



**Coimisiún na Scrúduithe Stáit  
State Examinations Commission**

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**LEAVING CERTIFICATE EXAMINATION, 2003**

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**MATHEMATICS — ORDINARY LEVEL**

**PAPER 1 (300 marks)**

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**THURSDAY, 5 JUNE — MORNING, 9:30 to 12:00**

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Attempt **SIX QUESTIONS** (50 marks each).

**WARNING: Marks will be lost if all necessary work is not clearly shown.**

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1. (a) A train leaves Cork at 09:05 and arrives in Dublin at 12:25.  
The distance from Cork to Dublin is 250 km.  
Find the average speed of the train in km/h.
- (b) The present reading on the electricity meter in John's house is 63792 units. The previous reading was 62942 units.
- (i) How many units of electricity were used since the previous reading?
- (ii) What is the cost of the electricity used, if electricity costs 9.52 cent per unit?
- (iii) A standing charge of €7.00 is added and VAT is then charged on the full amount. If John's total bill is €98.91, calculate the rate at which VAT is charged.
- (c) (i) When using a calculator to add 1.7 and 2.2, a student strikes the multiplication key instead of the addition key.  
Calculate the percentage error in the result, correct to one decimal place.
- (ii) What sum of money invested at 6% per annum compound interest will amount to €5000 in 7 years?  
Give your answer correct to the nearest euro.

2. (a) Given that  $3x - 2y = 4$ , find the value of  $y$  when  $x = -2$ .

- (b) (i) Evaluate  $9^{\frac{1}{2}}$ .
- (ii) Express  $\sqrt{8}$  in the form  $2^k$ ,  $k \in \mathbf{Q}$ .
- (iii) Solve for  $x$  the equation  $25^x = 5^{6-x}$ .

- (c) Solve for  $x$  the equation

$$\frac{3}{x+1} + \frac{1}{x-1} = 1.$$

Give your answers in the form  $a \pm \sqrt{b}$ , where  $a, b \in \mathbf{N}$ .

3. (a) Find the solution set of

$$5x - 3 < 12, \quad x \in \mathbf{N}.$$

(b) (i) Show that  $x + 2$  is a factor of  $x^3 + 3x^2 - 4x - 12$ .

(ii) Hence, or otherwise, solve the equation  $x^3 + 3x^2 - 4x - 12 = 0$ .

(c) (i) Simplify  $(x + \sqrt{a-x})(x - \sqrt{a-x})$ , where  $a - x \geq 0$ .

(ii) Given that  $x = 3$  is a solution of the equation  $(x + \sqrt{a-x})(x - \sqrt{a-x}) = 0$ , find the value of  $a$ .

(iii) Hence, find the other solution of the equation in part (ii), and verify your answer.

4. (a) Given that  $i^2 = -1$ , find the value of:

(i)  $i^8$

(ii)  $i^7$ .

(b) Let  $z_1 = 2 + 3i$  and  $z_2 = 5 - i$ .

(i) Plot  $z_1$  and  $z_2$  and  $z_1 + z_2$  on an Argand diagram.

(ii) Investigate whether  $|z_1 + z_2| > |z_1 - z_2|$ .

(c) Let  $w = 1 + i$ .

(i) Simplify  $\frac{6}{w}$ .

(ii)  $a$  and  $b$  are real numbers such that

$$a\left(\frac{6}{w}\right) - b(w+1) = 3(w+i).$$

Find the value of  $a$  and the value of  $b$ .

5. (a) The first term of a geometric sequence is 4 and the common ratio is 1.5. Write down the next three terms of the sequence.
- (b) The first two terms of a geometric series are  $32 + 8 + \dots$
- (i) What is the value of  $r$ , the common ratio?
- (ii) Find an expression for  $S_n$ , the sum of the first  $n$  terms.
- (iii) Find  $S_{10}$ , the sum of the first ten terms.  
Give your answer correct to four decimal places.
- (c) The fifth term of an arithmetic series is 21 and the tenth term is 11.
- (i) Find the first term and the common difference.
- (ii) Find the sum of the first twenty terms.
- (iii) For what value of  $n > 0$  is the sum of the first  $n$  terms equal to 0?
6. (a) Let  $g(x) = \frac{2x}{3} - 1$ .  
Find the value of  $x$  for which  $g(x) = 5$ .
- (b) Differentiate  $x^2 - 2x$  with respect to  $x$  from first principles.
- (c) Let  $f(x) = 3 - 5x - 2x^2$ ,  $x \in \mathbf{R}$ .
- (i) Find  $f'(x)$ , the derivative of  $f(x)$ , and hence find the co-ordinates of the local maximum point of the curve  $y = f(x)$ .
- (ii) Solve the equation  $f(x) = 0$ .
- (iii) Use your answers from parts (i) and (ii) to sketch the graph of  $f : x \rightarrow 3 - 5x - 2x^2$ , showing scaled and labelled axes.

7. (a) Differentiate with respect to  $x$ :

(i)  $x^3$

(ii)  $\frac{x^2 - x^4}{2}$ .

(b) (i) Differentiate  $(3x^3 - 2x^2 + 2)^4$  with respect to  $x$ .

(ii) Given that  $y = (5x^2 + 3)(4 - x^2)$ , find  $\frac{dy}{dx}$  when  $x = 1$ .

(c) A train is travelling along a track. Suddenly, the brakes are applied. From the time the brakes are applied ( $t = 0$  seconds), the distance travelled by the train, in metres, is given by

$$s = 30t - \frac{1}{4}t^2.$$

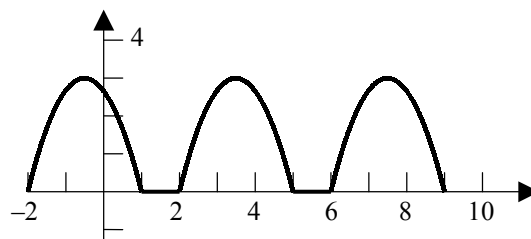
(i) What is speed of the train at the moment the brakes are applied?

(ii) How many seconds does it take for the train to come to rest?

(iii) How far does the train travel in that time?

8. (a) Part of the graph of a periodic function is shown.

Write down the period and range of the function.



(b) (i) The function  $g$  is defined for natural numbers by the rule:

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even.} \end{cases}$$

Find  $g(13) + g(14) + g(15)$ .

(ii) Given that  $h(x) = x^2$ , write down  $h(x + 3)$ .

Hence, find the value of  $x$  for which  $h(x) = h(x + 3)$ .

(c) Let  $f(x) = x^3 + 2x^2 - 1$ .

(i) Find  $f'(x)$ , the derivative of  $f(x)$ .

(ii)  $L$  is the tangent to the curve  $y = f(x)$  at  $x = \frac{-2}{3}$ .

Find the slope of  $L$ .

(iii) Find the two values of  $x$  at which the tangents to the curve  $y = f(x)$  are perpendicular to  $L$ .

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