Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2006

MATHEMATICS ─ ORDINARY LEVEL

PAPER 2 (300 marks)

### MONDAY, 12 JUNE – MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. **(a)** The diagram shows a rectangle of length 42 cm.
   The area of the rectangle is 966 cm^2.
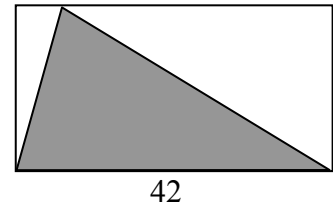
   (i) Find the height of the rectangle.

   (ii) Find the area of the shaded triangle.

   (b) Archaeologists excavating a rectangular plot *abcd* measuring 120 m by 60 m divided
   the plot into eight square sections as shown on the diagram. At the end of the first
   phase of the work the shaded area had been excavated. To estimate the area excavated,
   perpendicular measurements were made to the edge of the excavated area, as shown.
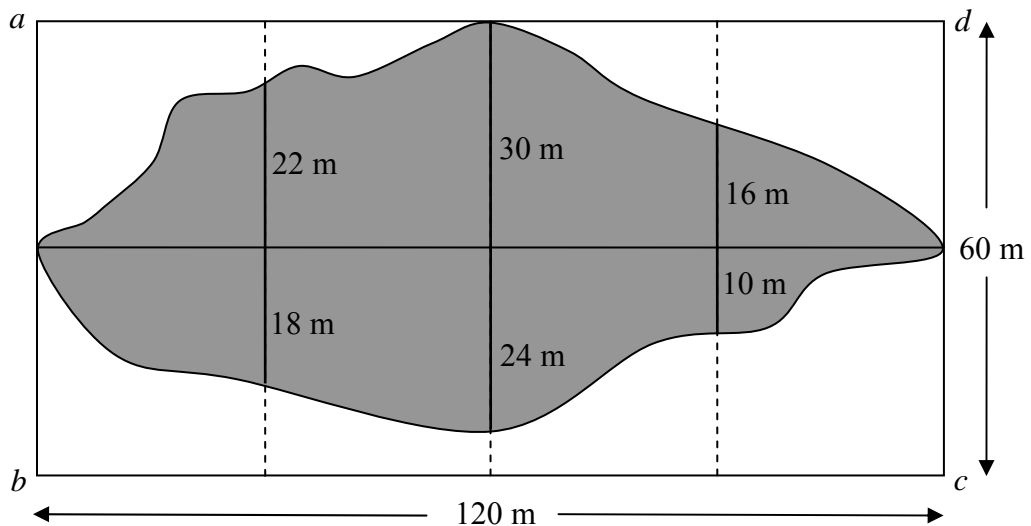

   (i) Use Simpson's Rule to estimate the area excavated.

   (ii) Express the excavated area as a percentage of the total area, correct to the nearest
   whole number.

   (c) **(i)** The volume of a hemisphere is 486π cm^3.
   Find the radius of the hemisphere.

   (ii) Find the volume of the smallest rectangular box that the hemisphere will fit into.

2. **(a)** $a(-2, 6)$ and $b(4, 3)$ are two points.

    (i) Plot a and b on a co-ordinate diagram.

    (ii) From your diagram, write down the co-ordinates of the point at which the line ab cuts the y-axis.

    (iii) Find the slope of ab.

    (iv) Calculate the area of the triangle abc, where the co-ordinates of c are $(1, -3)$.

(b) L is the line $3x + 2y + c = 0$.

    (i) $(3, -1)$ is a point on L. Find the value of c.

    (ii) The line K is parallel to L and passes through the point $(-2, 5)$.
    Find the equation of K.

    (iii) The lines L and K, together with the line $x = 3$ and the y-axis, form a parallelogram. Find the co-ordinates of the vertices of the parallelogram.

3. **(a)** The circle C has equation $x^2 + y^2 = 25$.
    The line L is a tangent to C at the point $(-3, 4)$.

    (i) Verify that the point $(-3, 4)$ is on C.

    (ii) Find the slope of L.

    (iii) Find the equation of L.

    (iv) The line T is another tangent to C and is parallel to L.
    Find the co-ordinates of the point at which T touches C.

(b) The vertices of a right-angled triangle are $p(1, 1)$, $q(5, 1)$ and $r(1, 4)$.
    The circle K passes through the points p, q and r.

    (i) On a co-ordinate diagram, draw the triangle pqr.
    Mark the point c, the centre of K, and draw K.

    (ii) Find the equation of K.

    (iii) Find the equation of the image of K under the translation $(5, 1) \rightarrow (1, 4)$.

**4.** **(a)** In the diagram $L \parallel K$.

Find the value of x.
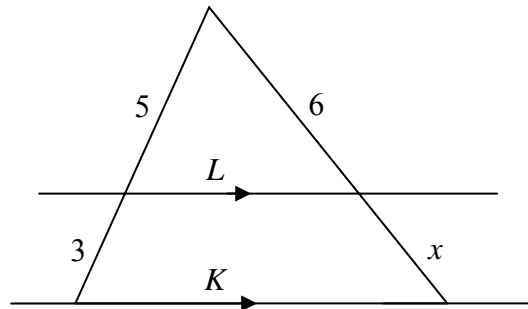
(b) Prove that if the lengths of two sides of a triangle are unequal, then the degree-measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

(c) **(i)** Construct a triangle abc in which $|ab| = 6.5$ cm, $|bc| = 2.5$ cm and $|ac| = 6$ cm.

(ii) Construct the image of the triangle abc under the enlargement of scale factor 1.8 and centre c.

(iii) Given that the area of triangle abc is 7.5 cm^2, find the area of the image triangle.

5. **(a)** The lengths of two sides of a right-angled triangle are shown in the diagram.
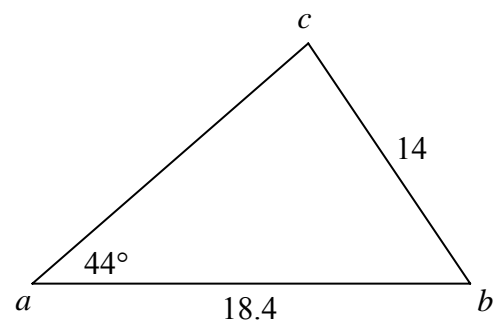
(i) Copy the diagram into your answer book and on it mark the angle A such that $\tan A = \dfrac{5}{8}$.
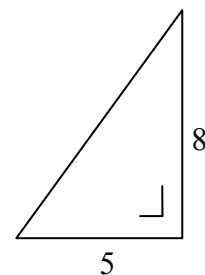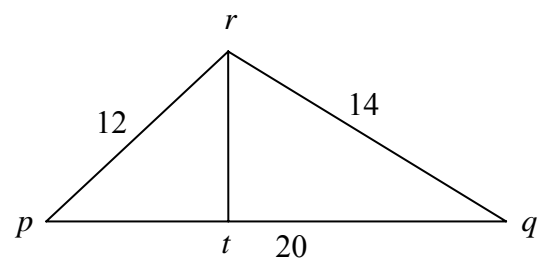
(ii) Find the area of the triangle.

(b) In the triangle abc,
$|ab| = 18.4$, $|bc| = 14$ and $|\angle cab| = 44°$.

(i) Find $|\angle bca|$, correct to the nearest degree.

(ii) Find the area of the triangle abc, correct to the nearest whole number.

(c) The lengths of the sides of the triangle pqr are
$|pq| = 20$, $|qr| = 14$ and $|pr| = 12$.

(i) Find $|\angle rpq|$, correct to one decimal place.

(ii) Find $|rt|$, where $rt \perp pq$. Give your answer correct to the nearest whole number.

6. **(a)** Evaluate $5\binom{8}{3} - 4\binom{8}{4}$.

(b) Niamh uses a password formed from one letter of her name followed by four of the digits from 1 to 9. She does not use any digit more than once.

   (i) How many such passwords can be formed?

   (ii) How many of the passwords begin with N?

   (iii) How many of the passwords end in an even digit?

   (iv) How many of the passwords begin with N and use only odd digits?

(c) Three coins are tossed. Each coin gives either a head or a tail.

   (i) Write down all the possible outcomes. For example, "H, T, H" or "head, tail, head" is one possible outcome.

   (ii) Find the probability that the result is three tails.

   (iii) Find the probability that the result includes no more than one head.

   (iv) Find the probability that the result has at least one head.

7. **(a)** The mean of the five numbers 2, 4, 7, 8, 9 is 6.

   Calculate the standard deviation of the five numbers, correct to one decimal place.

(b) The number of new cars in various price ranges sold by a retailer in one month is recorded in the following table:

Price (€1000's)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 50
Number sold	5	15	25	15	20

[Note: 15 – 20 means at least 15 but less than 20, etc.]

   (i) Draw a histogram to represent the data.

   (ii) By taking the data at the mid-interval values, calculate the mean price per car.

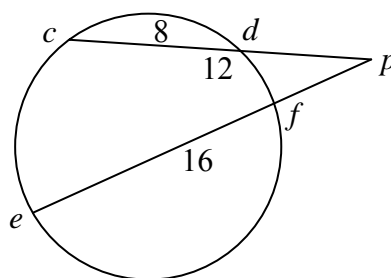   **(iii)** Copy and complete the following cumulative frequency table:

Price (€1000's)	< 15	< 20	< 25	< 30	< 50
Number sold					

   (iv) Draw the cumulative frequency curve (ogive).

   (v) Using your curve, estimate how many of the cars sold were priced between the mean and the median.

8.   **(a)** $[cd]$ and $[ef]$ are chords of a circle which, when produced, intersect at a point p outside the circle. $|cd| = 8$, $|cp| = 12$ and $|ep| = 16$.

   (i)   Find $|pd|$.

   (ii)   Find $|pf|$.

   (b)   Prove that an angle between a tangent ak and a chord $[ab]$ of a circle has degree-measure equal to that of any angle in the alternate segment.

   (c)   The lines pq and pr are tangents to the circle at the points a and b, respectively. c is a point on the circle. $|\angle acb| = 52°$ and $|\angle cbr| = 70°$.
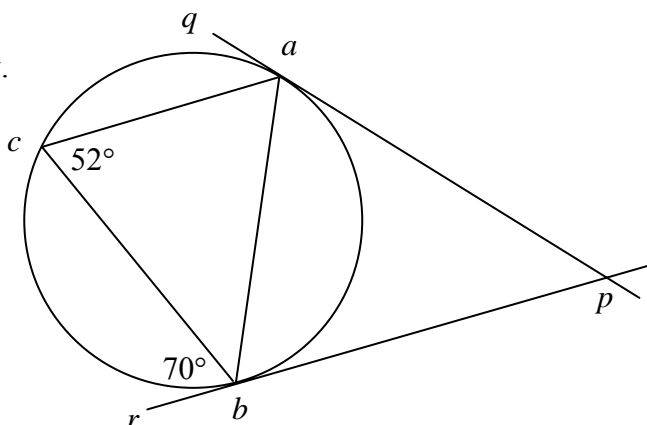
   (i)   Find $|\angle abp|$.

   (ii)   Find $|\angle bac|$.

   (iii)   Find $|\angle caq|$.

   (iv)   Find $|\angle bpa|$.

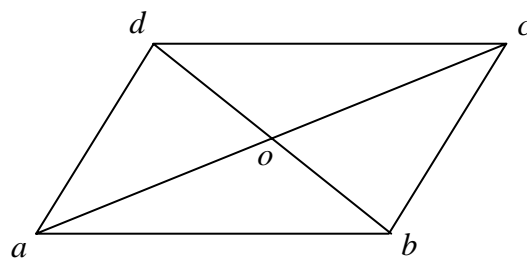**9.**   **(a)**   Let $\vec{v} = 3\vec{i} - 5\vec{j}$.

   (i)   Express $\vec{v}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

   (ii)   Express $\vec{v} + \vec{v}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

   (b)   Let $\vec{p} = 3\vec{i} - \vec{j}$ and $\vec{q} = 4\vec{i} + 2\vec{j}$.

   (i)   Express $5\vec{p} - 2\vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

   (ii)   Calculate $\vec{p} \cdot \vec{q}$, the dot product of $\vec{p}$ and $\vec{q}$.

   (iii)   Verify that $|\vec{q}| > |\vec{pq}|$.

   (c) $abcd$ is a parallelogram. The diagonals intersect at o, the origin.

   (i)   Express $\vec{ab} + \vec{bc}$ in terms of $\vec{c}$.

   (ii)   Express $\vec{ad} - \vec{bd}$ in terms of $\vec{a}$ and $\vec{b}$.

   (iii)   Show that $\vec{ad} - \vec{ac} + \vec{ab} = \vec{o}$.

   (iv)   Write $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ in its simplest form.

10. **(a)** Expand $(1-x)^5$ fully.

(b) A geometric series is $1 + \dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \dots$

    (i) Find the sum of the first 20 terms of the series.

    (ii) Find S, the sum to infinity of the series.

    (iii) The sum to infinity of another geometric series is also S.
   The first term of this series is 2. Find its common ratio.

(c) A company invests €P in new machinery.
The machinery depreciates at the rate of $r\,\%$ per annum.

    (i) Write down, in terms of P and r, the value of the machinery after 8 years.

    (ii) If the machinery depreciates to one-quarter of its original value after 8 years,
    find r, correct to the nearest whole number.

11. **(a)** The equation of the line L is $5x + 8y + 40 = 0$.
The equation of the line K is $10x - 7y - 35 = 0$.

Write down the 3 inequalities that together
define the shaded region in the diagram.
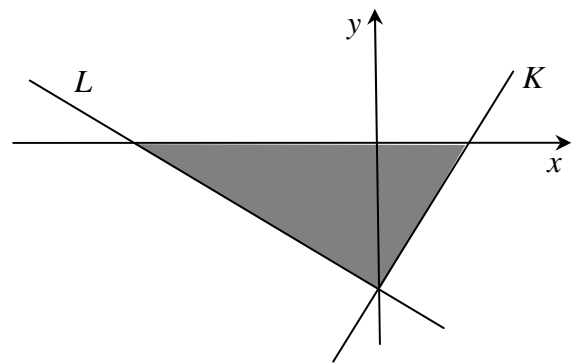
(b) Due to a transport disruption, a bus company is contracted at short notice to carry up to
1500 passengers to complete their journey. Passengers not carried by this company will
be carried by a taxi company.

The bus company has available standard buses and mini-buses. Each standard bus
carries 60 passengers and each mini-bus carries 30 passengers.

Each bus is operated by one driver and there are at most 30 drivers available.

    (i) Taking x as the number of standard buses and y as the number of mini-buses,
    write down two inequalities in x and y and illustrate them on graph paper.

    (ii) The operating profit for the journey is €80 for a standard bus and €50 for a mini-
    bus. How many of each type of bus should be used in order to maximise the
    profit?

    (iii) If the bus company paid each driver a bonus for working at short notice, the
    operating profit for each bus would be reduced by €30. By how much would this
    decrease the maximum profit available to the company?