
(a) (ii) Shows inequality	**5 marks**	**Att 2**

$6x - 5y + 30 \leq 0$

$6(0) - 5(0) + 30 = 30 \leq 0$

which is false.

The required half-plane does not contain the origin.



Blunders (-3)

B1 Switches x and y in substituting a point.

B2 Incorrect half-plane selected.

B3 Draws line $x = 2$ - may then be awarded 5 marks for correct inequality $x \geq 2$.

Slips (-1)

S1 One incorrect substitution in the inequality.

S2 Numerical slips to a maximum of 3.

Attempts (2 marks)

A1 Substitutes any point and stops or tests a point in the inequality.

A2 Draws the given diagram.

Worthless (0 marks)

W1 Any inequality involving an axis e.g. $x \geq 0$ or $y \leq 0$.

W2 Finds a point on the line, except (0, 2) or (2, 0) or $(0, -2)$.

Part (b) **35 (10, 10, 5, 5, 5) marks** **Att (4, 4, 2, 2, 2)**

A person is setting up a new taxi firm. The firm will use medium cars and large cars.
Each medium car costs €20 000 and each large car costs €30 000.
The person has at most €300 000 to purchase the cars.

At any given time there are at most 13 drivers available to operate the taxis.

(i) Taking x as the number of medium cars and y as the number of large cars, write down two inequalities in x and y and illustrate these inequalities on graph paper.

(ii) The estimate of the monthly profit on a medium car is €800 and on a large car is €900. How many of each type of car should the person buy to maximise profit?

(iii) On your graph, show the region where the monthly profit is at most €7200.

(b) (i) Inequalities **10 (5, 5) marks** **Att (2, 2)**

Cost: $20000x + 30000y \le 300\ 000$ or $2x + 3y \le 30$

Drivers: $x + y \le 13$.

* Accept correct multiples or fractions of inequalities or the use of different letters.

* Apply (-3), once, if no inequality sign or the incorrect inequality sign is written the first time it occurs.

	Medium x	Large y	Maximum
Cost	20 000	30 000	300 000
Drivers	1	1	13

Blunders (-3)

B1 Mixes up x's and y's (once if consistent error).

B2 Confuses rows and columns in table, e.g. $2x + y \le 30$ (once if consistent).

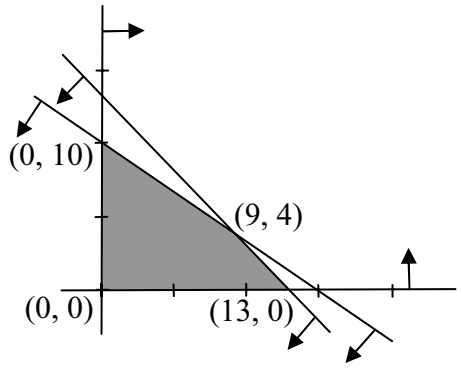
B3 Decimal blunder applies for error with zeros in equation, unless an obvious misreading.

Attempts (2 marks)

A1 Incomplete relevant data in table and stops e.g. x or $2x$ or ≤ 30 (each inequality).

A2 Any other correct inequality, e.g. $x \ge 0$, $y \ge 0$, (each time).

(b) (i) Graph **10 (5, 5) marks** **Att 2, 2**



* Points or scales required.

* Correct shading over-rules arrows or correct arrows overrule shading.

* Inequalities not written but correct graph drawn – award 0 + 10 marks.

* Two lines drawn and no shading indicated, only one of the following cases applies:

Case 1:	Two sets of arrows in expected direction	10 marks
Case 2:	Two sets of arrows in unexpected direction	10 marks
Case 3:	One set of arrows "correct", the other "incorrect"	7 (5 + Att2) marks

Case 4: One line with and the other without arrows 7 (5 + Att2) marks
Case 5: No arrows 4 (Att2 + Att2) marks
Case 6: Half-planes consistent with incorrect, penalised inequalities. 10 marks

Blunders (-3)
B1 Blunder in plotting a line or calculations.
B2 Incorrect shading e.g. one or both of the small triangles shaded.

Attempts (2 marks)
A1 Some relevant work towards a point on a line – apply to each line attempted.
A2 Draws scaled axes or axes and one line – a second line merits second attempt mark.

(b) (ii) Intersection of lines 5 marks Att 2

$$\begin{aligned} 2x + 3y &= 30 \\ 2x + 2y &= 26 \\ \hline y &= 4 \qquad \Rightarrow x = 9 \end{aligned}$$

* Accept candidate's own equations from previous sections.
* If solving incorrect equations, the point found may be outside the feasible set – award marks for correct work and accept in later sections.

Blunders (-3)
B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
B2 x or y value only found.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Correct or consistent answer without work or from a graph.
[Should get the *exact same* values from graph as if they had been found algebraically.]
A2 Any relevant step towards solving equations.

Worthless (0 marks)
W1 Incorrect answer without work and inconsistent with graph.

(b) (ii) Income 5 marks Att 2

Step 1	Vertices	$800x + 900y$	Profit
	(0, 0)	0 + 0	0
Step 2	(13, 0)	10400 + 0	10400
Step 3	(9, 4)	7200 + 3600	10800
Step 4	(0, 10)	0 + 9000	9000

Step 5 9 medium cars and 4 large cars to maximise profit.

* Information does not have to be in table form.
* Accept any correct multiple or fraction of $800x + 900y$ here.
* Accept work on a feasible set of points formed by axes and one line without further penalty.
* Accept only vertices consistent with previously accepted work, not arbitrary ones. If (15, 0) or (0, 13) is tested and result is used to give maximum income, award zero for step 5.
* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous work and also reward it here if the step is correct.
* Step 5 must be explicitly written to gain full marks.

* Testing only (9, 4) to get 10 800 merits Att 2 even if the candidate writes 9 medium-sized cars and 4 large cars i.e. no comparison means the attempt mark at most.

Award marks as follows:

5 marks: Answer is fully correct or consistent.
4 marks: The maximum value is identified but step 5 not stated.
2 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 marks)

A1 Any relevant work involving x or y and/or 800, 900 or similar.
A2 Any attempt at substituting co-ordinates into some relevant expression.
A3 Any step omitted, subject to the case for awarding 4 marks.

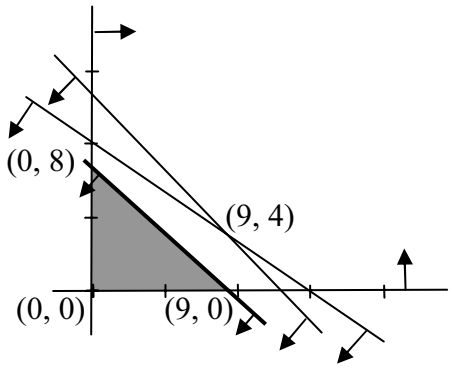
Worthless (0 marks)

W1 Writing €800 or €900 without further work.

(b) (iii) Profit **5 marks** **Att 2**

$800x + 900y \leq 7200$

or $8x + 9y \leq 72.$



* Accept correct multiples or fractions of the inequality.

Award marks as follows:

5 marks: Answer is fully correct or consistent, with conclusion.
2 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 marks)

A1 The point (9, 0) or (0, 8) plotted.
A2 A subset of the feasible set bounded by the axes shaded.

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0