



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2004

MATHEMATICS — ORDINARY LEVEL

PAPER 2 (300 marks)

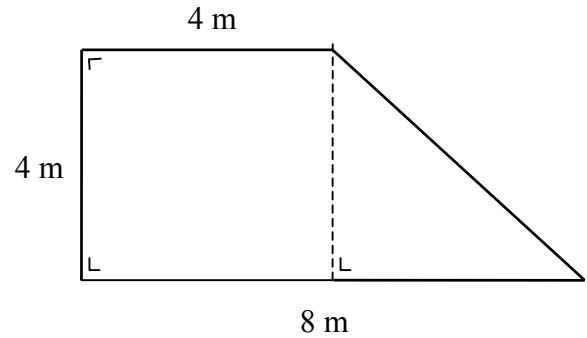
MONDAY, 14 JUNE — MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

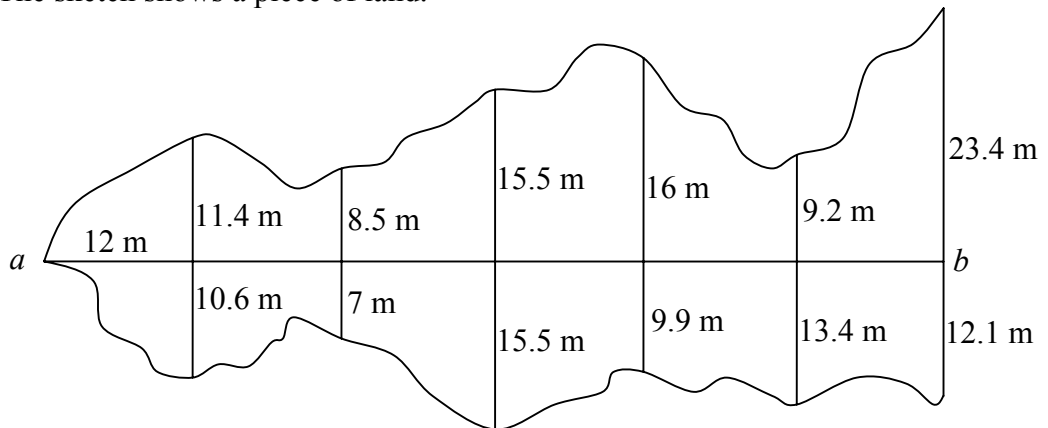
WARNING: Marks will be lost if all necessary work is not clearly shown.

SECTION A
Attempt FIVE questions from this section.

1. (a) Calculate the area of the figure in the diagram.



- (b) The sketch shows a piece of land.

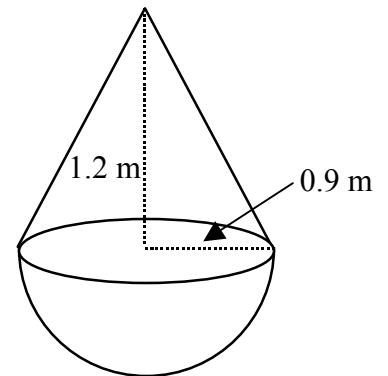


At equal intervals of 12 m along $[ab]$, perpendicular measurements are made to the boundary, as shown on the sketch.

Use Simpson's Rule to estimate the area of the piece of land.

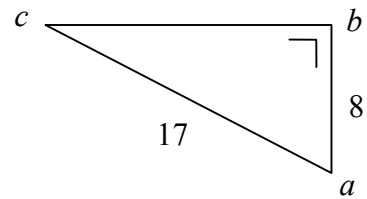
- (c) A buoy at sea is in the shape of a hemisphere with a cone on top, as in the diagram. The radius of the base of the cone is 0.9 m and its vertical height is 1.2 m.

- (i) Find the vertical height of the buoy.
(ii) Find the volume of the buoy, in terms of π .
(iii) When the buoy floats, 0.8 m of its height is above water. Find, in terms of π , the volume of that part of the buoy that is above the water.

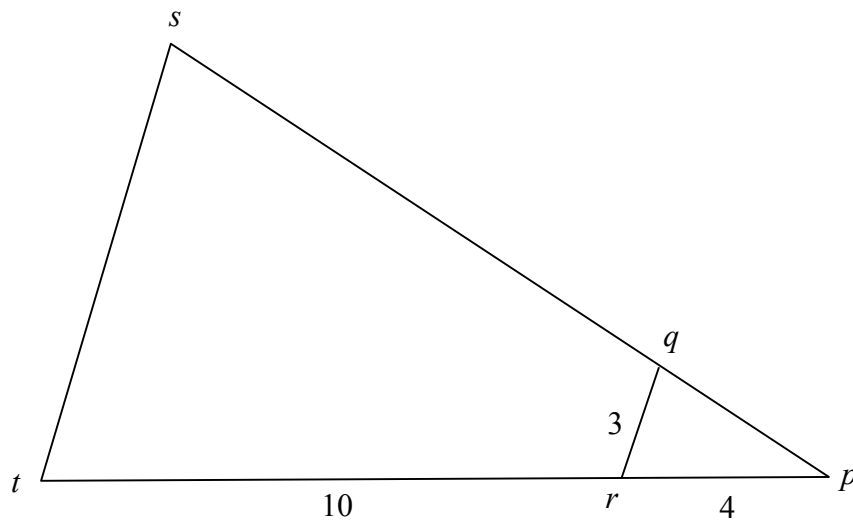


2. (a) $p(5, -8)$ and $q(11, 10)$ are two points.
Find the co-ordinates of the midpoint of $[pq]$.
- (b) $a(-1, -2)$, $b(3, 1)$, $c(0, 4)$ are three points.
- (i) Find the length of $[ab]$.
- (ii) Calculate the area of the triangle abc .
- (iii) The line L is parallel to ab and passes through the point c .
Find the equation of L .
- (iv) Show that the point $d(-4, 1)$ is on L .
- (v) Investigate whether $abcd$ is a parallelogram.
3. (a) The circle C has equation $x^2 + y^2 = 36$.
- (i) Write down the radius of C .
- (ii) The radius of another circle is twice the radius of C .
The centre of this circle is $(0, 0)$. Write down its equation.
- (b) A circle has equation $x^2 + y^2 = 13$.
The points $a(2, -3)$, $b(-2, 3)$ and $c(3, 2)$ are on the circle.
- (i) Verify that $[ab]$ is a diameter of the circle.
- (ii) Verify that $\angle acb$ is a right angle.
- (c) K is a circle with centre $(-2, 1)$. It passes through the point $(-3, 4)$.
- (i) Find the equation of K .
- (ii) The point $(t, 2t)$ is on the circle K .
Find the two possible values of t .

4. (a) In the triangle abc ,
 $|ab| = 8$, $|ac| = 17$ and $|\angle abc| = 90^\circ$.
 Find $|bc|$.



- (b) Prove that the opposite sides of a parallelogram have equal lengths.
- (c) The triangle pst is the image of the triangle pqr under an enlargement with centre p .

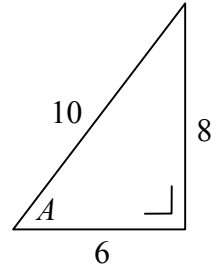


$|pr| = 4$, $|rt| = 10$ and $|qr| = 3$.

- (i) Find the scale factor of the enlargement.
- (ii) Find $|st|$.
- (iii) The area of the triangle pqr is 5 square units.
 Find the area of the quadrilateral $qstr$.

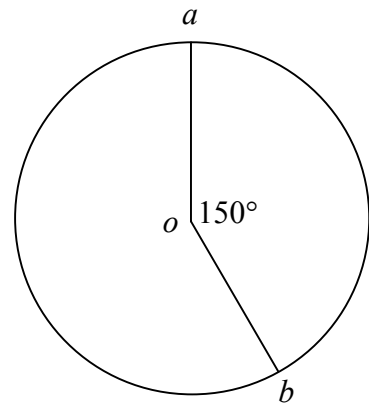
5. (a) The lengths of the sides of a right-angled triangle are shown in the diagram and A is the angle indicated.

- (i) Write down the value of $\cos A$.
- (ii) Hence, find the angle A , correct to the nearest degree.



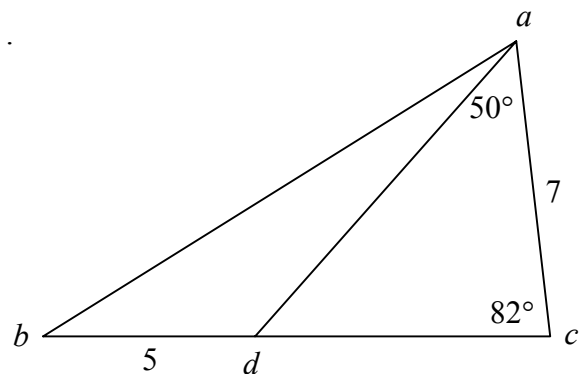
- (b) A circle has centre o and radius 4 cm. a and b are two points on the circle and $|\angle aob| = 150^\circ$.

- (i) Find the area of the circle, correct to the nearest cm^2 .
- (ii) Find the area of the sector aob , correct to the nearest cm^2 .
- (iii) Find the length of the shorter arc ab , correct to the nearest cm.



- (c) In the triangle abc , d is a point on $[bc]$.
 $|bd| = 5$ cm, $|ac| = 7$ cm,
 $|\angle dca| = 82^\circ$ and $|\angle cad| = 50^\circ$.

- (i) Find $|dc|$, correct to the nearest cm.
- (ii) Find $|ab|$, correct to the nearest cm.



6. (a) The letters of the word CUSTOMER are arranged at random.
- (i) How many different arrangements are possible?
 - (ii) How many of these arrangements begin with the letter C?
- (b) A committee of 3 people is selected from a group of 15 doctors and 12 dentists.
- In how many different ways can the 3 people be selected
- (i) if there are no restrictions
 - (ii) if the selection must contain exactly 2 doctors
 - (iii) if the selection must contain at least 1 doctor and at least 1 dentist
 - (iv) if the selection must contain one specific doctor and one specific dentist?
- (c) Four cards, numbered 2, 3, 4, 5 respectively, are shuffled and then placed in a row with the numbers visible.
- Find the probability that
- (i) the numbers shown are in the order: 5, 4, 3, 2
 - (ii) the first and second numbers are both even
 - (iii) the sum of the two middle numbers is 7.

7. (a) The mean of the set of numbers $\{1, 3, 7, 9\}$ is 5.
Find the standard deviation, correct to one decimal place.

- (b) The following table shows the time in minutes spent by customers in a cafeteria.

| | | | | |
|---------------------|--------|---------|---------|---------|
| Time in minutes | 0 – 10 | 10 – 20 | 20 – 40 | 40 – 70 |
| Number of customers | 80 | 100 | 160 | 60 |

[Note that 10 – 20 means at least 10 but less than 20 minutes etc.]

- (i) Find the total number of customers.
- (ii) Draw a histogram to represent the data.
- (iii) By taking the data at the mid-interval values, calculate the mean number of minutes per customer.
- (iv) What is the greatest number of customers who could have spent more than 30 minutes in the cafeteria?
- (v) What is the least number of customers who could have spent more than 30 minutes in the cafeteria?

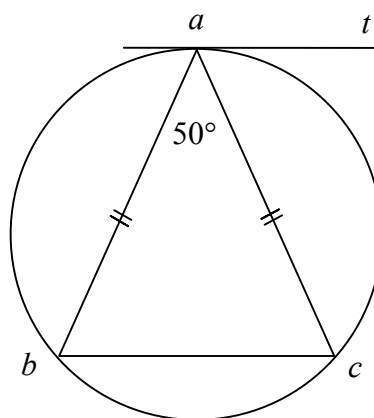
SECTION B
Attempt ONE question from this section.

8. (a) The points a, b and c lie on a circle.
 ta is a tangent to the circle.

$$|ab| = |ac| \text{ and } |\angle cab| = 50^\circ.$$

(i) Find $|\angle abc|$.

(ii) Find $|\angle tac|$.



- (b) Prove that if $[ab]$ and $[cd]$ are chords of a circle and the lines ab and cd meet at the point k , which is outside the circle, then $|ak| \cdot |kb| = |ck| \cdot |kd|$.

- (c) kt is a tangent to the circle, centre o .
 $[ab]$ is a diameter and $[cd]$ is a chord
of the circle.

ab and cd meet at the point k .

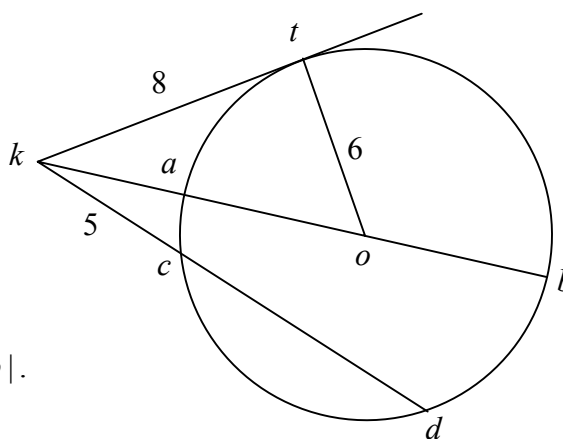
$$|kt| = 8, \quad |ot| = 6$$

$$\text{and } |kc| = 5.$$

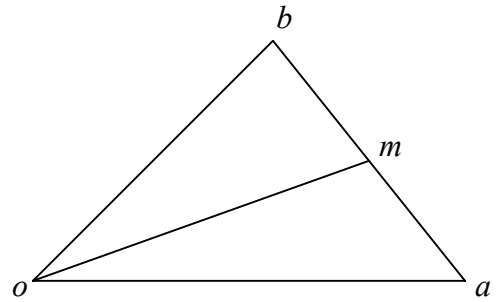
(i) Find $|ko|$.

(ii) Verify that $|kt|^2 = |ka| \cdot |kb|$.

(iii) Find $|cd|$.



9. (a) oab is a triangle.
 o is the origin and m is the midpoint of $[ab]$.



- (i) Express \vec{ba} in terms of \vec{a} and \vec{b} .
(ii) Express \vec{m} in terms of \vec{a} and \vec{b} .

- (b) Let $\vec{p} = 5\vec{i} + 2\vec{j}$ and $\vec{q} = 3\vec{i} - 6\vec{j}$.

- (i) Express $2\vec{p} - 3\vec{q}$ in terms of \vec{i} and \vec{j} .

- (ii) Find the scalars k and t such that

$$k(5\vec{i} + 2\vec{j}) + t(3\vec{i} - 6\vec{j}) = 7\vec{i} - 26\vec{j}.$$

- (c) Let $\vec{x} = 2\vec{i} + 3\vec{j}$ and $\vec{y} = 5\vec{i} + \vec{j}$.

- (i) Show that $|\vec{x} - \vec{y}| < |\vec{x}| + |\vec{y}|$.

- (ii) Write \vec{x}^\perp in terms of \vec{i} and \vec{j} .

Hence, calculate the dot product $\vec{y} \cdot (\vec{x} + \vec{x}^\perp)$.

10. (a) Expand $(1+x)^4$ fully.

- (b) The fourth term of an arithmetic sequence is -4 .
The seventh term of the sequence is -16 .

- (i) Find the common difference, d .

- (ii) Find the first term, a .

- (iii) Show that the difference between the fourth term and the twenty-ninth term is 100.

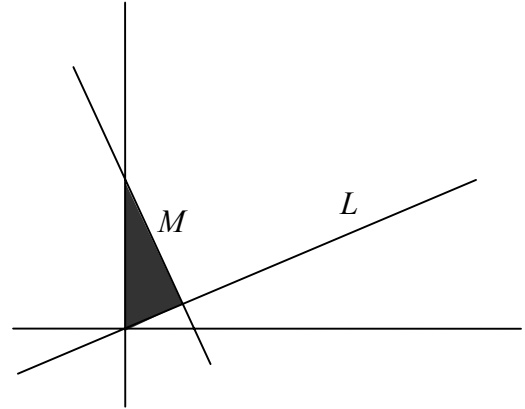
- (c) (i) The sum to infinity of a geometric series is 4.
The first term, a , is twice the common ratio, r .
Find r .

- (ii) €500 is invested at 7.5% per annum compound interest.
Show that after 10 years the value of the investment is greater than €1000.

11. (a) The equation of the line L is $x - 2y = 0$.

The equation of the line M is $2x + y = 4$.

Write down the three inequalities that together define the shaded region in the diagram.



(b) A shop-owner displays videos and DVDs in his shop.

Each video requires 720 cm^3 of display space and each DVD requires 360 cm^3 of display space. The available display space cannot exceed $108\,000 \text{ cm}^3$. The shop-owner buys each video for €6 and each DVD for €8. He does not wish to spend more than €1200.

(i) Taking x as the number of videos and y as the number of DVDs, write down two inequalities in x and y and illustrate these on graph paper.

During a DVD promotion the selling price of a video is €11 and of a DVD is €10. Assuming that the shop-owner can sell all the videos and DVDs,

(ii) how many of each type should he display in order to maximise his income?

(iii) how many of each type should he display in order to maximise his profit?

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