Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéimeanna Marcála	Scrúduithe Ardteistiméireachta, 2007
Matamaitic	**Gnáthleibhéal**

Marking Scheme	Leaving Certificate Examination, 2007
Mathematics	**Ordinary Level**

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE MATHEMATICS

ORDINARY LEVEL

MARKING SCHEME

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Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2007*
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MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2007

MATHEMATICS – ORDINARY LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error -3(-4) = -12
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error :e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$ *or* $4x = 12 \Rightarrow x = 8$ each time.
- Distribution error (once per term, unless directed otherwise) e.g. $3(2x+4) = 6x + 4$ *or* $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly: apply once unless directed otherwise, e.g. $(2x-3)(x+4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once $2x^2 - 2x - 3 = (2x-1)(x+3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formulae: e.g. $T_n = 2a + (n-1)d$
- Central sign error in *uv* or *u/v* formulae
- Omission of $\div v^2$ or division not done in *u/v* formula (apply once)
- Vice-versa substitution in *uv* or *u/v* formulae (apply once)

Slips (-1)

- Numerical slips: 4 + 7 = 10 or 3 ×6 = 24 *but* 5 + 3 = 15 is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off is penalised as a slip each time,
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per section (a), (b) and (c) of each question.(Deduct at first non-zero or non attempt mark section, where applicable.

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so MR(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

* Incorrect or omitted units: penalise as per guidelines

Part (a)	**10 marks**	**Att 3**

Convert 164 miles to kilometres, taking 5 miles to be equal to 8 kilometres.

(a)	**10 marks**	**Att 3**

5 miles = 8 km	[3m]	*or*	164 /5	*or*	164×8	*or*	
1 mile = $^8/_5$ or [1·6]	[4m]		32·8		1312		
164 miles = $^8/_5$ ×164 km	[7m]		32·8×8		1312÷5		164 × $^8/_5$
= 262·4 km	[10m]		262·4 km		262·4 km		262·4 km.

* Accept correct answer without work.

Blunders (-3)

B1 An incorrect numerator, e.g. 164×5 and continues.

B2 An incorrect denominator, e.g. 164/8 and continues.

Note: $\frac{164\times 5}{8}$ = 102.5 is 1×B(inversion); (if then not calculated: 2×B = 4m).

B3 Incorrect or no simplification, or simplification not possible.

Slips (-1)

S1 A numerical slip.

Attempts (3 marks)

A1 Mentions 8 times and stops

A2 Mentions $^1/_5$ and stops

A3 164 × correctly by a spurious number

Worthless (0)

W1 Mentions 1/8 and stops

W2 Incorrect answer with no work

Part (b) **20 (10, 10) marks** **Att (3, 3)**

€8500 was invested for 2 years at compound interest.

(i) The rate of interest for the first year was 4%.
Find the amount of the investment at the end of the first year.

(ii) The amount of the investment at the end of the second year was €9237·80.
Find the rate of interest for the second year.

(b)(i) **10 marks** **Att 3**

$100 + 4 = 104$ [3m] $\frac{8500 \times 104}{100}$ [7m] $= €8840$ [10m]	€8500 × 1.04 = €8840	$8500 \times {}^{4}/_{100}$ [3m] = 340 [7m] 8500+340 [9m] = €8840 [10m]

* Correct answer without work: full marks.
* €340 without work 7m

Blunders (-3)
B1 Uses ${}^{100}/_{104}$ and finishes (= 8173.0769..) or similar
B2 Subtracts the 4% (= 8160)
B3 Calculates simple interest/amount for 2 years (I=680; A = 9180)

Slips (-1)
S1 Each numerical slip.

Attempts (3 marks)
A1 Mentions 104 or ${}^{4}/_{100}$ and stops
A2 Mentions 100 or 100% and stops.
A3 Relevant formula and stops

(b)(ii) **10 marks** **Att 3**

9237.80 - €8840 = €397·8 [3m] $\frac{397.8 \times 100}{8840}$ [7m] $= 4 \cdot 5\%$ [10m]	$\frac{9237.80 \times 100}{8840}$ [4m] $= 104 \cdot 5$ [7m] [104·5-100] = 4·5%. [10m]	$A = P(1 + {}^{r}/_{100})^{n}$ [3m] $9237.80 = 8840(1 + {}^{r}/_{100})^{1}$ [4m] = 1·045 [7m] 4·5% [10m]

* Accept candidate's answer from part (b)(i).
* Correct answer without work Att.

Blunders (-3)
B1 Inverted fraction and continues
B2 Incorrect denominator e.g. 8500

Attempts (3 marks)
A1 9237·80 – candidate's answer and stops
A2 Relevant formula and stops

Worthless(0)
W1 Finds 4% of 8840

Part (c) **20 (10, 10) marks** **Att (3, 3)**

The table shows the hours Alan worked over four days.

Day	Thursday	Friday	Saturday	Sunday
Hours worked	9	9	9.5	*h*

Alan's basic rate of pay is €15·60 per hour.
He is paid one and a half times the basic rate for work on Saturday and Sunday.

(i) Calculate Alan's total pay for Thursday, Friday and Saturday?

(ii) Alan was paid a total of €702 for the four days' work.
Find *h*, the number of hours Alan worked on Sunday.

(c)(i) **10 marks** **Att 3**

$\underbrace{9\times 15{\cdot}60 \text{ and } 9\times 15{\cdot}60}_{[4m]}$ and $\underbrace{9{\cdot}5\times 15{\cdot}6\times 1{\cdot}5}_{[7m]}$ $= €503{\cdot}10$ [10m]

interchangeable

$(9 + 9 + 9{\cdot}5 \times 1{\cdot}5)$ [4m] $\times$ $15{\cdot}6$ [7m] $= €503{\cdot}10$ [10m]

* Correct answer without work: Att3.

Blunders (-3)
B1 Ignores or mishandles 1·5
B2 Adds in Sunday (using 9·5)

Slips (-1)
S1 Each numerical slip.

Attempts (3 marks)
A1 140·40 only and stops.
A2 15·60×1.5 and /or 23·40 and stops
A3 9·5×1·5 and/or 14·25 and stops

Worthless(0marks)
W1 Incorrect answer without work (other than specific attempts)

(c)(ii) **10 marks** **Att 3**

Pay for Sunday = €702 - €503·10 = €198·90 [3m]
$\frac{198\cdot 9}{15\cdot 60\times 1\cdot 5}$ or $\frac{198\cdot 9}{23\cdot 4}$ [7m]
$= 8\cdot 5$ hours [10m]

* Correct answer without work: Att 3
* Accept candidate's value for (15·60×1·5) from (i)
* Accept candidate's answer from (i)

Blunders (-3)
B1 Ignores or mishandles 1·5 (giving 12·75 hours)
B2 Uses $^{702}/_{4}$ = (175·5) and continues

Slips (-1)
S1 Each numerical slip.

Attempts (3 marks)
A1 23·40 appearing in this part and stops.
A2 702– ans from (i) and stops.

Worthless(0)
W1 175·50 or 702÷4 appearing and stops.
W2 Using 9·5 hours for *h* and 'calculating' pay

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Find the solution set of $4x - 15 < 1$, $x \in$ **N.**

(a) **10 marks** **Att 3**

$4x - 15 < 1 \quad \Rightarrow \quad 4x < 16$ [4m] $\Rightarrow \quad x < 4$. [7m]

$= \{0, 1, 2, 3\}$ [10m]

* Do not penalise omission of 0 in solution

* Accept {0, 1, 2, 3} or {1, 2, 3} without work

Blunders (-3)

B1 Algebraic errors each time

B2 Inequality ignored i.e. $x = 4$ and stops

B3 $x \in R$ or $x \in Z$

Slips (-1)

S1 4 included and/or each element of solution set $\in$ N, each time to max -3

Attempts (3 marks)

A1 Any correct transposition

A2 Any use of N with T + E

A3 {0, 1, 2, 3, 4} or {1, 2, 3, 4} without work

Worthless(0 marks)

W1 Any incorrect answer without work other than A3

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

(i) Find the value of $\dfrac{x+3y+5}{2x+2y}$ when $x=\dfrac{5}{2}$ and $y=\dfrac{1}{3}$.

(ii) Find the value of x for which $2^{x+3}=4^x$.

(b)(i)	**10 marks**	**Att 3**

$$\frac{5/2+3(1/3)+5}{2(5/2)+2(1/3)}\ [3m] \quad = \frac{17/2}{17/3}\ [7m] \quad = \frac{3}{2}\ [10m].$$

* Correct answer without work: Att 3 marks

Blunders (-3)
B1 Error in substitution, if not an obvious misreading
B2 Mathematical errors

Slips (-1)
S1 Use of decimals if it affects answer.

Attempts (3 marks)
A1 Any correct substitution

Worthless(0 marks)
W1 Cancelling *x* and *y* pre-substitution

b(ii)	**10 marks**	**Att 3**

$$2^{x+3}=(2^2)^x\ [3m] = 2^{2x}\ [4m] \Rightarrow x+3=2x\ [7m] \Rightarrow x=3\ [10m].$$

* Correct answer by T+E, verified fully: full marks

Blunders (-3)
B1 Error with indices, each time

Attempts (3 marks)
A1 Any correct use of indices e.g. $4=2^2$ and stops
A2 Incorrect equation solved correctly
A3 Incomplete T + E

Part (c) **20 (10, 10) marks** **Att (3, 3)**

(c) **(i)** Solve the equation $x - \frac{1}{x} = 2$ and write your solutions in the form $a \pm \sqrt{b}$, where $a,\ b \in \mathbf{N}$.

(ii) Verify **one** of your solutions.

(c)(i) **10 marks** **Att 3**

$$x^2 - 1 = 2x \quad [3m] \quad \Rightarrow \quad x^2 - 2x - 1 = 0.\ [4m]$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \quad [7m] = \frac{2 \pm \sqrt{8}}{2} \quad \text{or} \quad \frac{2 \pm 2\sqrt{2}}{2} \quad [9m] = 1 \pm \sqrt{2} \quad [10m].$$

* No use of quadratic formula in solving $x^2 - 2x - 1 = 0$: 4marks

Blunders (-3)

B1 Errors in multiplying by x

B2 Error in formula and / or substitution to max 2×B

B3 Error in surd

Slips (-1)

S1 Answer given as decimal

Attempts (3 marks)

A1 Any use of c.d.

A2 Correct quadratic formula only and stops

A3 1 error in formula with some substitution

A4 Linear equation e.g. $x - 1 = 2x$, and continues correctly

A5 If equation formed can be factorised: Att at most

(c)(ii) **10 marks** **Att 3**

$$1 + \sqrt{2} - \frac{1}{1+\sqrt{2}} \quad [3m] \quad = 1 + \sqrt{2} - \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} \quad [7m]$$

$$= 1 + \sqrt{2} - \frac{1-\sqrt{2}}{1-2} \quad = 1 + \sqrt{2} + 1 - \sqrt{2} \quad [9m] \quad = 2 \quad [10m]$$

* If no surd used: Att at most

* Apply same structure if $1 - \sqrt{2}$ substituted

Blunders (-3)

B1 Error in handling surds

B2 Error in substitution

B3 Substitution into equation other than original e.g. $x^2 - 2x - 1 = 0$

Attempts (3 marks)

A1 No use of surd

A2 Any correct substitution of any answer from (i)

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Solve $2x = 3(5-x)$.

(a) **10 marks** **Att 3**

$2x = 15 - 3x$ [3m] $\Rightarrow$ $2x + 3x = 15$ $\Rightarrow$ $5x = 15$ [7m] $\Rightarrow$ $x = 3$ [10m].

$\frac{2}{3}x = 5 - x$ [3m] $\Rightarrow$ $\frac{2}{3}x + x = 5$ $\Rightarrow$ $\frac{5}{3}x = 5$ [7m] $\Rightarrow$ $x = 3$ [10m]

* Correct answer with no work for full marks

Blunders (-3)
B1 Algebraic errors, once per step

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 Unsuccessful T + E
A2 Any correct relevant transposition or multiplication.

Worthless (0 marks)
W1 Incorrect answer without work

Part (b) **20 marks** **Att 7**

Solve the simultaneous equations.

$$\frac{x}{4} - \frac{y}{3} = \frac{5}{6}$$
$$2x - 6 = 3y.$$

(b) **20 marks** **Att 7**

$$\left.\begin{array}{l} 12\left(\frac{x}{4}\right) - 12\left(\frac{y}{3}\right) = 12\left(\frac{5}{6}\right) \Rightarrow 3x - 4y = 10 \\ 2x - 6 = 3y \Rightarrow 2x - 3y = 6 \quad [11\text{m}] \end{array}\right\} \text{interchangeable} \quad \left.\begin{array}{l} 3x = 4y + 10 \\ 2x = 3y + 6 \end{array}\right\}$$

$$\begin{array}{l} 6x - 8y = 20 \\ 6x - 9y = 18 \end{array} \quad \text{or similar} \quad [14\text{m}] \qquad \begin{array}{l} 6x = 8y + 20 \\ 6x = 9y + 18 \end{array}$$

$y = 2$ [17m] $\qquad \Rightarrow y = 2$

$3x - 4(2) = 10 \Rightarrow x = 6$ [20m] $\qquad 3x - 4(2) = 10 \Rightarrow x = 6$

* Correct answer without work: Att 7
* Note: Maximum 2 ×B per equation for initial simplification of each equation

Blunders (-3)
B1 Distribution error
B2 Sign error (each time)
B3 Transposition errors (each time)
B4 Incomplete multiplication of equations
B5 Error in eliminating a variable
B6 Fails to find value of second variable.
B7 Finds x but substitutes back into y (or vice versa)

Attempts (7 marks)

A1 Effort at isolating x or y

A2 Indicates correct c.d.

A3 Having found the first variable with work of no value substitutes to find the second variable.

Worthless (0)

W1 Incorrect values without work or from T and E

W2 Invented values substituted, and continues, e.g. $y = 0 \Rightarrow x = 3$ or some such.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $f(x) = 2x^3 + 11x^2 + 4x - 5$

(i) Verify that $f(-1) = 0$.

(ii) Solve the equation

$$2x^3 + 11x^2 + 4x - 5 = 0.$$

(c)(i) **10 marks** **Att3**

$$f(-1) = 2(-1)^3 + 11(-1)^2 + 4(-1) - 5 \quad [4m]$$
$$= -2 + 11 - 4 - 5 \quad [9m]$$
$$= 0 \quad [10m]$$

Blunders (-3)

B1 $f(1)$ evaluated

B2 Mathematical errors, each time if different

Slips(-1)

S1 Arithmetic error

Attempts (3 marks)

A1 Some correct substitution into $f(x)$

A2 Shows, or attempts to show that $x + 1$ is a factor

Worthless (0)

W1 $f(0)$, $f(x-1)$, $f(x+1)$ whether evaluated or not

(c)(ii) **10 marks** **Att 3**

$$[\, f(-1) = 0 \Rightarrow x+1 \text{ is a factor}] \Rightarrow$$

$$2x^3 + 11x^2 + 4x - 5 = (x+1)(2x^2 + Ax - 5)$$

$$-5 + A = 4 \Rightarrow A = 9$$

or $2 + A = 11 \Rightarrow A = 9$

OR

$$\begin{array}{r} 2x^2 + 9x - 5 \\ x+1 \overline{)\, 2x^3 + 11x^2 + 4x - 5} \\ \underline{2x^3 + 2x^2} \qquad\qquad \\ 9x^2 + 4x \qquad \\ \underline{9x^2 + 9x} \qquad \\ -5x - 5 \\ \underline{-5x - 5} \\ 0 \end{array}$$

$\therefore \; [(x+1)](2x^2 + 9x - 5) = 0 \Rightarrow$ [4m]

$[(x+1)](2x-1)(x+5) = 0 \Rightarrow$ [7m]

$x = -1, \quad x = \tfrac{1}{2}, \quad x = -5$. [10m]

* Synthetic division is acceptable

Blunders (-3)

B1 Incorrect initial divisor/factor
B2 Error in division/ finding quadratic factor to max 2×B
B3 Incorrect linear factors
B4 Failure to find, or error in finding roots from factors (once only)
NOTE: If quadratic formula used apply blunders as per guidelines

Slips (-1)

S1 $x = -1$ not given as root in this part.
S2 Arithmetic errors

Attempts (3 marks)

A1 Attempt at division
A2 $(x + 1)$ and stops
A3 $x = -1$ and stops
A4 Correct quadratic formula and stops
A5 *f(k)* ; $k \in$ R with some substitution

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a)	**10 marks**	**Att 3**

Given that $i^2 = -1$, simplify

$$3(2-4i) + i(5-6i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$

(a)	**10 marks**	**Att 3**

$6 - 12i + 5i - 6i^2$ [4m] $= 6 - 12i + 5i + 6$ [7m] $= 12 - 7i$ [10m]

Blunders (-3)
B1 Error in multiplying out bracket
B2 $i^2 \neq -1$
B3 Sign error
B4 Equates real and imaginary parts e.g. $12 = 7i$ if not rectified later

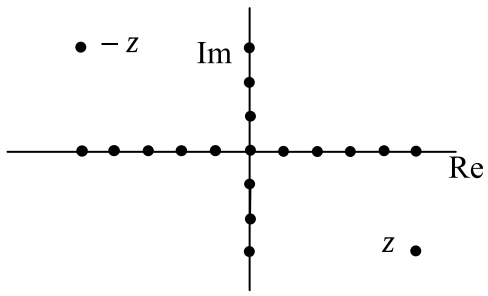
Attempts (3 marks)
A1 Any correct relevant multiplication
A2 Correct answer without work

Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Let $z = 5 - 3i$.
(i) Plot z and $-z$ on an Argand diagram.
(ii) Calculate $|z-1|$.
(iii) Find the value of the real number k such that $ki + 4z = 20$.

(b)(i)	**5 marks**	**Att 2**

$-z = -5 + 3i$



* Accept reversed axes if clearly identified, otherwise B(-3)
* Unlabelled axes, assume horizontal is real
* One unnamed point only plotted, assume it is z.

Blunders (-3)
B1 Point incorrectly plotted

Slips(-1)
S1 Labels swapped

Attempts (2 marks)
A1 Scaled axes
A2 Any correct step in finding $-z$ e.g. $-z = 5 + 3i$

(b)(ii) **5 marks** **Att 2**

$\lvert z-1\rvert = \lvert 5-3i-1\rvert$ or $\lvert 4-3i\rvert$ [2m] $=\sqrt{4^2+(-3)^2}$ or $\sqrt{16+9}$ or $\sqrt{25}$ or 5 [5m]

* Accept use of distance formula

Blunders (-3)

B1 Error in modulus formula
B2 Mathematical errors
B2 Errors in substitution into formula e.g. $(-3i)^2$

Attempts (2 marks)

A1 Substitutes for z into z -1, and stops
A2 $\sqrt{a^2+b^2}$ or distance formula correct and stops
A3 Mod formula / distance formula with 1 error and some correct substitution, and stops

Worthless (0)

W1 Incorrect formula (other than A3) with /without substitution

(b)(iii) **10 marks** **Att 3**

$ki+4(5-3i)=20$ [3m] $\Rightarrow ki+20-12i=20$ [4m]
$\Rightarrow k-12=0$ or $ki=12i$ [7m] $\Rightarrow k=12$ [10m]

Blunders (-3)

B1 Algebraic errors once per step
B2 Real and imaginary parts confused

Attempts (3marks)

A1 4(5 – 3i) and stops

Part (c) **20(10, 10) marks** **Att (3, 3)**

Let $u = 3 + 2i$. **(i)** Find the value of $u^2 + \bar{u}^2$, where $\bar{u}$ is the complex conjugate of u. **(ii)** Investigate whether $\dfrac{13}{u} = \bar{u}$.

(c)(i) **10 marks** **Att 3**

$u = 3 + 2i \quad \Rightarrow \quad \bar{u} = 3 - 2i \quad [3m]$

$(3+2i)^2 + (3-2i)^2 \;\; [4m] = 9 + 12i + 4i^2 + 9 - 12i + 4i^2 \quad [7m] = 9 - 4 + 9 - 4 \quad [9m] \quad = 10 \quad [10m]$

Blunders (-3)

B1 Incorrect conjugate

B2 $i^2 \neq -1$

B3 Each omitted or incorrect term when squaring, to a maximum of 2×B

B4 Real and imaginary terms mixed up

Slips(-1)

S1 Numerical slips

Attempts (3 marks)

A1 $\bar{u}$ correct and stops

A2 u^2 correct or partially correct and stops

(c)(ii) **10 marks** **Att 3**

(I) $\dfrac{13}{3+2i} = 3 - 2i \quad [3m] \quad \Rightarrow 13 = (3+2i)(3-2i) \quad [4m]$

$= 9 - 6i + 6i - 4i^2 \quad [7m] \quad = 9 + 4 \quad [9m] = 13 \quad [10m].$

or

(II) $\dfrac{13}{3+2i} \quad [3m] \quad = \dfrac{13}{3+2i} \times \dfrac{3-2i}{3-2i} \quad [4m] \quad = \quad \dfrac{39-26i}{9-4i^2} = \dfrac{39-26i}{13} \quad [7m] \quad = 3 - 2i = \bar{u} \quad [10m].$

* No penalty if numerator not multiplied out in (II)

Blunders (-3)

B1 Incorrect conjugate

B2 $i^2 \neq -1$

B3 Inverts fraction

Attempts (3 marks)

A1 Substitutes correctly for u and /or $\bar{u}$ and stops

A2 Correct conjugate and stops

QUESTION 5

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 4**

The *n*th term of a sequence is given by $T_n = 1 - n$.

(i) Find T_5, the fifth term.

(ii) Find $T_5 - T_{10}$ where T_{10} is the tenth term.

(a) (i) **5 marks** **Att 2**

$T_5 = 1 - 5 = -4$

* Accept correct answer without work
* Ignore notation

Blunders(-3)
B1 $Tn = 1 + n$ (oversimplify)
B2 $1 - n = 5$ and continues to $n = -4$
B3 Error in formula $Tn = a + (n-1)d$, if used
B4 Sign error

Attempts(2marks)
A1 $n = 5$ and stops
A2 $a = 0$ and / or $d = -1$ and stops
A3 $Tn = a + (n-1)d$ and stops
A4 Finds another term other than T_5 with work

Worthless (0 marks)
W1 Incorrect answer with no work

(a) (ii) **5 marks** **Att 2**

$T_5 - T_{10} = -4 - (1 - 10)$ or $-4 + 9$ [2m] $= 5$ [5m]

* Accept candidate's answer from (i)
* Ignore notation

Blunders(-3)
As in (i) if applicable

Misreading(–1)
M1 $T_{10} - T_5$ ($= -5$)

Attempts(2marks)
A1 $n = 10$ and stops
A2 $a = 0$ and / or $d = -1$ and stops
A3 $Tn = a + (n-1)d$ and stops
A4 Correct answer without work

Worthless (0 marks)
W1 $5 - 10 = -5$ or $10 - 5 = 5$

(b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

The first term of an arithmetic series is 3 and the common difference is 4. **(i)** Find, in terms of *n*, an expression for T_n, the *n*th term **(ii)** How many terms of the series are less than 200? **(iii)** Find the sum of these terms .

(b)(i) **5 marks** **Att 2**

$a = 3$ and /or $d = 4$ and/or $T_n = a + (n-1)d$ [2m] $= 3 + (n-1)4$ or $3 + 4n - 4$ or $4n - 1$ [5m]

* Errors, if any, in simplifying *Tn* are penalised in part (ii)

Blunders (-3)

B1 Incorrect *a*

B2 Incorrect *d* but *a* and *d* interchanged 1×B

B3 Error in formula

B4 Finds *Sn* of A.P.(= $^n/_2\{2(3)+(n-1)4\}$ or $2n^2 + n$)

Attempts (2 marks)

A1 *Tn* of G.P. formula with value for *a* correctly substituted

A2 *Sn* of A.P. formula with some correct substitution

A3 $T_1 = a$

A4 $T_n = S_n - S_{n-1}$ and stops

Worthless (0)

W1 3 and/or 4 written

(b)(ii) **5 marks** **Att 2**

(I) $3 + (n-1)4 < 200 \Rightarrow$ $4n - 1 < 200 \Rightarrow 4n < 201$ [2m] $\Rightarrow$ $n < \frac{201}{4}$ or 50.25 [4m] $\Rightarrow$ $n = 50$ [5m]	(II) List 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, ... 179, 183, 187, 191, 195, 199 [2m] = 50 terms [5m]

* No inequality sign used : Ignore unless it leads to incorrect answer.

Blunders (-3)

B1 Mathematical errors

B2 In List method ans = 49 or 51 , otherwise Att

B3 Incorrect inequality sign used

Slips(-1)

S1 Stops at $n < 50\frac{1}{4}$

Attempts (2 marks)

A1 Minimum of 2 consecutive terms correct in list method

A2 $n = 200$ and continues (= 799)

A3 $\frac{200}{4} = 50$ or $200 - 3$ or 50 without work

A4 $S_n \leq 200$ with some correct substitution

(b)(iii)	**10 marks**	**Att 3**

(I) $S_{50} = \frac{n}{2}\{2a + (n-1)d$ [3m] $= \frac{50}{2}\left(2(3) + (50-1)4\right)$ [7m] $= 5050$ [10m]

(II) correct list added = 5050 [10m]

* Method (I) Accept candidates answer from (ii), but note B4
* Method (II) Fully correct list and correct total: 10marks ; otherwise Att3 at best

Blunders (-3)
B1 Error in formula (not more than 1 error, otherwise attempt at best)
B2 Error in substitution (once if consistent)
B3 Mathematical error in calculation to max B×2
B4 $n \notin N$

Slips (-1)
S1 (I) Numerical errors

Attempts (3 marks)
A1 Attempt at adding terms
A2 Formula for S_n of G.P. with a substituted or Tn of A.P. with some correct substitution

(c)	**20 (5, 10, 5)**	**Att (2, 3, 2)**

The first two terms of a geometric series are $\frac{1}{3} + \frac{1}{9} + ...$

(i) Find r, the common ratio.
(ii) Find an expression for S_n, the sum of the first n terms .
Write your answer in the form $\frac{1}{k}\left(1 - \frac{1}{3^n}\right)$ where $k \in \mathbf{N}$.
(iii) The sum of the first n terms of the geometric series $\frac{p}{3} + \frac{p}{9} + ...$is $1 - \frac{1}{3^n}$, where $p \in \mathbf{N}$.
Find the value of p .

(c)(i)	**5 marks**	**Att 2**

$r = \frac{1/9}{1/3}$ [2m] $= \left[\frac{1}{9} \times \frac{3}{1}\right] = \frac{1}{3}$ [5m]

* Accept correct answers without work i.e. r = ⅓ or ⅓ on its own.

Blunders (-3)
B1 Mathematical errors
B2 $T_1 \div T_2$ (=3)

Attempts (2 marks)
A1 $a = ⅓$ and / or $ar = 1/9$ and stops
A2 $T_2 \div T_1$ or similar and stops
A3 some correct substitution
A4 $\left[\frac{1}{3}, \frac{1}{9}\right], \frac{1}{27}$ or $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ or similar

Worthless (0marks)
W1 $\frac{1}{9} \pm \frac{1}{3}$ or similar
W2 3 without work

(c)(ii) **Formula and substitution: 5 marks** **Att 2**
Finish: 5 marks **Hit/Miss**

$$S_n = \frac{a(1-r^n)}{1-r} \quad [2m] \quad = \frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} \quad [5m]$$

$$= \frac{1}{2}\left(1-\frac{1}{3^n}\right) \quad [5m]$$

* Accept r from (i) provided it does not oversimplify the question

Blunders (-3)
B1 Error in formula
B2 Error in substitution, once if consistent

Attempts (2 marks)
A1 Writes out next 2 terms (at least) $^1/_{27}$, $^1/_{81}$
A2 Some correct substitution

Worthless (0marks)
W1 1 further term only written

(c)(iii) **5 marks** **Att 2**

(I) $r = \frac{1}{3}$ $\quad S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{p}{3}(1-\frac{1}{3^n})}{1-\frac{1}{3}} = \frac{p}{2}\left(1-\frac{1}{3^n}\right)$ $\quad [2m] \Rightarrow \frac{p}{2} = 1 \Rightarrow p = 2$ $\quad [5m]$

(II) Series = p(Series in (ii)) $\Rightarrow p = 2$

Blunders (-3)
B1 Error in formula
B2 Error in substitution
B3 Mathematical errors

Attempts (2 marks)
A1 Correct answer without work
A2 $r = \frac{1}{3}$
A3 $r = T_2 \div T_1$ or similar and stops
A4 $p(\frac{1}{3}+\frac{1}{9}+...)$ and stops
A5 Correct formula written

Worthless(0)
W1 Incorrect answer with no work

QUESTION 6

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Let $g(x) = x^2 - 6x, \quad x \in \mathbf{R}.$

(i) Write down $g'(x)$, the derivative of $g(x)$.

(ii) For what value of x is $g'(x) = 0$?

(a)(i) **5 marks** **Att 2**

$g'(x) = 2x - 6$

* Accept correct answer without work or notation
* If done from 1st principles, ignore errors in procedure – just mark the answer.

Blunders (-3)

B1 Differentiation errors (see W1)

Attempts (2 marks)

A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + h$ on R.H.S., ‘limit’ mentioned, $\Delta x \longrightarrow 0$, $g(x + h)$, etc.

A2 Writes down the notation ‘*dy/dx*’ and stops.

Worthless(0)

W1 No term differentiated correctly

(a) (ii) **5 marks** **Att 2**

$[[g'(x)] = 2x - 6 = 0$ [2m] $\Rightarrow$ $2x = 6$ $\Rightarrow$ $x = 3$ [5m]

* Accept candidate’s answer from (i)
* Accept correct answer without work

Blunders (-3)

B1 Transposition errors

Attempts (2 marks)

A1 Finds $g'(0)$

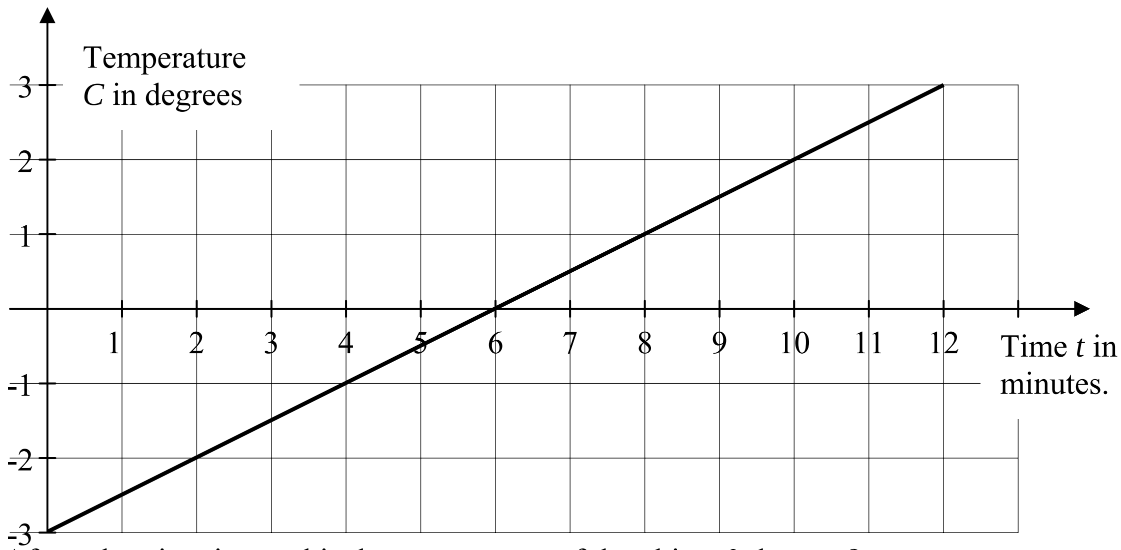
Worthless(0marks)

W1 *g(x)* = 0 whether continues or not

W2 Incorrect answer without work

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

A cold object is placed in a warm room.
Its temperature C degrees after time t minutes is shown in the following graph.



(i) After what time interval is the temperature of the object 0 degrees?

(ii) What is the rise in temperature of the object in the first 10 minutes?

(iii) The relationship between the temperature C and the time t is given by $C = \frac{1}{2}(t+k)$.

Find the value of k.

* Units: Penalise as per guidelines

(b)(i) **5 marks** **Att 2**

6 minutes

* Incorrect answer with no work: 0 marks

(b)(ii) **5 marks** **Att 2**

5 degrees

* Accept correct answer without work * Incorrect answer and no work: 0marks

Blunders (-3)

B1 Not finding or error in finding difference between –3 and +2

Attempts (2 marks)

A1 Copies graph and shows relevant work

(b)(iii) **10 marks** **Att 3**

(6, 0) on line [3m] $C = \frac{1}{2}(t+k) \Rightarrow 0 = \frac{1}{2}(6+k)$ [7m] $\Rightarrow k = -6$ [10m]

intercept on y-axis = -3 $C = \frac{1}{2}t + \frac{1}{2}k$ [3m] $\frac{1}{2}k = -3$ [7m] $\Rightarrow k = -6$ [10m]

* Substituting any correct point is acceptable

Blunders (-3)

B1 Values of C and t reversed

B2 Mathematical errors

Attempts (3 marks)

A1 Lists one correct point on line and stops A2 Tries to isolate k

A3 $y-y_1 = m(x-x_1)$ or $(y_2 - y_1)/(x_2 - x_1)$

Part (c) **20 (5, 15) marks** **Att (2, 5)**

Let $f(x) = (5x-2)^4$ for $x \in$ **R.**
(i) Find $f'(x)$, the derivative of $f(x)$.
(ii) Find the co-ordinates of the point on the curve $y = f(x)$ at which the slope of the tangent is 20.

(c)(i) **5 marks** **Att 2**

(I) $f'(x) = 4(5x-2)^3(5) = [20(5x-2)^3]$

(II) $f(x) = (5x-2)^4 = 625x^4 - 1000x^3 + 300x^2 - 160x + 16 \Rightarrow f'(x) = 2500x^3 - 3000x^2 + 600x - 160$

* Treat $4(5x–2)^3$ and (5) as separate parts

* (I) Errors, if any, in simplifying are penalised in part (ii)

Blunders (-3)

B1 Differentiation errors, once per term

B2 Mathematical errors (II)

Attempts (2 marks)

A1 If power ignored…oversimplification

A2 $u = 5x - 2$ and stops

A3 Some correct element of chain rule e.g. index = 3 or coefficient = 4

A4 At least 1 term multiplied out correctly.(II)

A5 Any correct differentiation

A6 Writes down the notation '*dy/dx*' and stops.

Worthless (0marks)

W1 *uv* or *u/v* written and stops

(c)(ii) **15 marks** **Att 5**

$4(5x-2)^3(5) = 20$ [5m] $\Rightarrow$ $(5x-2)^3 = 1$ $\Rightarrow$ $5x-2=1$ [6m]

$\Rightarrow$ $5x = 3$ [9m] $\Rightarrow$ $x = \frac{3}{5}$ [12m]

$f(\frac{3}{5}) = (5(\frac{3}{5})-2)^4 = (3-2)^4 = 1$ [15m] $[(\frac{3}{5}, 1)]$

* Accept candidate's $f'(x)$ from (i), unless it oversimplifies the question

Blunders (-3)

B1 Mathematical errors

B2 Fails to find *y*-ordinate

Attempts (5 marks)

A1 *f'(x)* or $\frac{dy}{dx}$ mentioned

A2 Slope of tangent = derivative or similar

Worthless (0marks)

W1 No use of derivative e.g. *f(x)* = 20

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

(a) Differentiate $6x^4 - 3x^2 + 7x$ with respect to x.

(a) **10 marks** **Att3**

$$\left[\frac{dy}{dx}\right] = 24x^3 - 6x + 7$$

* Correct answer without work or notation: full marks, 10m.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated, award 4 marks.

Blunders (-3)

B1 Differentiation error, once per term.

Attempts (3 marks)

A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., 'limit' mentioned, $\Delta x \longrightarrow 0$, $f(x + h)$, etc.

A2 Writes down the notation '*dy/dx*' or '$f'(x)$' and stops.

Worthless (0)

W1 No term differentiated correctly, but check attempts first

Part (b) **20 (10, 10) marks** **Att 6**

(b)	**(i)**	Differentiate $(x^2+9)(4x^3+5)$ with respect to x.
	(ii)	Given that $y = \dfrac{3x}{2x+3}$, find $\dfrac{dy}{dx}$.
		Write your answer in the form $\dfrac{k}{(2x+3)^n}$, where $k,\ n \in \mathbf{N}$.

* Apply penalties as in guidelines
* No penalty for omission of brackets if multiplication implied. (Decide by later work).
* No marks for writing down *u/v* or *u.v* formula from Tables, and stopping

(b)(i) **10 marks** **Att 3**

(I)	(II)	
$(x^2+9)(12x^2) + (4x^3+5)(2x)$ [10m]	$y = 4x^5 + 36x^3 + 5x^2 + 45$	[3m]
	$\dfrac{dy}{dx} = 20x^4 + 108x^2 + 10x$	[10m]

Blunders (-3)
B1 Differentiation errors, once per term
B2 (II) Errors in expanding brackets to max 2×B

Attempts (3 marks)
A1 *u* and/or *v* correctly identified and stops
A2 Any correct differentiation
A3 At least one term multiplied out correctly (II)

(b)(ii) **10 marks** **Att 3**

$$\frac{dy}{dx} = \frac{(2x+3)(3) - 3x(2)}{(2x+3)^2} \text{ [7m]} = \frac{6x+9-6x}{(2x+3)^2} \text{ [9m]} = \frac{9}{(2x+3)^2} \text{ [10m]}$$

Blunders (-3)
B1 Differentiation errors, once per term

Attempts (3 marks)
A1 *u* and/or *v* correctly identified and stops
A2 Any correct differentiation

Worthless (0 marks)
W1 *uv* or *u/v* written and stops

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

A car starts from rest at the point a.

The distance of the car from a, after t seconds, is given by

$$s = 2t^2 + 2t$$

where s is in metres.

(i) Find the speed of the car after 2 seconds.
(ii) Find the acceleration of the car.
(iii) The distance from a to the point b is 24 metres. After how many seconds does the car reach the point b?

* Units: Penalise as per guidelines

(c)(i) **5 marks** **Att 2**

$$\frac{ds}{dt} = 4t + 2 \quad [2m] \quad = 4(2) + 2 = 10\,\text{m/s} \quad [5m].$$

* Correct answer without work: Att 2
* No retrospective marking
* No penalty for incorrect notation.
* If the parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii).

Blunders (-3)
B1 Differentiation errors
B2 Incorrect or no value for t substituted into ds/dt equation

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 ds/dt or dy/dx or $f'(x)$ mentioned
A2 $\frac{d^2s}{dt^2} = 4$

Worthless (0 marks)
W1 $t = 2$ substituted into original equation
W2 Incorrect answer without work
W3 States speed = d^2s/dt^2 and stops.
W4 Effort to use Speed = Distance ÷ Time.

(c)(ii) **5 marks** **Att 2**

$$\frac{d^2s}{dt^2} = 4\ m/s^2$$

* Accept correct answer without work
* Accept $v = u + at$ or equivalent with correct values for *u, v* and corresponding *t* calculated and used.
* Accept use of *ds/dt* from (i) provided expression contains '*t*'. Otherwise Att at most.
* Unit written as *m/s/s* or ms^{-2} is acceptable

Blunders (-3)
B1 Differentiation errors

Attempts (2 marks)
A1 $\frac{d^2s}{dt^2}$ or $\frac{dv}{dt}$ or similar written.
A2 Finds or attempts to find *ds/dt* in this part.

Worthless (0 marks)
W1 Incorrect answer without work.

(c)(iii) **10 marks** **Att 3**

$24 = 2t^2 + 2t$ [3m] $\Rightarrow 2t^2 + 2t - 24 = 0$ or $t^2 + t - 12 = 0$ [4m]
$\Rightarrow (t+4)(t-3) = 0$ [7m] $\Rightarrow t = -4$ or $t = 3$ [9m] $\Rightarrow$ $t = 3$ seconds [10m]

Blunders (-3)
B1 Mathematical errors as per guidelines

Slips (-1)
S1 Does not exclude negative answer

Case:
$2(3)^2 + 2(3) = 24 \Rightarrow t = 3$seconds: Award 6 marks
If $\Rightarrow t = 3$ Award 5 marks

Attempts (3 marks)
A1 Correct answer without work: Accept 3 or 3seconds or t = 3
A2 Writes s = 24 and stops

Worthless (0 marks)
W1 Incorrect answer without work
W2 $t = 24$ substituted into original equation
W3 $\frac{ds}{dt} = 24$ whether continues or not

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Let $f(x) = \frac{1}{4}(6-2x)$ for $x \in$ **R.** Evaluate $f(5)$.

(a) **10 marks** **Att 3**

$f(5) = \frac{1}{4}(6-2(5))$ [3m] $= \frac{1}{4}(6-10)$ $=$ $\frac{1}{4}(-4)$ [7m] $=$ -1 [10m]

* Accept correct answer without work:

Blunders(-3)
B1 Precedence errors
B2 Mathematical error

Misreadings(-1)
M1 *f*(-5) and continues (= 4)

Attempts (3 marks)
A1 *f*(any number), substituted.
A2 *f(x)* = 5 or –5 and continues

Part (b) **20 marks** **Att 7**

Differentiate $x^2 - 3x$ with respect to x from first principles.

(b) **20 marks** **Att 7**

$f(x) = x^2 - 3x$

$f(x+h) = (x+h)^2 - 3(x+h)$ [8m]

$= x^2 + 2xh + h^2 - 3x - 3h$ $\underline{or}$

$f(x+h) - f(x) = (x+h)^2 - 3(x+h) - (x^2 - 3x)$ [11m]

$= x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x$

$= 2xh + h^2 - 3h$ [14m]

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2 - 3h}{h} = 2x + h - 3$ [17m]

$\lim\limits_{h \to 0}\left(\dfrac{f(x+h) - f(x)}{h}\right) = 2x - 3$ [20m]

$y + \Delta y = (x + \Delta x)^2 - 3(x + \Delta x)$ [8m]

$= x^2 + 2x\Delta x + (\Delta x)^2 - 3x - 3\Delta x$ [11m]

$y = x^2 \qquad - 3x$

$\Delta y = \quad 2x\Delta x + (\Delta x)^2 - 3\Delta x$ [14m]

$\dfrac{\Delta y}{\Delta x} = 2x + \Delta x - 3$ [17m]

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = 2x - 3$ [20m]

* Accept $h = 0$ in limit or *dy/dx* instead of lim$\Delta y/\Delta x$

Blunders(-3)
B1 Mathematical errors-once per term e.g. $x^2 + 2x\Delta x + (\Delta x)^2 - 3\underline{x + 3}\Delta x$
B2 Omits *f(x + h)* and / or *f(x + h) – f(x)* on LHS or equivalent
B3 Omits { *f(x + h) – f(x)* }/ *h* on LHS or equivalent
B4 Indication of limit on LHS omitted or incorrect
B5 Error in evaluating candidate's limit
B6 Uses $x^2 + 3x$ or $x^2 \pm 3$

Attempts (7marks)
A1 *f(x +h)* on LHS; or *x + h* or *x – h* substituted somewhere for *x* on RHS or equivalent
A2 Linear function used (oversimplification)

Worthless (0 marks)
W1 Answer 2*x* – 3 without work

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

Let $f(x) = \dfrac{1}{x+7}$, $x \in$ **R,** $x \neq -7$

(i) Given that $f(k) = 1$, find k.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Show that the curve $y = f(x)$ has no turning points.

(c)(i) **5 marks** **Att2**

$f(k) = 1 \Rightarrow \dfrac{1}{k+7} = 1$ [2m] $\Rightarrow 1 = k + 7$ $\Rightarrow k = -6$ [5m]

* No penalty if *f(x)* = 1 is solved correctly

Blunders(-3)

B1 Transposition errors

Attempts(2 marks)

A1 $f(1) = k \Rightarrow k = {}^1/_8$

A2 Correct answer without work

(c)(ii) **5 marks** **Att 2**

(I) $f'(x) = -1(x+7)^{-2} = \left[\dfrac{-1}{(x+7)^2}\right]$ (II) $\dfrac{(x+7)(0)-(1)(1)}{(x+7)^2} = \left[\dfrac{-1}{(x+7)^2}\right]$

* Apply penalties as in guidelines

* No penalty for omission of brackets if multiplication implied.

* No marks for writing down u/v or u.v formula from Tables, and stopping

* Errors in simplification, if any, are penalised in (iii)

Attempts (2marks)

A1 $(x + 7)^{-1}$ and stops

A2 *u* and/or *v* correctly identified and stops

A3 Any correct differentiation

(c)(iii) **10marks** **Att 3**

$\dfrac{-1}{(x+7)^2} = 0$ [4m] $\Rightarrow$ $-1 = 0$ which is impossible [7m] $\therefore$ no turning pt [10m]

* Accept candidate's answers from (ii)

Case:

$\dfrac{-1}{(x+7)^2} \neq 0$ [7m] $\Rightarrow$ no turning point [10m]

$\dfrac{-1}{(x+7)^2} = 0$ impossible [7m] $\Rightarrow$ no turning point [10m]

$\dfrac{-1}{(x+7)^2} < 0$ [7m] $\Rightarrow$ no turning point /decreasing function [10m]

$\dfrac{-1}{(x+7)^2} = 0$ [4m]

Attempts(3marks)

A1 Sketch of *f(x)*

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2007

MATHEMATICS – ORDINARY LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLICATION OF PENALTIES THROUGHOUT SCHEME

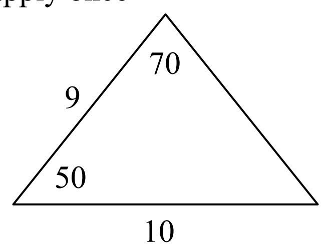
Penalties are applied subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ *or* $5p \times 4p = 20p$
- Sign error: $-3(-4) = -12$ *or* $(-3)^2 = 6$.
- Fraction error: Incorrect fraction inversion etc. apply once
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$
  or $4x = 12 \Rightarrow x = 8$ each time.
- Distributive law errors (once per term, unless otherwise directed)
  $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ *or* $-(4x + 3) = -4x + 3$ *or* $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly (apply once unless directed otherwise)
  $(2x - 3)(x + 4) = 8x^2 - 12x$
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once
  $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

  e.g. $2x^2 + 3x + 4 = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(3)}$

  or $\dfrac{10}{\sin 70} = \dfrac{9}{\sin 50}$



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example: $\dfrac{y_2 + y_1}{x_2 + x_1}$ *or* $A = P\left(1 + \dfrac{n}{100}\right)^r$ *or* $a^2 = b^2 + c^2 + bc\cos A$

  or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Note: A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the Tables.

Slips – examples are as follows:

- Numerical slips such as: $4 + 7 = 10$ *or* $3 \times 6 = 24$ *but* $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to either the incorrect accuracy or an early round-off are penalised as a slip once in each section. This applies to **Q5** (a), (b) (ii) and (iii), (c) (i) and (ii), **Q10** (c) (i) and (ii).
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once per part (a), (b) and (c) of each question where appropriate and at the first place where it matters. This applies to **Q1** (a), (b) and (c) and to **Q5** (a), (b) and (c).

Misreadings

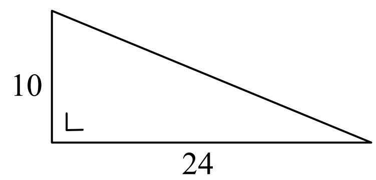
- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

QUESTION 1

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

The right-angled triangle shown in the diagram has sides of length 10 cm and 24 cm.



(i) Find the length of the third side.

(ii) Find the length of the perimeter of the triangle.

(a) (i) **5 marks** **Att 2**

$$x^2 = 10^2 + 24^2 \downarrow = 100 + 576 = 676 \Rightarrow x = \sqrt{676} \text{ or } 26 \text{ cm.}\downarrow$$

[*2 marks*] [*5 marks*]

* Accept correct answer without work.
* Any error other than an obvious slip merits the attempt mark at most.
* Accept a correct trigonometric method.

Blunders (-3)
B1 $10^2 = 20$.

Attempts (2 marks)
A1 Some relevant work, e.g. squares one value or 24 + 10 or indicates hypotenuse on a diagram.
A2 Statement of Theorem of Pythagoras.

Worthless (0)
W1 Incorrect answer without work, except 100 or 576.
W2 Area of triangle calculated.

(a) (ii) **5 marks** **Att 2**

Perimeter = $10 + 24 + 26\downarrow =$ 60 cm. $\downarrow$

[*4 marks*] [*5 marks*]

* Accept correct answer without work.
* Accept answers consistent with section (i).
* Any error other than an obvious slip merits the attempt mark at most.
* $\sqrt{676} \neq 26$ is penalised in final mark.

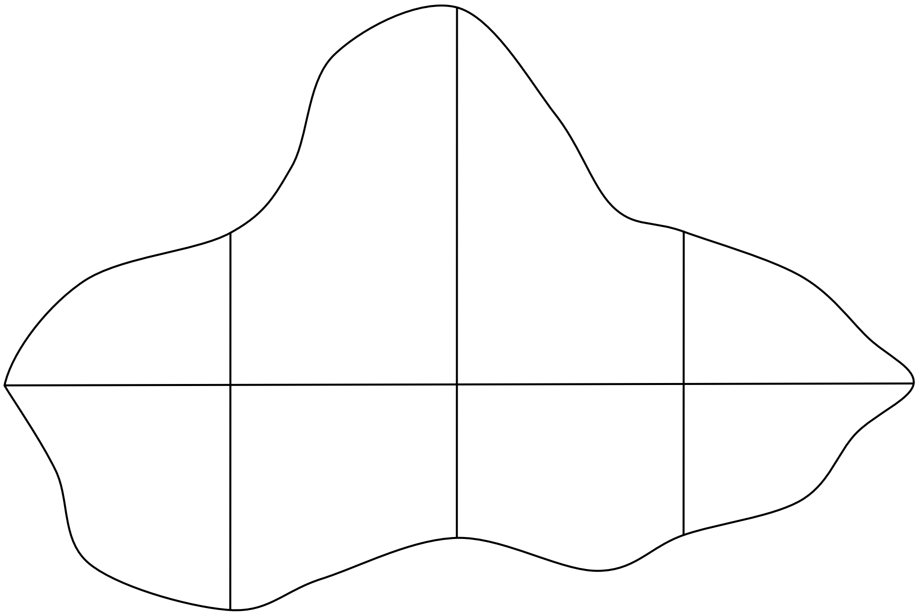
Slips (-1)
S1 Each slip, including units penalty, to a maximum of 3.

Attempts (2 marks)
A1 Statement of, or correct use of, any relevant result.

Worthless (0 marks)
W1 Incorrect answer without work.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

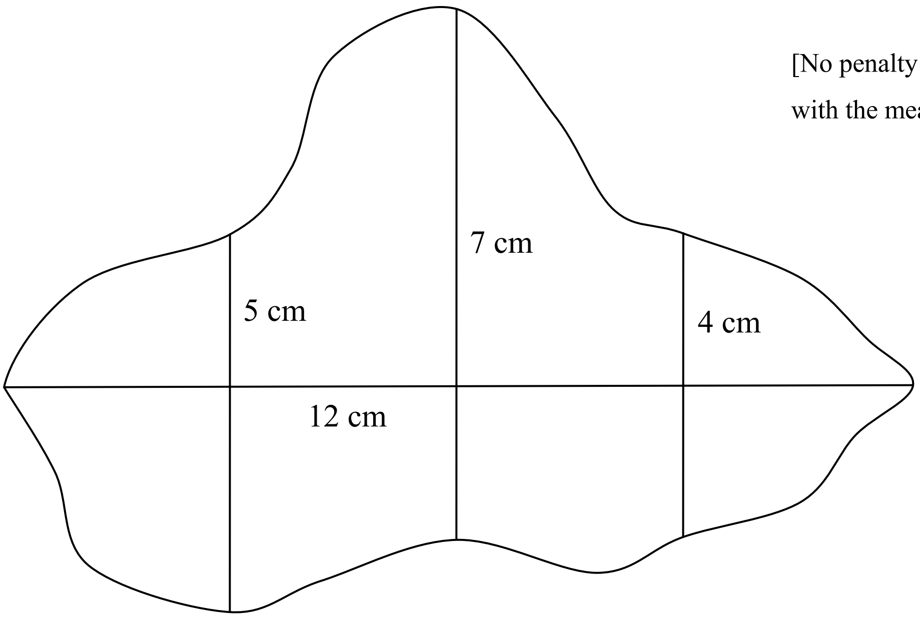
In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.



(i) Measure the horizontal line and the offsets, in centimetres.
Make a rough sketch of the shape in your answerbook and record the measurements on it.

(ii) Use Simpson's Rule with these measurements to estimate the area of the shape.

(b) (i) **10 marks** **Att 3**



[No penalty for not writing units with the measurements]

- Allow tolerance of ± 0.1 cm, on candidate's measurements.
- Accept measurements written on the question paper but penalise (–1)×2 if sketch not drawn.
- Award 2 marks for sketch and 2 marks for each correct measurement, hit or miss subject to the attempt mark. If offsets measured in two parts award 1 + 1 for each part correct.
- If candidate measures offsets and horizontal from his/her sketch rather than the given diagram, mark as above and apply (-3) for the error.
- Accept 3 or 12 as the measurement of the horizontal line in this section.
- If mm used apply penalty of (–3), if inches used award the attempt mark, subject to tolerance.

Attempts (3 marks)

A1 Some relevant work e.g. sketch drawn or one correct measurement.

(b) (ii) **10 marks** **Att 3**

$h = 12 \div 4 = 3$

$\text{Area} = {}^{h}/_{3}\ \{F + L + 2(\text{odds}) + 4(\text{evens})\}$

$= {}^{3}/_{3}\{0 + 0 + 2(7) + 4(5 + 4)\}$

$= 1(14 + 36) = 50\ \text{cm}^2.$

or

$\text{Area} = {}^{3}/_{3}\{0 + 0 + 2(5) + 4(2 + 2)\} + {}^{3}/_{3}\{0 + 0 + 2(2) + 4(3 + 2)\}$

$= 1(10 + 16) + 1(4 + 20)$

$= 26 + 24 = 50\ \text{cm}^2.$

- * Allow ${}^{h}/_{3} = \{F + L + \text{TOFE}\}$ and penalise in calculations if formula not used correctly.
- * Accept correct TOFE *or* TOFE consistent with candidates F and L.
- * If section (i) not answered explicitly, candidate may be awarded the attempt mark for section (i) for some correct measurement used in this section.
- * No more than 3 marks may be lost for errors in calculations.

Blunders (-3)

B1 Incorrect ${}^{h}/3$ (once).
B2 Incorrect F and / or L or extra terms with F and / or L (once).
B3 Incorrect TOFE (once), if not consistent with candidates F and L.
B4 E **or** O omitted (once).
B5 Calculates top or bottom area only.

Slips (-1)

S1 Each slip to a maximum of 3.

Attempts (3 marks)

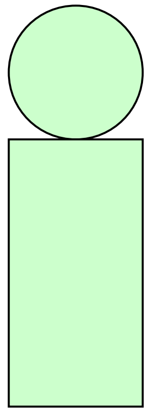
A1 Some relevant step, e.g. identifies F and / or L or odds or evens and stops.
A2 Statement of Simpson's Rule not transcribed from tables.
A3 E and O omitted (candidate may be awarded attempt at most).
A4 Some correct calculation only.
A5 Correct answer without work.

Worthless (0)

W1 Incorrect answer without work.
W2 Formula transcribed from tables and stops.

Part (c) **20 (5, 15) marks** **Att (2, 5)**

A team trophy for the winners of a football match is in the shape of a sphere supported on a cylindrical base, as shown.
The diameter of the sphere and of the cylinder is 21 cm.



(i) Find the volume of the sphere, in terms of π.

(ii) The volume of the trophy is $6174\pi \text{ cm}^3$
Find the height of the cylinder.

(c) (i) **5 marks** **Att 2**

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{21}{2}\right)^3 \downarrow = 1543.5\pi \text{ cm}^3 . \downarrow$
[*2 marks*] [*5 marks*]

* Accept volume of sphere read as $\frac{4}{8}\pi r^3$. [Answer is $578.8125\pi \text{ cm}^3$].

Blunders (-3)

B1 Incorrect relevant sphere formula e.g. πr^3, $\frac{1}{3}\pi r^3$, with substitution.

B2 Radius taken as 21 cm.

B3 Omits π from answer, or uses an obvious value for π outside of the range $3{\cdot}1 < \pi < 3{\cdot}2$.

Slips (-1)

S1 Each slip to a maximum of 3.

S2 Inserts a value for π in the range $3{\cdot}1 \le \pi \le 3{\cdot}2$.

Attempts (2 marks)

A1 Some relevant step e.g. $r = \frac{21}{2}$ or diagram with additional work.

A2 Correct answer without work.

(c) (ii) **15 marks** **Att 5**

Volume of cylinder $= 6174\pi - 1543.5\pi = 4630.5\pi \text{ cm}^3$.

Volume of cylinder $= \pi r^2 h = \pi\left(\frac{21}{2}\right)^2 h = 4630.5\pi \Rightarrow h = \dfrac{4630.5 \times 2^2}{21^2} = 42$ cm.

* Accept candidates volume of sphere from section (i).

Award: 15 marks for a fully correct or consistent answer but deduct -1 if units not given [if not already penalised in section (c) (i)].

5 marks for some correct work – including correct answer without work shown.

0 marks for worthless attempt.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 marks**	**Att 9**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 marks** **Att 3**

Find the co-ordinates of the mid-point of the line segment joining the points $(2, -3)$ and $(6, 9)$.

(a) **10 marks** **Att 3**

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)_{\downarrow [3\ marks]} = \left(\frac{2+6}{2}, \frac{-3+9}{2}\right)_{\downarrow [7\ marks]} = \left(\frac{8}{2}, \frac{6}{2}\right) \text{ or } (4, 3)_{\downarrow [10\ marks]}$$

* Accept (4, 3) without work.

* If the correct formula is not written, any sign or substitution error is at least a blunder – *apply to all sections of this question*.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ or $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$ and continues.

B2 Two or more incorrect substitutions if formula written.

B3 Switches x and y in substituting – *apply to all sections of this question*.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 One incorrect sign in formula if formula written.

S3 One incorrect substitution in formula if formula written.

Attempts (3 marks)

A1 Some relevant step e.g. $(2, -3)$ with x_1 and y_1 identified, or point plotted.

A2 Correct relevant formula written and stops.

A3 Incorrect relevant formula, partially substituted – *apply to all sections of this question*.

A4 Diagram with correct mid-point indicated.

Worthless (0 marks)

W1 Irrelevant formula, even if completed, e.g. distance formula, but subject to A1.

Part (b) **25 (5, 5, 10, 5) marks** **Att (2, 2, 3, 2)**

The line L intersects the x-axis at $(-4, 0)$ and the y-axis at $(0, 6)$.

(i) Find the slope of L.

(ii) Find the equation of L.

The line K passes through (0, 0) and is perpendicular to L.

(iii) Show the lines L and K on a co-ordinate diagram.

(iv) Find the equation of K.

(b) (i) **5 marks** **Att 2**

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{0+4} = \frac{6}{4} \textbf{ or } \frac{3}{2} \textbf{ or } 1.5$$

[2 marks] [4 marks] [5 marks]

* Accept correct answer without work.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\frac{y_2 + y_1}{x_2 + x_1}$ or $\frac{y_2 - y_1}{x_1 - x_2}$ or $\frac{x_2 - x_1}{y_2 - y_1}$ and continues.

B2 Two or more incorrect substitutions, if formula is written.

Misreadings (-1)

M1 If candidate uses the points given in part (a) to find the slope – apply once.

Slips (- 1)

S1 One incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$ part of formula.

S2 One incorrect substitution, if formula is written.

Attempts (2 marks)

A1 Some relevant step, e.g. (−4, 0) with x_1 and y_1 identified.

A2 $m = \tan\theta$ or m = vertical/horizontal.

(b) (ii) **5 marks** **Att 2**

$$y - y_1 = m(x - x_1) \Rightarrow y + 0 = \tfrac{3}{2}(x+4) \Rightarrow 2y = 3x + 12 \Rightarrow 3x - 2y + 12 = 0.$$

[2 marks] [5 marks]

or

$$y - y_1 = m(x - x_1) \Rightarrow y - 6 = \tfrac{3}{2}(x-0) \Rightarrow 2y - 12 = 3x \Rightarrow 3x - 2y + 12 = 0.$$

[2 marks] [5 marks]

or

$$y = mx + c \Rightarrow y = \tfrac{3}{2}x + 6 \Rightarrow 3x - 2y + 12 = 0.$$

[2 marks] [5 marks]

or

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y-0}{6-0} = \frac{x+4}{0+4} \Rightarrow 4y = 6x + 24 \Rightarrow 3x - 2y + 12 = 0.$$

[2 marks] [5 marks]

* Do not penalise for errors in simplifying *L*.
* Accept a correct answer without work shown.

Blunders (-3)

B1 Uses an arbitrary point for the line.
B2 Uses incorrect and inconsistent slope.
B3 Incorrect relevant formula e.g. $y + y_1 = m(x + x_1)$ [Both signs incorrect].
B4 Two or more incorrect substitutions or signs in formula.

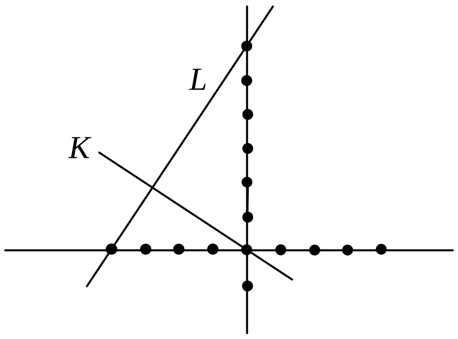
Slips (-1)

S1 One incorrect sign in line formula, if formula written.
S2 One incorrect substitution in line formula, if formula written.

Attempts (2 marks)

A1 Gives correct relevant formula and stops.

(b) (iii) **10 marks** **Att 3**



* Accept a vertical x-axis and a horizontal y-axis.
* Intervals should be indicated or implied.

Blunders (-3)
B1 Scales unreasonably inconsistent (to the eye).
B2 Different scales on x and y axes.
B3 Incorrect intercept.
B4 Measure of angle between L and K outside tolerance of $\pm 10°$.
B5 K does not pass through (0, 0).

Misreadings (-1)
M1 Plots $(0, -4)$ and $(6, 0)$.

Attempts (3 marks)
A1 Draws scaled axes and stops.

Worthless (0 marks)
W1 Draws arbitrary line L with K not perpendicular to L, subject to A1.

(b) (iv) **5 marks** **Att 2**

Slope of $L = \frac{3}{2} \quad \Rightarrow \quad$ Slope of $K = -\frac{2}{3}$ [*2 marks*]
$K : y - 0 = -\frac{2}{3}(x - 0)$ [*5 marks*]
$\Rightarrow 3y - 0 = -2x - 0 \quad \Rightarrow \quad 2x + 3y = 0$.

or

$y = mx + c$ [*2 marks*]
$K : y = -\frac{2}{3}x + c$ [*4 marks*]
$\Rightarrow 0 = -\frac{2}{3}(0) + c \quad \Rightarrow \quad c = 0$ [*5 marks*]
Hence, $K : y = -\frac{2}{3}x + 0 \quad or \quad 3y = -2x \quad or \quad 2x + 3y = 0$.

or

$L \perp K \Rightarrow K : 2x + 3y + k = 0$ [*2 marks*]
$(0, 0) \in K \Rightarrow 2(0) + 3(0) + k = 0$ [*4 marks*]
$\Rightarrow \quad 0 + 0 + k = 0 \Rightarrow \quad k = 0$. [*5 marks*]
Hence, $K : 2x + 3y = 0$.

* Accept correct answer without work.

Blunders (-3)
B1 Incorrect slope of K, i.e. $m_1 m_2 \neq -1$, e.g. $\frac{2}{3}$ or $-\frac{3}{2}$.
B2 Two or more incorrect signs or substitutions in formula.
B3 Uses an arbitrary point.

Attempts (2 marks)
A1 Correct formula and stops e.g. $m_1 m_2 = -1$.

Part (c)	**15 (5, 10) marks**	**Att (2, 3)**

$a(-4, 3)$, $b(6, -1)$ and $c(2, 7)$ are three points.

(i) Find the area of the triangle abc.

(ii) $abcd$ is a parallelogram in which $[ac]$ is a diagonal.
Find the co-ordinates of the point d.

(c) (i)	**5 marks**	**Att 2**

$(-4, 3) \rightarrow (0, 0)$; $(6, -1) \rightarrow (10, -4)$; $(2, 7) \rightarrow (6, 4)$.

Area $= \frac{1}{2}|x_1y_2 - x_2y_1| = \frac{1}{2}|10\times 4 - 6\times -4| = \frac{1}{2}|40+24| = \frac{1}{2}|64| = 32.$

or

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}|-4(-1-7) + 6(7-3) + 2(3+1)|$

$= \frac{1}{2}|32+24+8| = \frac{1}{2}|64| = 32.$

or

Area $= \frac{1}{2}[x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_3y_2 - x_2y_1]$

$= \frac{1}{2}|-4\times -1 + 6\times 7 + 2\times 3 + 4\times 7 - 2\times -1 - 6\times 3|$

$= \frac{1}{2}|4+42+6+28+2-18| = \frac{1}{2}|64| = 32.$

* $\frac{1}{2}|-64| = -32$ incurs no penalty.

Award: 5 marks for a fully correct answer.
2 marks for some correct work – including correct answer without work shown.
0 marks for a worthless attempt e.g. irrelevant formula or stops at $\frac{1}{2}$ on its own.

(c) (ii)	**10 marks**	**Att 3**

By $(6, -1) \rightarrow (2, 7)$, $a(-4, 3) \rightarrow d(-8, 11)$.

or

By $(6, -1) \rightarrow (-4, 3)$, $c(2, 7) \rightarrow d(-8, 11)$.

or

Mid-point of $[ac]$ $= \left(\frac{-4+2}{2}, \frac{3+7}{2}\right) = (-1, 5)$.

Mid-point of $[bd]$ $= \left(\frac{6+x}{2}, \frac{-1+y}{2}\right) = (-1, 5) \Rightarrow x = -8,\ y = 11$.

Award: 10 marks for fully correct answer including correct answer without work.
3 marks for some correct work.
0 marks for worthless attempt.

QUESTION 3

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

A circle C, has centre (0, 0) and radius 4.

(i) Write down the equation of C.

(ii) Verify that the point (3, 2) lies inside the circle C.

(a) (i) **5 marks** **Att 2**

$$x^2 + y^2 = r^2 = 4^2 \text{ or } 16.$$

* Accept correct answer without work.

Blunders (- 3)

B1 Radius not squared or $r^2 = 2r$ or $4^2 \neq 16$.

B2 Incorrect relevant formula with substitution e.g. $x^2 - y^2 = r^2$.

B3 Omission of squares e.g. $x + y = 16$.

Attempts (2 marks)

A1 Correct relevant circle formula and stops e.g. $x^2 + y^2 = r^2$ or $(x-h)^2 + (y-k)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$.

A2 Accurate diagram drawn.

(a) (ii) **5 marks** **Att 2**

$$x^2 + y^2 = 16 \quad \Rightarrow \quad \underbrace{3^2 + 2^2\downarrow}_{[2\ marks]} = 9 + 4 = \underbrace{13\downarrow}_{[4\ marks]} < \underbrace{16.\downarrow}_{[5\ marks]}$$

or

$$\underbrace{\sqrt{(0-3)^2 + (0+2)^2}\downarrow}_{[2\ marks]} = \underbrace{\sqrt{13}\downarrow}_{[4\ marks]} < \underbrace{\sqrt{16} \text{ or } 4.\downarrow}_{[5\ marks]}$$

* Accept "distance from (3, 2) to (0, 0) is $\sqrt{13}$ which is less than the length of the radius".

* Any error other than an obvious slip merits the attempt mark at most.

Attempts (2 marks)

A1 Some relevant step e.g. 3^2 or mentions (0, 0).

A2 Point $(3, 2)$ with x_1 and y_1 identified, or point plotted – *apply to all sections*.

A3 Any formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.

A4 Accurate diagram drawn, with (3, 2) shown inside.

A5 States or refers to Theorem of Pythagoras.

Part (b) **20 (15, 5) marks** **Att (5, 2)**

The line $x - 3y = 0$ intersects the circle $x^2 + y^2 = 10$ at the points a and b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) Show that $[ab]$ is a diameter of the circle.

(b) (i) **15 marks** **Att 5**

$x - 3y = 0 \quad \Rightarrow \quad x = 3y$

$x^2 + y^2 = 10 \quad \Rightarrow \quad (3y)^2 + y^2 = 10$

$\Rightarrow \quad 9y^2 + y^2 = 10 \quad \Rightarrow \quad 10y^2 = 10$

$\Rightarrow \quad y^2 = 1 \quad \Rightarrow \quad y = \pm 1$

$\Rightarrow \quad x = \pm 3$

Points (3, 1) and (−3, −1).
Similarly for using $x - 3y = 0 \quad \Rightarrow \quad y = \frac{1}{3}x$.

* Accept two correct points verified correctly in both line and circle.

Award: 15 marks for a fully correct answer: $x = \pm 3$, $y = \pm 1$.

5 marks for some correct work – including one correct point, or a correct answer without work or an accurate graphical solution.

0 marks for worthless attempt.

(b) (ii) **5 marks** **Att 2**

Mid-point of [*ab*] $= \left(\frac{3-3}{2}, \frac{1-1}{2}\right) = (0, 0)$

Centre of circle $= (0, 0)$ Thus, [*ab*] a diameter.

or

$|ab| = \sqrt{(3+3)^2 + (1+1)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

Radius of circle $= \sqrt{10}$ Thus, [*ab*] a diameter.

or

Centre of circle (0,0)
Image of (3, 1) under central symmetry in (0, 0) is (−3, −1). [*ab*] a diameter.

or

Equation of *ab* is $y - 1 = \frac{1}{3}(x - 3) \quad \Rightarrow \quad x - 3y = 0$.
or states *a* and *b* belong to the line $x - 3y = 0$.
Substitute (0, 0): $0 + 3(0) = 0 \quad \Rightarrow \quad 0 = 0$ Thus, [*ab*] a diameter.

Award: 5 marks for a fully correct or consistent answer.
2 marks for some correct work.
0 marks for a worthless attempt.

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

The circle *K* has equation $(x-5)^2 + (y+1)^2 = 34$.

(i) Write down the radius of *K* and the co-ordinates of the centre of *K*.

(ii) Verify that the point $(10, -4)$ is on the circle.

(iii) *T* is a tangent to the circle at the point $(10, -4)$.
S is another tangent to the circle and *S* is parallel to *T*.
Find the co-ordinates of the point at which S is a tangent to the circle.

(c) (i) **5 marks** **Att 2**

	$(x-5)^2+(y+1)^2=34$.	
	Radius $\sqrt{34}$.	[2 *marks*]
	Centre $(5,-1)$ or $h=5,\ k=-1$.	[5 *marks*]
or		
	$x^2-10x+25+y^2+2y+1=34 \quad\Rightarrow\quad x^2+y^2-10x+2y-8=0$.	
	Radius $\sqrt{g^2+f^2-c} = \sqrt{25+1+8} = \sqrt{34}$.	[2 *marks*]
	Centre $(-g,-f)=(5,-1)$.	[5 *marks*]

* Accept correct answer without work shown.

Blunders (-3)
B1 Centre $(-5,1)$ or $(-1,5)$.
B2 Radius given as 34 or 17.

Slips (– 1)
S1 One sign incorrect in co-ordinates of centre.

Attempts (2 marks)
A1 Correct relevant formula and stops.
A2 Attempt at a graphical solution with some correct work.

(c) (ii) **5 marks** **Att 2**

	$(x-5)^2+(y+1)^2=34$
	$(10-5)^2+(-4+1)^2 = 5^2+(-3)^2 = 25+9=34$.
or	
	$\sqrt{(10-5)^2+(-4+1)^2} = \sqrt{5^2+(-3)^2} = \sqrt{34} =$ radius.

* Accept "distance from $(10,-4)$ to $(5,-1)$ is $\sqrt{34}$ which is the length of the radius".

Award: 5 marks for a fully correct or consistent answer.
2 marks for some correct work.
0 marks for a worthless attempt.

(c) (iii) **10 marks** **Att 3**

	By $(10,-4) \to (5,-1)$
	$(5,-1) \to (0,2)$, the point of tangency of *S*.
or	
	$\frac{1}{2}(10+x)=5, \quad \frac{1}{2}(-4+y)=-1$
	$10+x=10 \Rightarrow x=0$ and $-4+y=-2 \Rightarrow y=2$.

Award: 10 marks for a fully correct answer or an answer consistent with the candidate's answer to section (i) [accept if work not shown].
3 marks for some correct work.
0 marks for a worthless attempt.

QUESTION 4

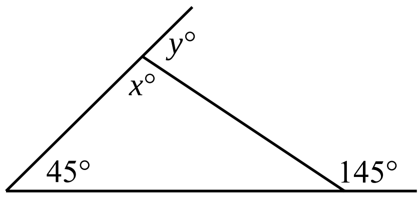
Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

In the diagram, two sides of the triangle are produced.

(i) Find x.

(ii) Find y.



(a) (i) **5 marks** **Att 2**

$x° + 45° = 145° \downarrow \Rightarrow \quad x° = 145° - 45° \downarrow \ = 100°. \downarrow$

[*2 marks*] [*4 marks*] [*5 marks*]

or

$x° + 45° + 35° = 180° \downarrow \Rightarrow \quad x° = 180° - 45° - 35° \downarrow = 100°. \downarrow$

[*2 marks*] [*4 marks*] *5 marks*]

(a) (ii) **5 marks** **Att 2**

$x° + y° = 180° \quad \Rightarrow \quad 100° + y° = 180° \downarrow \Rightarrow \quad y° = 180° - 100° \downarrow \ = 80°. \downarrow$

[*2 marks*] [*4 marks*] [*5 marks*]

or

$y° = 45° + 35° \downarrow \ = 80°. \downarrow$

[*4 marks*] [*5 marks*]

* Accept correct answer without work shown.

* Allow candidate's value for x in finding y.

* Accept x and y in any order, based on candidate's work – if work not shown award 0 and 5 if $x + y = 180°$.

Blunders (-3)

B1 Incorrect geometrical result e.g. sum of three angles $\neq 180^{o}$.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (2 marks each part)

A1 Some relevant step or statement.

Worthless (0 marks)

W1 Incorrect answer without work.

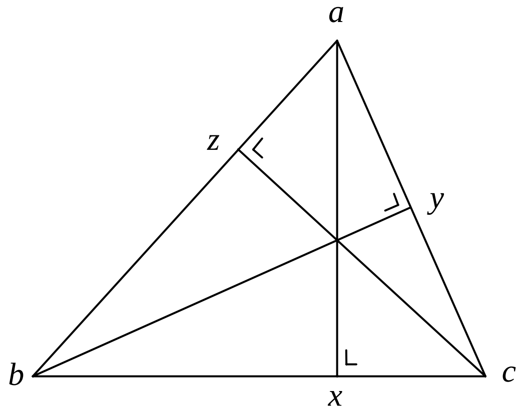
W2 "Alternate" angles.

Part (b) **20 marks** **Att 7**

Prove that the products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

Part (b) **20 marks** **Att 7**

In the triangle abc, $[ax]$, $[by]$, $[cz]$ are altitudes.
To prove: $|ab|.|cz| = |ac|.|by| = |bc|.|ax|$.



Proof:
In Δaby and Δcaz

$|\angle yab| = |\angle caz|$ … same angle [*10 marks*]

$|\angle bya| = |\angle azc|$ … right angles

Hence, Δaby and Δcaz are equiangular [*13 marks*]

Hence, $\dfrac{|ab|}{|ac|} = \dfrac{|by|}{|cz|}$ [*16 marks*]

Hence, $|ab|.|cz| = |ac|.|by|$ [*19 marks*]

Similarly $|ab|.|cz| = |bc|.|ax|$ [*20 marks*]

* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

Blunders (-3)
B1 Each step omitted, incorrect or incomplete, except the last.
B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

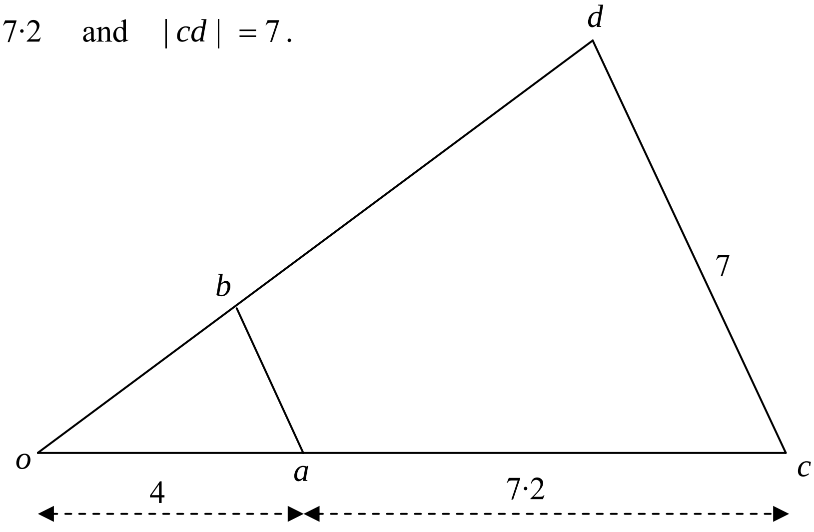
Attempts (7 marks)
A1 Any relevant step, stated or indicated, e.g. triangle with additional relevant information.
A2 States or illustrates a special case, e.g. measuring the sides and altitudes.

Worthless (0 marks)
W1 Any irrelevant theorem, subject to the attempt mark.
W2 Triangle only.

Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

The triangle *ocd* is the image of the triangle *oab* under an enlargement with centre *o*.

$|oa| = 4$, $|ac| = 7{\cdot}2$ and $|cd| = 7$.



(i) Find the scale factor of the enlargement.
(ii) Find $|ab|$.
(iii) The area of the triangle *oab* is 4·5 square units. Find the area of the triangle *ocd*.

(c) (i) **5 marks** **Att 2**

$$|oc| = k\,|oa| \;\Rightarrow k = \frac{|oc|}{|oa|} \;\Rightarrow k = \frac{7.2+4}{4} = \frac{11.2}{4} \downarrow = 2.8\,.$$

[*4 marks*]

* Accept a correct answer without work shown.

Blunders (-3)

B1 Incorrect ratio in finding scale factor.
B2 Incorrect centre of enlargement.
B3 Scale factor given as 1.8.

Attempts (2 marks)

A1 Relevant step, e.g. 7.2 + 4.
A2 Attempt at ratio e.g. $|oa| : |oc|$.
A3 Gives 11.2 and stops.

(c) (ii) **10 marks** **Att 3**

$$\frac{7}{|ab|} = 2.8 \;\Rightarrow\; |ab| = \frac{7}{2.8} \downarrow = 2.5\,.$$

[*9 marks*]

or

$$\frac{|oa|}{|oc|} = \frac{|ab|}{|cd|} \;\Rightarrow\; \frac{4}{11.2} = \frac{|ab|}{7} \;\Rightarrow\; |ab| = \frac{4\times 7}{11.2} \downarrow = 2.5\,.$$

[*7 marks*]

* Accept candidate's scale factor from (i).
* Accept correct or consistent answer without work.

Blunders (-3)

B1 Incorrect or inconsistent scale factor or ratio.

Slips (-1)

S1 Error in calculating length of side of image, each time.

(c) (iii) **5 marks** **Att 2**

Area of $\Delta ocd = 4.5 \times 2.8^2 \downarrow = 35.28 \downarrow$ square units.

[*2 marks*] [*5 marks*]

* Accept correct answer without work.
* Accept answer consistent with earlier sections.

Blunders (-3)

B1 Does not square scale factor. (Answer 12.6).
B2 $4.5 * 2.8^2$ where $*$ is an operation other than multiplication.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (2 marks)

A1 2.8^2 or $4.5 \div 2.8^2$ or $2.8^2 \div 4.5$ or $4.5 \div 2.8$ or $2.8 \div 4.5$.
A2 Some substitution into a correct area formula.

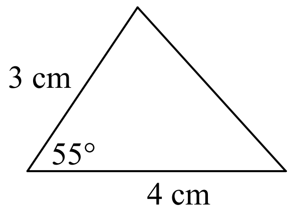
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 5**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Calculate the area of the triangle shown.

Give your answer correct to one decimal place.



(a) **10 marks** **Att 3**

Area of triangle $= \tfrac{1}{2}ab\sin C = \tfrac{1}{2}\times 3\times 4\times \sin 55°\downarrow = 6(0.8192)\downarrow = 4.91 = 4.9\text{ cm}^2.\downarrow$

[*4 marks*] [*7 marks*] [*10 marks*]

or

Area $= \tfrac{1}{2}(4)\times$ perp. height $= \tfrac{1}{2}(4)\times[3\sin 55°]\downarrow = 2[2.457]\downarrow = 4.91 = 4.9\text{cm}^2.\downarrow$

[*4 marks*] [*7 marks*] [*10 marks*]

Blunders (-3)

B1 $\tfrac{1}{2}a\sin C$ fully worked.

B2 Uses radians (or gradient) mode incorrectly – apply once in each part in which it occurs.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Some correct substitution into incorrect relevant formula e.g. $\tfrac{1}{2}a\sin C$.

A2 Some correct use of sine rule or cosine rule.

A3 Answer of 4.9 or 5 without work shown.

A4 Triangle treated as right-angled and answer given is $\tfrac{1}{2}\times 3\times 4$.

Worthless (0 marks)

W1 Incorrect answer without work.

W2 Writes formula from Tables and stops.

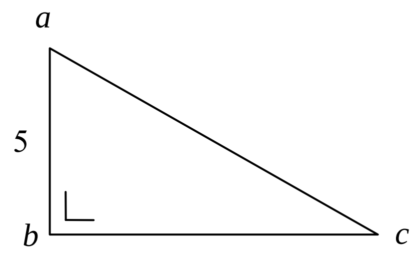
Part (b) **20 (5, 10, 5) marks** **Att(2, 3, -)**

In the right-angled triangle abc, $|ab| = 5$ cm.
The area of the triangle is 15 cm^2.

(i) Find $|bc|$.

(ii) Find $|\angle cab|$, correct to the nearest degree.

(iii) Find $|\angle bca|$, correct to the nearest degree.



(b) (i) **5 marks** **Att 2**

$$\tfrac{1}{2}|bc| \times 5 = 15 \downarrow \Rightarrow \quad |bc| = \frac{2 \times 15}{5} \downarrow = 6 \text{ cm.} \downarrow$$

[*2 marks*] [*4 marks*] [*5 marks*]

* Accept a correct answer without work shown.

Blunders (-3)
B1 Incorrect formula for area and continues.
B2 Incorrect substitution into formula.

Attempts (2 marks)
A1 Incorrect relevant formula with some correct substitution.

(b) (ii) **10 marks** **Att 3**

$$\tan \angle cab = \tfrac{6}{5} = 1.2 \quad \Rightarrow \quad |\angle cab| = 50.19° \quad = \quad 50°.$$

Award: 10 marks for a fully correct answer or an answer consistent with candidate's answer to section (i) subject to slip -1 for omitted or incorrect round-off.

3 marks for some correct work – including correct answer without work shown.

0 marks for a worthless attempt, including writing a formula from Tables and stopping.

(b) (iii) **5 marks** **-**

$$|\angle bca| = 90° - 50° \quad = \quad 40°.$$

or

$$\tan \angle bca = \tfrac{5}{6} \quad = 0.8333 \quad \Rightarrow \quad |\angle bca| = 39.8° \quad = \quad 40°.$$

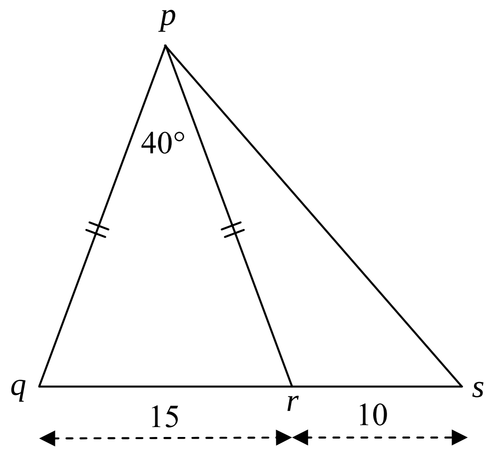
Award: 5 marks for a correct or consistent answer (work not required), subject to slip -1 for omitted or incorrect round-off, – otherwise 0 marks.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

In the triangle pqr,
$|pq| = |pr|$, $|qr| = 15$ cm
and $|\angle rpq| = 40°$.

(i) Find $|pr|$, correct to the nearest centimetre.

(ii) s is a point on qr such that $|rs| = 10$.
Find $|ps|$, correct to the nearest centimetre.



(c) (i) **10 marks** **Att 3**

$|\angle pqr| = \tfrac{1}{2}(180° - 40°) = 70°$

$$\frac{15}{\sin 40} = \frac{|pr|}{\sin 70} \Rightarrow |pr| = \frac{15\sin 70}{\sin 40} = \frac{15(0.9397)}{0.6428} = 21.9 = 22 \text{ cm.}$$

Award: 10 marks for a fully correct answer, subject to a slip -1 for no units and a slip -1 for an omitted or incorrect round-off, where relevant.

3 marks for some correct work – including correct answer without work shown.

0 marks for a worthless attempt, including treating the triangle as right-angled, or measurement from the diagram or sine rule stated without substitution.

(c) (ii) **10 marks** **Att 3**

$|\angle prs| = 180° - 70° = 110°$

$a^2 = b^2 + c^2 - 2bc\cos A \Rightarrow |ps|^2 = 22^2 + 10^2 - 2(22)(10)\cos 110°$

$\Rightarrow |ps|^2 = 484 + 100 - 440(-0.3420) = 584 + 150.48 = 734.48 \Rightarrow |ps| = 27.1 = 27$ cm.

Award: 10 marks for a fully correct or consistent answer, subject to a slip -1 for no units and a slip -1 for an omitted or incorrect round-off, where relevant.

3 marks for some correct work – including correct answer without work shown.

0 marks for a worthless attempt, including treating the triangle as right-angled, or measurement from the diagram or cosine rule stated without substitution.

QUESTION 6

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att -**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

One letter is chosen at random from the letters of the word EUCLID.

(i) Find the probability that the letter chosen is D.

(ii) Find the probability that the letter chosen is a vowel.

(a) **10 (5, 5) marks** **Att (2, 2)**

(i)	Probability of D = $\frac{1}{6}$.
(ii)	Probability of vowel = $\frac{3}{6}$ or $\frac{1}{2}$.

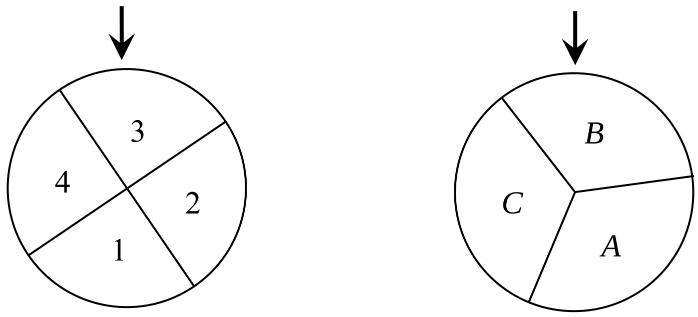
* Accept correct answer without work shown in each section (i) and (ii).
* Accept an answer for section (ii) consistent with section (i)

Attempts (2 marks)

A1 #(E) correctly identified or given as the numerator or
#(S) correctly identified or given as the denominator – *apply to parts (a) and (b).*
A2 The unsimplified correct answer inverted – *apply to parts (a) and (b).*
A3 Any relevant step such as sample space entries listed but word EUCLID written is worthless.
A4 Statement of probability theorem, awarded once, unless specifically adapted to each section.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The diagram shows two wheels.
The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.
The second wheel is divided into three equal segments labelled *A*, *B* and *C*.



A game consists of spinning the two wheels and noting the segment at the arrow on each when the wheels stop spinning. For example, the outcome shown is (3, *B*).

(i) Write down all the possible outcomes.

(ii) Find the probability that the outcome is (2, *C*)?

(iii) Find the probability that the outcome is an odd number with the letter *A*?

(iv) Find the probability that the outcome includes the letter *C*?

(b) (i) **5 marks** **Att 2**

(1, A); (1, B); (1, C); (2, A); (2, B); (2, C); (3, A); (3, B); (3, C); (4, A); (4, B); (4, C).

* Penalise -1 for each incorrect or omitted entry, subject to the attempt mark for at least one correct entry other than (3, B) or (2, C).
* Penalise excess entries -1 each to a maximum of 3, but only apply if subsequently used.
* If candidate gives 11 entries excluding (3, B), apply (-1) once in this section if 11 is used in subsequent sections.

(b) (ii) **5 marks** **Att 2**

P(2, C) = $\frac{1}{12}$.

* Accept correct answer without work in each of the following sections, including an answer in percentage form or decimal form (minimum of two decimal places).
* Accept an answer consistent with section (i) in each of the following sections.

Attempts (2 marks)

A1 Relevant step such as relevant entries in the sample space listed or indicated – apply to sections (iii) and (iv) also.

A2 Answer of 1 or 12 or $\frac{12}{1}$.

(b) (iii) **5 marks** **Att 2**

P(odd, A) = $\frac{2}{12}$ or $\frac{1}{6}$.

Attempts (2 marks)

A3 Answer of 2 or 12 or $\frac{12}{2}$.

Worthless (0 marks)

W1 1 or 6.

(b) (iv) **5 marks** **Att 2**

	P(includes C) = $\frac{4}{12}$ or $\frac{1}{3}$.
or	
	1 – P(C not included) = $1-\frac{8}{12} = \frac{4}{12} = \frac{1}{3}$.

Attempts (2 marks)

A4 Answer of 4 or 12 or $\frac{12}{4}$.

Worthless (0 marks)

W1 1 or 3.

Part (c) **20 (5, 5, 5, 5) marks** **Att (-, -, -, -)**

(i)	How many different three-digit numbers can be formed from the digits 2, 3, 4, 5, 6, if each of the digits can be used only once in each number.
(ii)	How many of the numbers are less than 400?
(iii)	How many of the numbers are divisible by 5?
(iv)	How many of the numbers are less than 400 and divisible by 5?

(c) (i) **5 marks** -

$5 \times 4 \times 3 = 60.$ ***or*** ${}^{5}P_{3} = 60.$

* If sections of (c) are not identified and it is not obvious which section is being attempted, treat each section in order.

Award: 5 marks in each part for a fully correct answer or an answer consistent with the candidate's work; penalise -1 for the multiplication written but not done, otherwise 0 marks.

(c) (ii) **5 marks** -

$2 \times 4 \times 3 = 24.$ ***or*** $2 \times {}^{4}P_{2} = 2 \times 12 = 24.$

* Accept $\frac{2}{5}$ of the candidate's answer from section (i).

(c) (iii) **5 marks** -

$4 \times 3 \times 1 = 12.$ ***or*** ${}^{4}P_{2} = 12.$ ***or*** $60 \div 5 = 12.$

* Accept candidate's answer from section (i) divided by 5.

(c) (iv) **5 marks** -

$2 \times 3 \times 1 = 6$ ***or*** $60 \div 10 = 6$

* Accept candidate's answer from section (i) divided by 10.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3**
Part (c)	**20 marks**	**Att 5**

Part (a) **10 marks** **Att 3**

Find the median of the numbers 5, 11, 3, 16, 8.

(a) **10 marks** **Att 3**

3, 5, 8, 11, 16. Median = 8.

* Accept correct answer without work shown.

Award marks as follows:

10 marks: Correct answer of 8.

7 marks: Five numbers ordered correctly but incorrect or no answer or
Five numbers incorrectly re-ordered but middle number selected.

4 marks: Answer 3 given without an attempt at re-ordering list.

3 marks: Attempt at finding the mean (8.6) or states there is no median (confusing median and mode) or gives answer of 5, 11 or 16 without work or defines median or gives $\frac{1}{2}(5+8)$ or $\frac{1}{2}(8+11)$ without work.

0 marks Worthless work.

Part (b) **20 (10, 5, 5) marks** **Att (3, - , -)**

The table below shows the time, in minutes, that customers were waiting to be served in a restaurant

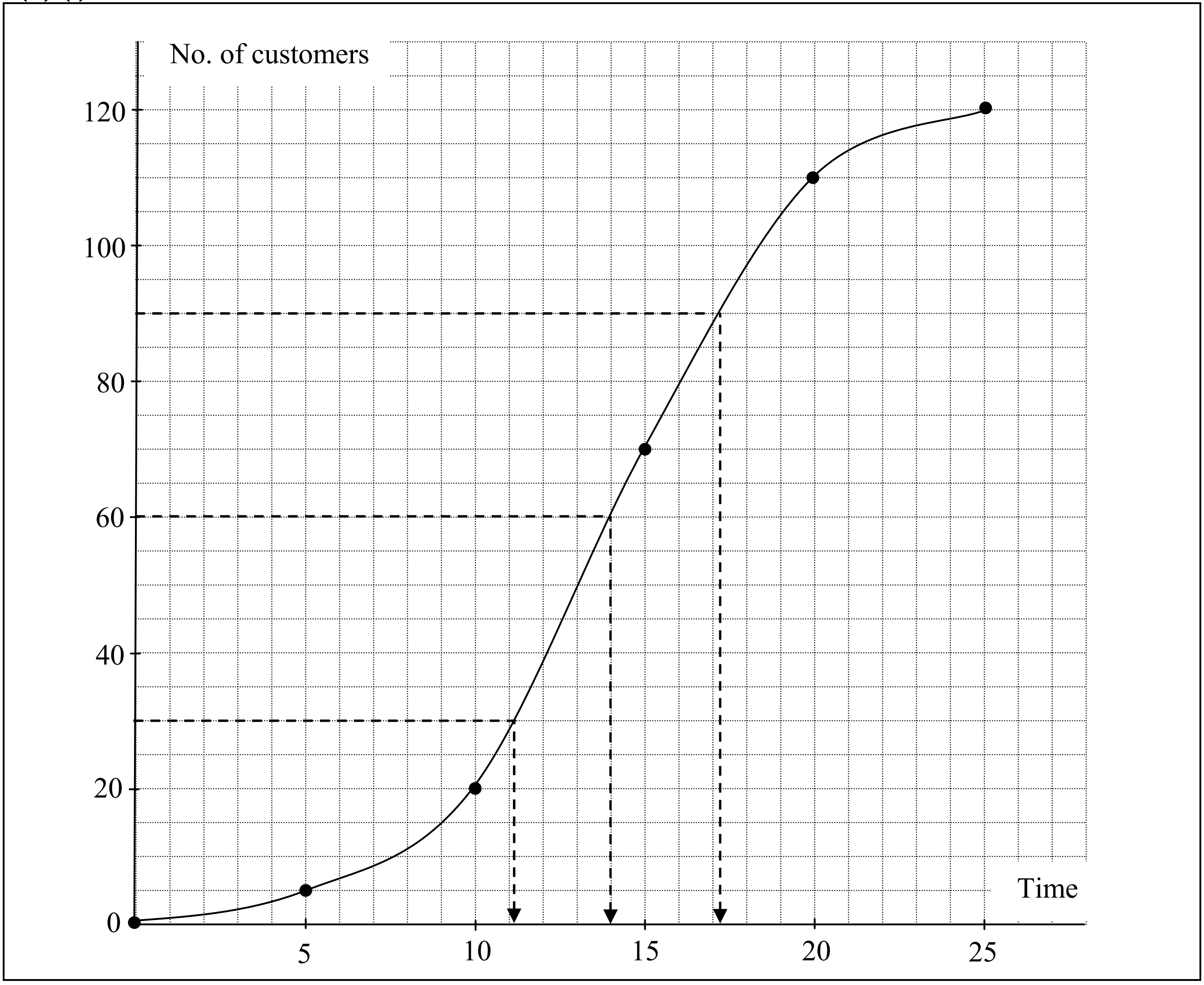
Time (minutes)	< 5	< 10	< 15	< 20	< 25
Number of customers	5	20	70	110	120

(i) Draw the cumulative frequency curve (ogive).

(ii) Use your curve to estimate the median waiting time.

(iii) Use your curve to estimate the interquartile range.

(b) (i) **10 marks** **Att 3**



* Accept frequency on the horizontal.

Blunders (-3)

B1 Draws a cumulative frequency polygon – apply slips also.
B2 Draws a cumulative cumulative curve – apply slips also.
B3 Scale irregular (apply once).

Slips (-1)

S1 Each point omitted or incorrectly plotted (to the eye) – B3 can also apply.
S2 Each pair of required points not joined – including (0, 0) to (5, 5).

Attempts (3 marks)

A1 Some correct step e.g. draws axes and stops.
A2 Draws histogram correctly instead of ogive.

(b) (ii) & (iii)	**(5, 5) marks**	**Hit/Miss**

(ii)	Median = 14 minutes.
(iii)	Interquartile range = 17 − 11 = 6 minutes.

Award: 5 marks for a correct numerical answer written from candidate's graph.
If work is shown, allow the tolerance of 20% of candidate's scale unit in reading answer from candidate's graph.
If no work is shown the answer must be exact from candidate's graph.
Otherwise 0 marks.

Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, -)**

The age of each person living in one street was recorded during a census.
The information is summarised in the following table:

Age (in years)	0 - 20	20 - 30	30 - 50	50 - 80
Number of people	16	12	32	12

(i) How many people were living in the street?

(ii) Using mid-interval values, calculate the mean age.

(iii) What is the greatest number of people who could have been aged under 40 years?

(c) (i)	**5 marks**	**Att 2**

$16 + 12 + 32 + 12 = 72.$

* Accept correct answer without work.

Slips (-1)
S1 Each value omitted from the addition.
S2 Writes the numbers with addition indicated without getting the total.

Attempts (2 marks)
A1 Identifies frequencies as the required numbers.

Worthless (0 marks)
W1 Incorrect answer without work, subject to A1.

(c) (ii) **10 marks** **Att 3**

Mid-interval values 10, 25, 40, 65

Mean $\bar{x} = \dfrac{10\times16+25\times12+40\times32+65\times12}{16+12+32+12}$

$= \dfrac{160+300+1280+780}{72} = \dfrac{2520}{72}$ *or* 35.

or

Interval	Mid-interval (x)	f	fx
0 – 20	10	16	160
20 – 30	25	12	300
30 – 50	40	32	1280
50 – 80	65	12	780
		72	2520

Mean $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{2520}{72}$ *or* 35.

* Accept correct answer or an answer consistent with candidate's answer to section (i) without work i.e. uses calculator.

Award: 10 marks for a fully correct or consistent answer.
3 marks for some correct work.
0 marks for a worthless attempt.

(c) (iii) **5 marks** **-**

16 + 12 + 32 = 60.

Award: 5 marks for a fully correct answer, apply slip -1 for 16 + 12 + 32 written but not added, otherwise 0 marks

QUESTION 8

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

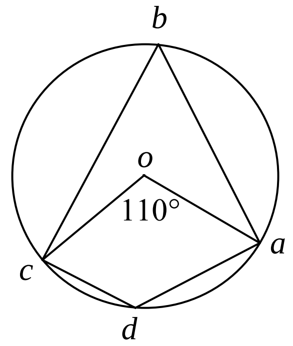
Part (a) **10 (5, 5) marks** **Att (2, 2)**

The points a, b, c and d lie on a circle, centre o.

$|\angle aoc| = 110°$.

(i) Find $|\angle abc|$.

(ii) Find $|\angle cda|$.



(a) (i) **5 marks** **Att 2**

$|\angle abc| = \tfrac{1}{2}(110°) \downarrow = 55°.\downarrow$

[4 *marks*] [5 *marks*]

* Accept correct answer without work or answer clearly indicated on a diagram.

Blunders (-3)

B1 $|\angle abc| = 2|\angle aoc| = 220°$.

Attempts (2 marks)

A1 States relevant geometrical result without applying it.

A2 Indicates $|\text{reflex } \angle aoc| = 250°$.

Worthless (0 marks)

W1 Incorrect answer without work – both sections.

(a) (ii) **5 marks** **Att 2**

$|\angle cda| = \tfrac{1}{2}(360° - 110°)\downarrow = \tfrac{1}{2}(250°)\downarrow = 125°.\downarrow$

[2 *marks*] [4 *marks*] [5 *marks*]

or

$|\angle cda| = 180° - 55°\downarrow = 125°.\downarrow$

[4 *marks*] [5 *marks*]

* Accept candidates answer from section (i).

* Accept correct answer without work or answer clearly indicated on a diagram.

Blunders (-3)

B1 Error in calculating reflex angle.

Attempts (2 marks)

A1 States relevant geometrical result without applying it.

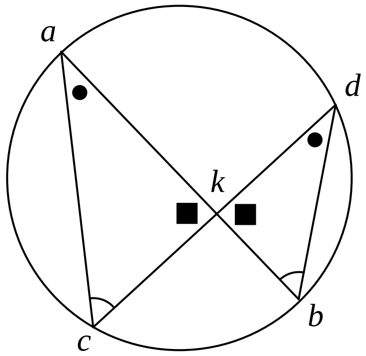
A2 Indicates $|\text{reflex } \angle aoc| = 250°$.

Part (b) **20 marks** **Att 7**

Prove that if $[ab]$ and $[cd]$ are chords of a circle and the lines ab and cd meet at the point k which is inside the circle, then $|ak|.|kb| = |ck|.|kd|$.

(b) **20 marks** **Att 7**

$[ab]$ and $[cd]$ are chords of the circle intersecting at k. To prove: $|ak|.|kb| = |ck|.|kd|$.



Construction:
Join a to c and b to d. [7 *marks*]

Proof:

$|\angle kac| = |\angle bdk|$ … angles on same arc [10 *marks*]

$|\angle ack| = |\angle kbd|$ … angles on same arc [13 *marks*]

$|\angle cka| = |\angle dkb|$ … vertically opposite angles

Δkac and Δkbd are equiangular [16 *marks*]

Hence, $\dfrac{|ak|}{|kd|} = \dfrac{|ck|}{|kb|}$ [19 *marks*]

Hence, $|ak|.|kb| = |ck|.|kd|$. [20 *marks*]

* Accept steps clearly marked on a diagram, subject to B2.

Blunders (-3)

B1 Incorrect step or step omitted where appropriate.
B2 Steps written in an illogical order. [Penalise once only.]
B3 Incomplete steps in proof.

Misreadings (-1)

M1 External case proved.

Attempts (7 marks)

A1 Outline diagram and stops. (Minimum required - circle and intersecting chords).
A2 States a relevant result or step.

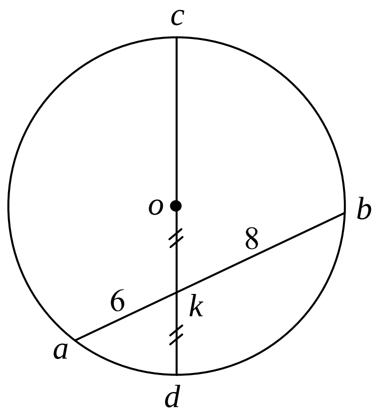
Part (c) **20 marks** **Att 7**

$[ab]$ and $[cd]$ are chords of the circle, centre *o*.

$[ab]$ bisects $[od]$ at the point *k*.

$|ak| = 6$ and $|kb| = 8$.

Find the length of the radius of the circle.



(c) **20 marks** **Att 7**

$\lvert ak \rvert \times \lvert kb \rvert \quad = \quad \lvert ck \rvert \times \lvert kd \rvert$	[*7 marks*]
$\Rightarrow \quad 6 \times 8 \ = \tfrac{3}{2} r \times \tfrac{1}{2} r$	[*10 marks*]
$\Rightarrow \quad \tfrac{3}{4} r^2 = 48$	[*13 marks*]
$\Rightarrow \quad r^2 = 64$	[*16 marks*]
$\Rightarrow \quad r = \pm 8$	[*19 marks*]
Radius = 8	[*20 marks*]

* Accept an answer where candidate writes $r = 2x$ and continues with $48 = 3x^2$ etc.

Blunders (-3)

B1 Incorrect application of theorem e.g. $|ak| \times |kb| \quad = \quad |co| \times |ok|$.

B2 $|ck| = 2r^2$ gives radius of 5.76.

Attempts (7 marks)

A1 Effort to link radius to a segment of diameter or chord.

A2 Writes 6×8 or 48.

A3 Relevant step towards applying theorem.

QUESTION 9

Part (a)	**10 marks**	**Att -**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att 6**

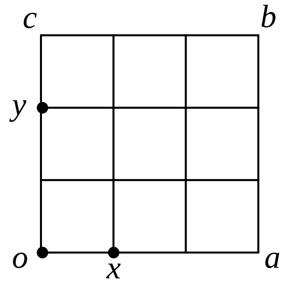
Part (a)	**10 (5, 5) marks**	**Att (- , -)**

oabc is a square divided into nine small squares.
o is the origin and *x* and *y* are the points shown.

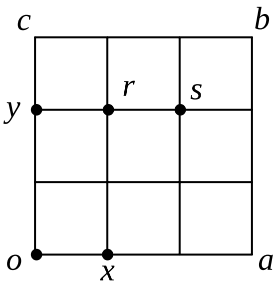
Copy the diagram and on it show

(i) the point *r* such that $\vec{r} = \vec{x} + \vec{y}$.

(ii) the point *s* such that $\vec{s} = 2\vec{x} + \vec{y}$.



(a)	**10 (5, 5) marks**	**Hit/Miss**



Point *r* [5 *marks*]

Point *s* [5 *marks*]

* Accept $\vec{s} = \vec{r} + \vec{x}$ for section (ii) from candidate's $\vec{r}$.

Award: 5 marks for each correct or consistent point plotted – otherwise 0 marks.

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Let $\vec{p} = 2\vec{i} - \vec{j}$ and $\vec{q} = -3\vec{i} + 4\vec{j}$.

(i) Find $|\vec{p}|$.

(ii) Express $5\vec{p} - \vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Express $\overrightarrow{pq}$ in terms of $\vec{i}$ and $\vec{j}$.

(iv) Calculate $\vec{p} \cdot \vec{q}$, the dot product of $\vec{p}$ and $\vec{q}$.

(b) (i)	**5 marks**	**Att 2**

$$|\vec{p}| = \sqrt{2^2 + (-1)^2} \downarrow = \sqrt{4+1} = \sqrt{5} . \downarrow$$

[2 *marks*] [5 *marks*]

* Accept correct answer without work shown in sections (i) and (ii).

Blunders (-3)

B1 Blunder in formula e.g. square root omitted or squares omitted or $-$ instead of $+$.

Attempts (2 marks)

A1 Finds 2^2 or $(-1)^2$ and stops.

A2 Effort at use of square root.

(b) (ii) **5 marks** **Att 2**

$5\vec{p} - \vec{q} = 5(2\vec{i} - \vec{j}) - (-3\vec{i} + 4\vec{j})$	[*2 marks*]
$= 10\vec{i} - 5\vec{j} + 3\vec{i} - 4\vec{j} = 13\vec{i} - 9\vec{j}$.	[5 *marks*]

Slips (-1)

S1 Finds $5\vec{q} - \vec{p}$.

Attempts (2 marks)

A1 Some effort at scalar multiplication or combining components.

A2 $13\vec{i}$ or $-9\vec{j}$ without work shown.

A3 Plots one or more of the vectors.

Worthless (0 marks)

W1 Incorrect answer without work.

(b) (iii) **5 marks** **Att 2**

$\vec{pq} = \vec{q} - \vec{p} \downarrow = -3\vec{i} + 4\vec{j} - 2\vec{i} + \vec{j} = -5\vec{i} + 5\vec{j} . \downarrow$

[*2 marks*] [*5 marks*]

Blunders (-3)

B1 $\vec{pq} = \vec{q} + \vec{p}$ or $\vec{p} - \vec{q}$ or $\vec{p} \cdot \vec{q}$ and continues.

Attempts (2 marks)

A1 Relevant work on a diagram.

Worthless (0 marks)

W1 Incorrect answer without work.

(b) (iv) **5 marks** **Att 2**

$\vec{p} \cdot \vec{q} = (2\vec{i} - \vec{j}) \cdot (-3\vec{i} + 4\vec{j}) \downarrow = -6 - 4 = -10. \downarrow$

[*2 marks*] [*5 marks*]

* Accept a correct answer without work.

Blunders (-3)

B1 $\vec{i}^2 \neq 1$ or $\vec{j}^2 \neq 1$ or $\vec{i} \cdot \vec{j} \neq 0$, applied once.

B2 Incorrect relevant formula e.g. $|\vec{x}||\vec{y}|\sin\theta$ or $|\vec{x}| = \sqrt{a^2 - b^2}$.

Attempts (2 marks)

A1 Correct relevant formula and stops.

A2 Finds the length of one vector and stops.

A3 Some correct work in multiplication using *p* and/or *q*.

Worthless (0 marks)

W1 Incorrect answer without work.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $\vec{u} = 2\vec{i} + 5\vec{j}$ and $\vec{v} = 8\vec{i} + 10\vec{j}$.

(i) Find the scalars *h* and *k* such that $\vec{u} + h\vec{v} = k\vec{i}$.

(ii) Using your values for *h* and *k*, verify that $\vec{u}^{\perp} + h\vec{v}^{\perp} = k\vec{i}^{\perp}$.

(c) (i) **10 marks** **Att 3**

$$2\vec{i} + 5\vec{j} + h(8\vec{i} + 10\vec{j}) = k\vec{i}$$

$$\Rightarrow \quad 2\vec{i} + 5\vec{j} + 8h\vec{i} + 10h\vec{j} = k\vec{i}$$

$$\Rightarrow \quad 2 + 8h = k \text{ and } 5 + 10h = 0 \Rightarrow h = -\tfrac{1}{2}$$

$$\Rightarrow \quad 2 + 8(-\tfrac{1}{2}) = k \quad \Rightarrow k = -2.$$

Award: 10 marks for a fully correct answer.
3 marks for some correct work, including a correct answer without work.
0 marks for a worthless attempt.

(c) (ii) **10 marks** **Att 3**

$$\vec{u}^{\perp} = -5\vec{i} + 2\vec{j}$$

$$\vec{v}^{\perp} = -10\vec{i} + 8\vec{j}$$

$$\vec{u}^{\perp} + h\vec{v}^{\perp} = -5\vec{i} + 2\vec{j} + -\tfrac{1}{2}(-10\vec{i} + 8\vec{j})$$

$$= -5\vec{i} + 2\vec{j} + 5\vec{i} - 4\vec{j} = -2\vec{j}$$

$$= k\vec{i}^{\perp} \quad \text{or writes} \quad k\vec{i}^{\perp} = -2\vec{j}.$$

Award: 10 marks for a fully correct answer.
3 marks for some correct work.
0 marks for a worthless attempt.

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Find the sum to infinity of the geometric series $2 \quad + \quad \frac{2}{5} \quad + \quad \frac{2}{25} \quad + \; ...$

(a) **10 marks** **Att 3**

$r \;=\; \tfrac{2}{5} \div 2 \;=\; \tfrac{1}{5}$ [*3 marks*]

$S_\infty = \frac{a}{1-r} = \frac{2}{1-\tfrac{1}{5}} \downarrow \;=\; \frac{2}{\tfrac{4}{5}} \;=\; 2\times\frac{5}{4}\downarrow \;=\; \frac{5}{2}.\downarrow$

[*6 marks*] [*9 marks*] [*10 marks*]

or

$\underset{n\to\infty}{\text{Limit}}\, S_n \;=\; \underset{n\to\infty}{\text{Limit}}\, \frac{2}{\tfrac{4}{5}}\left(1-\frac{1}{5^n}\right)\downarrow \;=\; \underset{n\to\infty}{\text{Limit}}\, \frac{5}{2}\left(1-\frac{1}{5^n}\right)\downarrow \;=\; \frac{5}{2}.\downarrow$

[*6 marks*] [*9 marks*] [*10 marks*]

Blunders (-3)

B1 Incorrect a.
B2 Incorrect r.
B3 Blunder in fractions.
B4 Incorrect relevant formula e.g. $a/(1+r)$ giving answer of $1\tfrac{2}{3}$.

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 Correct relevant formula and stops.
A2 Some relevant step e.g. states the value for a or the value for r.
A3 Adds 2 or more of the given terms e.g $S_2 = 2\tfrac{2}{5} = 2.4$ or $S_3 = 2\tfrac{12}{25} = 2.48$.
A4 One correct step in adding relevant fractions.
A5 Treats as arithmetic series with further work, e.g. identifies a.
A6 Writes $T_n = ar^{n-1}$ or $2(\tfrac{1}{5})^{n-1}$.
A7 Gives $T_4 = \tfrac{2}{125}$.
A8 Correct answer without work.

Worthless (0 marks)

W1 Formula for arithmetic series and stops.
W2 $2+\tfrac{2}{5}+\tfrac{2}{25} = \tfrac{6}{30}$ or $\tfrac{6}{31}$.
W3 Incorrect answer without work.

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

(i)	Expand $(1+2x)^3$ fully .
(ii)	Given that $(1+2x)^3+(1-2x)^3=2(a+bx^2)$, find the value of a and the value of b.

(b) (i)	**10 marks**	**Att 3**

$$(1+2x)^3=\binom{3}{0}+\binom{3}{1}(2x)+\binom{3}{2}(2x)^2+\binom{3}{3}(2x)^3 \qquad [\textit{4 marks}]$$
$$=1+3(2x)+3(4x^2)+8x^3 \qquad [\textit{7 marks}]$$
$$=1+6x+12x^2+8x^3\,. \qquad [\textit{10 marks}]$$

* Accept long multiplication or Pascal's triangle and award 4 marks for 2 correct terms, 7 marks for 3 correct terms and 10 marks for all terms correct.
* Accept correct answer without work.

Blunders (-3)
B1 Incorrect power in a term.
B2 Blunder in working out binomial coefficients.
B3 Treats binomial coefficients as fractions e.g. $\frac{3}{2}(2x)^2$.
B4 Puts a + sign between coefficient and power of x.
B5 Expands $(1+x)^3$.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (3 marks)
A1 Any term, including first term, written down correctly.
A2 Answer of $1+2x^3$ is attempt mark at most.
A3 Gives part of Pascal's triangle or effort at Pascal's triangle.
A4 Gives coefficients only.
A5 Any step towards getting a binomial coefficient e.g. $\binom{3}{2}$.
A6 Any correct step towards long multiplication.

Worthless (0 marks)
W1 Writes $3(1+2x)^2$ or $3(1+2x)^2(2)$.

(b) (ii)	**10 marks**	**Att 3**

$$(1+2x)^3+(1-2x)^3 = 1+6x+12x^2+8x^3+1-6x+12x^2-8x^3 \qquad [\textit{4 marks}]$$
$$= 2+24x^2 = 2(1+12x^2) \qquad [\textit{7 marks}]$$
$$= 2(a+bx^2) \Rightarrow a=1, \quad b=12\,. \qquad [\textit{10 marks}]$$

Blunders (-3)
B1 Expands $(1-2x)^3$ incorrectly using Binomial or long multiplication – apply once.
B2 Writes $(1-2x)^3$ as $-1+6x-12x^2+8x^3$.
B3 States $a=2$ and $b=24$.

Slips (-1)
S1 Numerical slips to a maximum of 3.
S2 Works $(1+2x)^3-(1-2x)^3$.

Attempts (3 marks)

A1 Writes $(1-2x)^3$ as $-1-6x-12x^2-8x^3$.

A2 Writes $x^3+y^3=(x+y)(x^2-xy+y^2)$ and stops.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

> **(i)** €2000 is invested at 4% compound interest.
> Find the value of the investment at the end of six years, correct to the nearest euro.
>
> **(ii)** An investment account earns 4% per annum compound interest.
> At the beginning of each year for six consecutive years €2000 is invested in the account.
> Using the formula for the sum of the first *n* terms of a geometric series, find the total value of the investment at the end of the six years, correct to the nearest euro.

(c) (i) **10 marks** **Att 3**

$$A=P\left(1+\frac{r}{100}\right)^n = 2000\left(1+\frac{4}{100}\right)^6 \quad [\textit{3 marks}]$$

$$= 2000(1.04)^6 = 2000(1.2653) \quad [\textit{6 marks}]$$

$$= 2530.6 \quad [\textit{9 marks}]$$

$$= €2531. \quad [\textit{10 marks}]$$

or

Calculation on year by year basis:

Year 1:	€2000	+	€80				
Year 2:	€2080	+	€83.2				
Year 3:	€2163.2	+	€86.53				
Year 4:	€2249.73	+	€89.99				
Year 5:	€2339.72	+	€93.59				
Year 6:	€2433.31	+	€97.33	=	€2530.64	=	€2531.

Blunders (-3)

B1 Mathematical error e.g. $1.04^6=6.24$.

B2 Sign error in formula – uses 1 – 0.04.

B3 Subtracts in long method.

B4 Each year omitted in calculation on year by year basis.

Slips (-1)

S1 Numerical slips to a maximum of 3.

S2 Early rounding off that affects accuracy of answer – maximum of 3 in long method.

Attempts (3 marks)

A1 Mention of 0.04 or 1.04 or $^4/_{100}$ or $^{104}/_{100}$.

A2 4% of €2000 = €80 or 104% of €2000 = €2080.

A3 Correct answer without work.

Worthless (0 marks)

W1 €2000/4 = €500.

(c) (ii) **10 marks** **Att 3**

$S_6 = \quad 2000(1.04)^6 \quad + \quad 2000(1.04)^5 \quad + \ldots + \quad 2000(1.04)$	[*3 marks*]
$= \quad 2000[1.04 + 1.04^2 + 1.04^3 + 1.04^4 + 1.04^5 + 1.04^6]$	[*4 marks*]
$= \quad 2000\left(\dfrac{1.04(1.04^6 - 1)}{1.04 - 1}\right)$	[*7 marks*]
$= \quad 2000\left(\dfrac{1.04 \times 0.265319}{0.04}\right) = 2000 \times 6.898294 = 13796.588$ = €13 797.	[*10 marks*]

Blunders (-3)
B1 Incorrect *a*.
B2 Incorrect *r*.
B3 Incorrect relevant formula and continues.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (3 marks)
A1 Mention of 0.04, 1.04, 4/100, 104/100.
A2 4% of €2000 = €80 or 104% of €2000 = €2080 and stops.
A3 Attempt at calculation on year by year basis, even if fully correct.

QUESTION 11

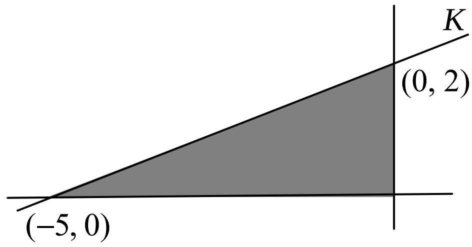
Part (a)	**15 marks**	**Att 5**
Part (b)	**35 marks**	**Att 10**

Part (a) **15 (10, 5) marks** **Att (3, 2)**

The line K cuts the x-axis at $(-5, 0)$ and the y-axis at $(0, 2)$.

(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.



(a) (i) **10 marks** **Att 3**

Slope of K $\quad \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{0+5} = \dfrac{2}{5}$ [*4 marks*]

Equation of K: $\quad y - y_1 = m(x - x_1)$ [*7 marks*]

$y - 0 = \frac{2}{5}(x+5)$ *or* $y - 2 = \frac{2}{5}(x-0)$ *or* $2x - 5y + 10 = 0$. [*10 marks*]

or

Equation of K: $\quad y = mx + c$ [*7 marks*]

$0 = \frac{2}{5}(-5) + c \Rightarrow c = 2.$ [*10 marks*]

* Accept $y - 0 = \frac{2}{5}(x+5)$ without work.

* Apply scheme for Q2 , Q3 where relevant.

Blunders (–3)

B1 Incorrect relevant formula and continues.

B2 Mixes up x's and y's when substituting.

B3 $y - 2 = m(x - 0)$ where m is not equal to $\frac{2}{5}$ without work.

B4 $y - y_1 = \frac{2}{5}(x - x_1)$ where (x_1, y_1) is not $(-5, 0)$ or $(0, 2)$ without work.

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 One, or more, correct, relevant formulae and stops.

Worthless (0 marks)

W1 An arbitrary line without work.

(a) (ii) **5 marks** **Att 2**

$x \le 0,$ $\quad y \ge 0,$ $\quad 2x - 5y + 10 \ge 0$ or equivalent

* Accept correct inequalities without work.

* Accept $<$ for $\le$ and $>$ for $\ge$.

* Accept an inequality consistent with candidate's K.

* Award 2 marks for one correct inequality, 4 marks for 2 correct and 5 marks for 3 correct.

Slips (- 1)

S1 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Substitutes any point and stops.
A2 $x \geq 0$ or $y \leq 0$ and stops, (without work).
A3 Incorrect or no conclusion e.g. $2x - 5y + 10 = 0 \Rightarrow 2(0) - 5(0) + 10 = 0$.
A4 Mathematical error in testing a point (e.g. sign error).
A5 Some relevant step e.g. $x = 0$.

Worthless (0 marks)
W1 Writes equation of K and stops.
W2 Draws the given diagram.

Part (b) **35 (20, 10, 5) marks** **Att 10 (8, 2, -)**

A developer is planning a holiday complex of cottages and apartments.
Each cottage will accommodate 3 adults and 5 children and each apartment will accommodate 2 adults and 2 children.
Other facilities in the complex are designed for a maximum of 60 adults and a maximum of 80 children.

(i) Taking x as the number of cottages and y as the number of apartments, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental income per night for a cottage will be €65 and for an apartment will be €40, how many of each should the developer include in the complex to maximise potential rental income?

(iii) If the construction costs are €200 000 for a cottage and €120 000 for an apartment, how many of each should the developer include in the complex to minimise construction costs?

(b) (i) Inequalities **10 (5, 5) marks** **Att (2, 2)**

Adults: $3x + 2y \leq 60$
Children: $5x + 2y \leq 80$

Also accept

	Cottages	Apartments	Maximum
Adults Children	3 5	2 2	60 80

or

3	2	60
5	2	80

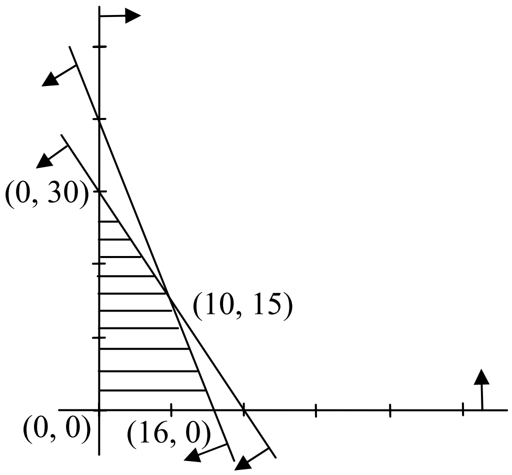
Award 10 marks but penalise in graph if link-up is incorrect

* Accept correct multiples or fractions of inequalities or use of different letters.
* Do not penalise here for an incorrect or for no inequality sign. Penalise in graph if used.

Blunders (-3)
B1 Mixes up x's and y's (once if consistent error).
B2 Confuses rows and columns in table, e.g. $3x + 5y \leq 60$ (once if consistent).

Attempts (2 marks for each inequality)
A1 Incomplete relevant data in table and stops (each inequality).
A2 Any other correct inequality, e.g. $x \geq 0$, $y \geq 0$, (each time).
A3 Some variable $\leq$ 60 or $\leq$ 80 (each time).
A4 $3x$ and / or $2y$ and stops (1 × Att 2).
A5 $5x$ and / or $2y$ and stops (1 × Att 2).

(b) (i) Graph **10 (5, 5) marks** **Att (2, 2)**



Each half-plane
[*5 marks*]

* Points or scales required.
* Half-planes required but no penalty for not indicating intersection if half-planes are indicated.
* If half-planes are indicated correctly, do not penalise for incorrect shading.
* Accept correct shading of intersection for half-planes but candidates may shade out areas that are not required and leave intersection blank.
* Correct shading over-rules arrows.
* Two lines drawn and **no shading** indicated, only one of the following applies :

Case 1:	Two sets of arrows in expected direction	10 marks
Case 2:	Two sets of arrows in unexpected direction	10 marks
Case 3:	One set of arrows "correct" and the other "incorrect"	7 marks (5+Att 2)
Case 4:	One line with and the other without arrows	7 marks (5+Att 2)
Case 5:	No arrows	4 marks (Att 2, Att 2)

Blunders (-3)

B1 No half-plane indicated (each time).
B2 Blunder in plotting a line or calculations (each line).
B3 Incorrect shading (once) e.g. one or both of the small triangles shaded.

Attempts (2 marks each half-plane)

A1 Some relevant work towards a point on a line, i.e. 2 marks for each line attempted.
A2 Draws axes or axes and one line (1 × Att 2).
A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2).

(b) (i) Intersection of lines **5 marks** **Att 2**

$$\begin{aligned} 3x + 2y &= 60 \\ 5x + 2y &= 80 \\ \hline 2x \quad &= 20 \quad \Rightarrow x = 10 \quad \Rightarrow y = 15 \end{aligned}$$

* Accept candidate's own equations from previous sections.
* x is calculated, accept consistent value for y without further work and vice versa.

Blunders (-3)

B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
B2 x or y value only found.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Correct or consistent answer without work or from a graph.
[Should get the *exact same* values from graph as if they had been found algebraically.]
A2 Any relevant step towards solving equations.

Worthless (0 marks)
W1 Incorrect answer without work and inconsistent with graph.

(b) (ii) Potential rental income **5 marks** **Hit/Miss**

Step 1	Vertices	$65x + 40y$	Income
Step 2	(16, 0)	1040 + 0	1040
Step 3	(10, 15)	650 + 600	1250
Step 4	(0, 30)	0 + 1200	1200

Step 5 10 cottages and 15 apartments to maximise rental income.

* Accept point of intersection from previous work.
* Accept work on a feasible set of points formed by axes and one line without further penalty.
* Information does not have to be in table form.
* Accept only vertices consistent with previously accepted work, not arbitrary ones. If (20, 0) or (0, 40) is tested and result is used to give maximum income, award 0, otherwise ignore.
* Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
* Accept any correct multiple or fraction of $65x + 40y$ here.
* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous work and also reward it here if the step is correct.

Award: 5 marks for a correct or consistent answer – see 4th and 5th * above – otherwise 0 marks.

(b) (iii) Construction costs **5 marks** **Hit/Miss**

Step 1	Vertices	$200x + 120y$	Costs (in 000's)
Step 2	(0, 0)	0 + 0	0
Step 3	(16, 0)	3200 + 0	3200
Step 4	(10, 15)	2000 + 1800	3800
Step 5	(0, 30)	0 + 3600	3600

Step 6 0 cottages and 0 apartments to minimise construction costs

* Mark as in (b) (ii) above.
* If candidate gives correct answer with a valid reason, without using the table, award 5 marks.

Marcanna Breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn thar 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéar ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás marcanna suas go 225. (e.g. 198 marks × 5% = 9.9 ⇒ bónas = 9 marc.)

Thar 225, is féidir an bónas a ríomh de réir na foirmle seo: [300 – bunmharc] × 15%, (agus an marc sin a shlánú **síos**). In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0