Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2003*
Matamaitic	*Gnáthleibhéal*
Marking Scheme	*Leaving Certificate Examination, 2003*
Mathematics	*Ordinary Level*

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MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2003

MATHEMATICS

ORDINARY LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

7. The phrase “and stops” means that no more work is shown by the candidate

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 marks**	**Att 8**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 marks** **Att 3**

(a) A train leaves Cork at 09:05 and arrives in Dublin at 12:25.
The distance from Cork to Dublin is 250 km.
Find the average speed of the train in km/h.

(a) speed **10 marks** **Att 3**

I: 12:25 – 9:05 = 3:20 …3m Av. Speed = 250 ÷ 31/3 …7m = 75 ...10m	II: 12:25 – 9:05 = 200m …3m Av. Sp. = 250 ÷ 200 …7m = 1.25 km/min = 75 km/hr. ...10m	III: 745 – 545 = 200 m …3m Av. Sp. = 250 ÷ 200 …7m = 1.25 km/min = 75 km/hr. ...10m

* Allow 3.33 (or better) for 3⅓. If 3 is used instead of 3⅓, apply B1.

Blunders (–3)
B1 3 hr 20 min not 3⅓ hrs or 200 min, e.g. 250 ÷ 3.20 = 78.125, or 78, but 78.135 is B1 + S1 (6marks).
B2 Adds the times.
B3 Average speed ≠ Distance ÷ Time, or equivalent, e.g. 250 x time, or time ÷ 250.
B4 Decimal error (other than S2 below).

Misreadings (–1)
M1 Misreads a time

Slips (–1)
S1 Numerical error, other than those caused by 1 hour ≠ 60 mins.
S2 3⅓ taken as 3.3.

Attempts (3 marks)
A1 12:25 = 12hr 25 min; or 9:05 = 9 hr 5 mins; and stops.
A2 12:25 = 12(60) + 25 = 745 mins; or 9:05 = 9(60)+5 = 545 mins.
A3 Speed = Distance ÷Time, or "S = D / T", or any correct variation, and stops.

Worthless (0)
W1 12:25 = 12.25 and/or 9:05 = 9.05 and stops.

Part (b) 25 marks (10, 5, 10) Att 8 (3, 2, 3)

(b) The present reading on the electricity meter in John's house is 63792 units. The previous reading was 62942 units.

(i) How many units of electricity were used since the previous reading?

(ii) What is the cost of the electricity used, if electricity costs 9.52 cent per unit?

(iii) A standing charge of €7.00 is added and VAT is then charged on the full amount. If John's total bill is €98.91, calculate the rate at which VAT is charged.

(b) (i) Units 10 marks Att 3

63792 – 62942 …7m

= 850 …10m

* Allow correct answer without work (10m).

Blunders (–3)

B1 Adds instead of subtracts.

Misreading(–1)

M1 Misreads number, e.g. 69242.

Slips (–1)

S1 Numerical error in subtraction.

Worthless (0)

W1 Incorrect answer without work.

(b) (ii) Cost 5 marks Att 2

I:	850 × 9.52	…2m	II:	63792 × 9.52 – 62942 × 9.52	…2m
				= 607299.84 – 599207.48	
	= 8092 or 80.92	…5m		= 8092 or 80.92	…5m

* Accept candidate's total from (i) without penalty.

* Correct answer without work: 5 marks.

* If (b)(i) is not done, but 850 appears in (b)(ii), allow 5m for (b)(i).

* If (b)(i) is not done, but (b)(ii) is fully correct, with or without work, allow 5m + 5m.

Blunders(–3)

B1 Decimal error in currency, e.g. €8092, or €809.2, or 80.92 c.

B2 850 × 9 = 7650, but 7650 without work gets no marks.

Slips (–1)

S1 Numerical error in multiplication or subtraction.

Attempts (2 marks)

A1 "Cost = units × 9.52", and stops.

Worthless (0)

W1 Incorrect answer without work.

(b) (iii) Rate **10 marks** **Att 3**

I: 80.92 + 7 = 87.92	II: 98.91 – 7 = 91.91	III: 80.92 + 7 = 87.92	…3m
Vat = 98.91 – 87.92 = 10.99	91.91 – 80.92 = 10.99	$\frac{98.91}{87.92} = 1.125$	…7m
$R = \frac{10.99}{87.92} \text{ x } 100 = 12.5\%$	$R = \frac{10.99}{87.92} \text{ x } 100 = 12.5\%$	x100 = 112.5 - 100 = 12.5% 1.125 – 1 = .125 x 100 = 12.5%	..10m

* Allow candidate to use incorrect answer from (b)(ii), if penalised already.

* Method IIA: 98.91 – 80.92 first, then subtracts the 7. Use the same steps (3, 7, 10m).

Blunders (–3)

B1 Decimal error, e.g. 8092 in (b)(i) ≠ 80.92 in this part, or an equivalent error.
B2 7 not added in I or III, 7 not subtracted in II, or 80.92 not subtracted in IIA.See note 2.
B3 Incorrect relevant fraction × 100, e.g. (10.99 / 98.91) × 100, or (87.92 / 98.91) × 100, or (10.99 / 80.92) × 100, or (87.92 / 10.99) × 100.
B4 Not multiplied by 100.

Part (c) **15 marks (10, 5)** **Att 5 (3, 2)**

(c) **(i)** When using a calculator to add 1.7 and 2.2, a student strikes the multiplication key instead of the addition key.
Calculate the percentage error in the result, correct to one decimal place.

(ii) What sum of money invested at 6% per annum compound interest will amount to €5000 in 7 years?
Give your answer correct to the nearest euro.

(c)(i) % error **10 marks** **Att 3**

1.7 + 2.2 = 3.9 1.7 × 2.2 = 3.74	…3m	1.7 + 2.2 = 3.9 1.7 × 2.2 = 3.74	…3m
Diff = 0.16	…4m		
% error = $\frac{0.16}{3.9} \times 100$	…7m	=> $\frac{3.74}{3.9} \times 100$	…7m
= 4.1025641 = 4.1	..10m	= 95.897 and 100 – 95.897… = 4.103 = 4.1	..10m

Blunders (–3)

B1 (0.16 / 3.74) × 100 = 4.27.. = 4.3.
B2 (3.74 / 3.9) × 100 = 0.9589.. × 100 = 95.897 and stops.
B3 Not multiplied by 100.

Slips (–1)

S1 Rounding incorrect or not done.

Attempts (3 marks)

A1 Full line 1 or full line 2 and stops.

(c) (ii) P. 5 marks Att 2

I: $A = P(1.0R)^n$	II: .. $(1.06)^7 = 1.5036303$ …2m
$5000 = P(1.06)^7$ …2m	
$5000 / 1.5036303 = P$	$P = 5000 / 1.5036303$
3325. 2855… = P …4m	P = 3325. 2855 …4m
3325 = P ..5m	P = 3325 ..5m

III:	Principal (P)	…………of which…		Interest (I)	Prev. P = P – I	
	5000	now		283.01 88679		…2m
÷ 1.06 =	4716.98 1132	…1 year ago	… 2m	266.99 8932	4716.98 1132	
÷ 1.06 =	4449.98 22	…2 years ago		251.88 57849	4449.98 22	
÷ 1.06 =	4198.09 6415	…3 years ago		237.62 8099	4198.09 6415	
÷ 1.06 =	3960.46 8316	…4 years ago		224.17 74518	3960.46 8316	
÷ 1.06 =	3736.29 0864	…5 years ago		211.48 81621	3736.29 0864	
÷ 1.06 =	3524.80 2702	…6 years ago		199.51 71341	3524.80 2702	
÷ 1.06 =	3325.28 5568	…7 years ago	…4m	---	3325.28 5568	…4m
Answer	3325		..5m	Answer	3325	…5m

IV: $A = P(1.0R)^n$

Consider €100: $A = 100(1.06)^7$ …2m

A = 150.36303 and 5000 / 150.36303 = 33.2528

.: P = 33. 2528 x 100 = 3325.28... …4m

P = 3325 ...5m

* Correct answer without work: full marks (5m).

* In Method III, repeated multiplication by 100/106 or division by 1.06 gives previous P. This is more easily done on a calculator without recording the yearly sums.

* Each year's interest, found by P x 6/106 or P x 0.056603773, can be deducted from each year's Principal to achieve the same purpose (less efficiently). See RHS of Method III.

Blunders (–3)

B1 $(1.06)^7 \neq 1.5036303$ if not S1.

B2 5000 x 1.5036303. If continued to 7518, still 2m.

B3 Incorrect rate, ≠ 1.06.

B4 Incorrect time, n ≠ 7, e.g. in method III stops the process too soon or continues too long, e.g. 3525. If not rounded, S1 also applies (award 2m).

B5 Deducts 6% of *P* each year, e.g. 5000–300 = 4700; 4700–282 = 4418, etc. Or (0.94)×P. Results: 4700; 4418; 4152.92; 3903.74..; 3669.52..; 3449.34..; 3242.38.. Apply once.

Slips (–1)

S1 No rounding, or incorrect or premature rounding, e.g. $(1.06)^7 \neq 1.5036$ at least.

Attempts (2 marks)

A1 Compound interest formula and stops, e.g. $A = P(1 + r/100)^n$, or $A = P.R^n$ and stops.

A2 1.06 or $(1.06)^7$ anywhere.

A3 Calculates correctly 6% of any stated amount.

A4 Use of Simple Interest formula. (See W1).

A5 5000 = P x (relevant no., e.g. 6 or 7 or 42), or P = 5000 ÷ (relevant no. e.g. 6, 7 or 42).

Worthless (0)

W1 States Simple Interest formula and stops.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

(a) Given that $3x - 2y = 4$, find the value of y when $x = -2$.

(a) y value **10 marks** **Att 3**

3(–2) – 2y = 4	–2y = –3x + 4	–2y = –3x + 4	3x = 2y + 4	3x – 4 = 2y	3m
–6 – 2y = 4 –2y = 4+6 –2y = 10	–2y = –3(–2) + 4 –2y = 6 + 4 –2y = 10	$y = \frac{3x-4}{2}$ $y = \frac{3(-2)-4}{2}$ $y = \frac{-6-4}{2}$	$x = \frac{2y+4}{3}$ $-2 = \frac{2y+4}{3}$ –6 = 2y + 4 –10 = 2y	$\frac{3x-4}{2} = y$ $\frac{3(-2)-4}{2} = y$ $\frac{-6-4}{2} = y$	7m
y = –10/2	y = – 10/2	y = –10/2	–10/2 = y	– 10/2 = y	10m

* Correct answer without work: full marks.

Blunders (–3)

B1 Transposition or sign errors, each time.

B2 Last step not completed correctly, e.g. – y = 5, or –2y = 10 => y = – 20.

B3 Incorrect substitution, (if not M1). e.g. x = –2 => x + 2 => 2 subst.; or y = – 2 subst.

Misreadings (–1)

M1 Substitutes x = 2.

Slips (–1)

S1 Numerical error if signs not involved.

Attempts (3 marks)

A1 A correct substitution of x = –2, i.e. line 1 of method I.

A2 A correct transposition, i.e. line 1 of methods II – V.

Part (b) Evaluate **20 marks (5, 5, 10)** **Att 7 (2, 2, 3)**

(b) **(i)** Evaluate $9^{\frac{1}{2}}$.

(ii) Express $\sqrt{8}$ in the form 2^k, $k \in \mathbf{Q}$.

(iii) Solve for x the equation $25^x = 5^{6-x}$.

(b) (i) **5 marks** **Att 2**

$\pm\sqrt{9}$ … 2m $= \pm 3$ …5m	$9^{1/2} = \pm 3$

- * Correct answer without work: full marks.
- * Allow full marks for $\sqrt{9} = 3$ or $9^{1/2} = 3$.
- * Answer 3^2, without work: treat as $9 = 3^2$ and award 2 marks.

Blunders (–3)

B1 $9^{1/2}$ = the square root of an incorrect number, e.g. $9^{1/2} = \sqrt{3}$.

B2 $\sqrt{9}$ incorrectly evaluated, e.g. $\sqrt{9} = 81$, or $\sqrt{9} = 4.5$.

B3 Index error, e.g. $9^{1/2} = (3^3)^{1/2} = 3^{3/2}$.

Attempts (2 marks)

A1 $9^{1/2} = \sqrt{9}$ and stops.

A2 Recognises 3 or –3 as the base, e.g. 3^2 on its own, or 3^3 on its own , or –3 on its own.

Worthless (0)

W1 Other incorrect answer without work, e.g. $9^{1/2} = 4.5$.

(b) (ii) 2^k **5 marks** **Att 2**

$\sqrt{2^3} = (2^3)^{1/2}$	$8^{1/2} = (2^3)^{1/2}$	$\sqrt{4}.\sqrt{2} = 2.\sqrt{2} = 2.2^{1/2}$	$\sqrt{2}.\sqrt{2}.\sqrt{2} = (2^{1/2})^3$	...2m
$= 2^{3/2}$	$= 2^{3/2}$	$= 2^{3/2}$	$= 2^{3/2}$	...5m

- * Correct answer without work: 5 marks. Incorrect answer without work: no marks (e.g. 2^8).
- * Trial and Error using $\sqrt{8} = 2.82$ with a calculator to get k = 1.5: full marks. See W1, W2.

Blunders (–3)

B1 Index error, e.g. $(2^3)^{1/2} = 2^{3.5}$

Attempts (2 marks)

A1 Any correct relevant statement,

e.g. $\sqrt{8} = 8^{1/2}$; or $\sqrt{8} = \sqrt{2^3}$ or $\sqrt{4(2)}$ or $\sqrt{4}.\sqrt{2}$ or $\sqrt{2.2.2}$; or $8 = 2.2.2$ or 2^3.

or any mention of 3 as a power of 2, e.g. 2^3, or 2^{-3} or $2^{1/3}$, etc.

Worthless (0)

W1 $\sqrt{8} = 2.82$ and stops.

W2 Trial and error, using 2.82 without success.

(b) (iii) Index eqn. 10 marks Att 3

I: $(5^2)^x = 5^{6-x}$		II: $25^x = (25^{1/2})^{6-x}$	
$5^{2x} = 5^{6-x}$	…3m	$25^x = 25^{3-(1/2).x}$	…3m
$2x = 6 - x$	…7m	$x = 3 - (1/2)x$	…7m
		$2x = 6 - x$	
$3x = 6$		$3x = 6$	
$x = 2$	..10m	$x = 2$	..10m

- * Correct answer without work: full marks. Incorrect answer without work: 0 marks.
- * Correct answer by trial and error: full marks; but if incorrect/inconclusive, 0 marks, but see if A1 applies, e.g. $25^1 = 25$.

Blunders (–3)

B1 Each index error.
B2 Incorrect index equation formed.
B3 Transposition error.

Slips (–1)

S1 Numerical error, apart from index blunders and other blunders.

Attempts (3 marks)

A1 Any correct relevant index statement, e.g. $25 = 5^2$, or $5 = 25^{1/2}$, or line 1 of I or II..

Worthless (0)

W1 $25^x = 5^{6-x} \Rightarrow 5^x = 6 - x$; *or* $25x = 30 - 5x$.

Part (c) 20 marks (10, 10) Att 6 (3, 3)

(c) Solve for *x* the equation

$$\frac{3}{x+1} + \frac{1}{x-1} = 1.$$

Give your answers in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

(c) '(i)' Equation 20 (10, 10) marks Att 6 (3, 3)

$\frac{(x+1)(x-1).3}{x+1} + \frac{(x+1)(x-1).1}{x-1} = (x+1)(x-1)$	$\frac{3(x-1)+1(x+1)}{(x+1)(x-1)} = 1$	…3m
$(x-1).3 + (x+1).1 = (x+1)(x-1)$	$3(x-1) + 1(x-1) = (x+1)(x-1)$	…4m
$3x - 3 + x + 1 = x^2 - 1$	$3x - 3 + x + 1 = x^2 - 1$	…7m
$x^2 - 4x + 1 = 0$ *or* $x^2 - 4x = -1$	$x^2 - 4x + 1 = 0$ *or* $x^2 + 1 = 4x$	..10m

- * No additional marks from point where equation becomes linear, apart from A1 in (c)(ii).
- * Apply two blunders for $\frac{4}{(x+1)(x-1)} = 1 \Rightarrow x^2 - 1 = 4 \Rightarrow x^2 - 5 = 0$.
- * Apply two blunders in Method I if RHS is not multiplied by Common Denominator (CD).

Blunders (–3)

B1 Incorrect CD <u>or</u> incorrect use of correct CD; <u>or</u> incorrect multiplier used, each time. Apply at most two blunders.
B2 Cross-multiplication error.
B3 Error multiplying out of brackets. Apply at most two blunders.
B4 Cancellation error. Apply at most two blunders.

Attempts (3 marks)

A1 Correct CD and stops.
A2 Any correct transposition.
A3 Adds L.H.S. correctly and stops.
A4 Remainder theorem tried.
A5 3(x – 1) or 3x – 3 and stops.

(c) '(ii)' Solving Quadratic 10 marks Att 3

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ …3m

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$ …7m => $x = \dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2} = \dfrac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$ ..10m

* If equation is not a quadratic (or higher) then zero marks in this section, apart from A1 below.
* If the quadratic generated in first part is of the form $x^2 + k = 0$ (i.e. oversimplified), then a max of Att 3 is attainable for solving correctly for one or two roots.
In such cases, the roots may be left in $\pm\sqrt{-k}$ form, e.g. $\pm\sqrt{-5}$ from $x^2 + 5 = 0$.
* For 2 ± 1.7, or 3.7 and 0.3, allow 10m in (c)'(ii)'.
* For an incorrect quadratic (e.g. $x^2 - 4x + 3 = 0$) correctly solved (x = 1 and 3), allow 10m in (c)'(ii)'.

Blunders (–3)

B1 Incorrect quadratic formula or error in use, to a max of 2. e.g (4 ± √12)/2 = 2 ± √12.
B2 Incorrect factor(s) used. Apply once. See A3
B3 Incorrect roots from factors used. Apply once.

Attempts (3 marks)

A1 Quadratic formula and stops.
A2 Incorrect quadratic formula, with a max of one error, and with some correct substitution, and stops.
A3 Tries to find factors, e.g. (x …?)(x …?).

Worthless (0)

W1 Solves linear equation.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) Inequality **10 marks** **Att 3**

(a) Find the solution set of

$$5x - 3 < 12 \quad , \qquad x \in \mathbf{N} .$$

(a) **10 marks** **Att 3**

I:			II:		
	$5x < 15$	…3m		$5x < 15$	…3m
	$x < 3$	…7m		$x < 3$	…7m
	$\{0, 1, 2\}$	..10m		$\{ x / x < 3,\ x \in N \}$	..10m

* 0,1,2 without work: allow full marks.
* Solution of x < 3 correctly graphed: allow full marks.
* Trial and error, giving correct answer: full marks (10m); otherwise, see B1 and A1.
* 0,1,2,3 without work, or 1,2 without work: treat these as B1.
* In method II, (rule method), notation must be full and proper.

Blunders (–3)

B1 Each incorrect or omitted value if Trial and Error method used.
B2 Transposition or sign error.
B3 Inequality sign error.
B4 Last step missing or incorrect, e.g. $x < 3 \Rightarrow \{1,2\}$ or, $x < 3 \Rightarrow \{0,1,2,3\}$.
B5 $n \notin N$, e.g. if graphing the solution.

Misreadings (–1)

M1 Each misreading which does not oversimplify, e.g. $5x – 3 \leq 12$, or $5x – 3 > 12$.

Attempts (3 marks)

A1 One step or part of a step or an effort at trial and error.
A2 Any correct transposition, e.g. $5x < 12 + 3$ and stops.
A3 $x – 3 < 12$ solved correctly.
A4 $5x – 3 = 12$ solved or some correct work towards its solution.

Worthless (0)

W1 Incorrect answer without work, other than those in note 4 above.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

(b)	**(i)**	Show that $x+2$ is a factor of $x^3+3x^2-4x-12$.
	(ii)	Hence, or otherwise, solve the equation $x^3+3x^2-4x-12=0$.

(b) (i) Factor **10 marks** **Att 3**

x = –2 or f(–2) ...3m = $(-2)^3 + 3(-2)^2 - 4(-2) - 12$ ...7m = –8 + 12 + 8 – 12 ...10m	II: $x^2+ x-6$ $x+2 \overline{)\, x^3+3x^2 -4x-12}$ ...set up 3m $\underline{x^3+2x^2}$ ...7m x^2-4x $\underline{x^2+2x}$ $-6x-12$ $\underline{-6x-12}$ ..10m

* Check (b)(ii) first:
For (b)(ii), method I, II, III as answer to both parts of (b): allow 20 marks.
i.e. (b)(i) not attempted, or no distinction between (b)(i) and (b)(ii).

In (b)(ii), method IV needs f(–2) worked out correctly for the full 20m.

* If (b)(i) and (b)(ii) done separately, then mark each part individually.

* If only (b)(i) done, or (b)(i) and (b)(ii) are indistinguishable, then:
x + 2 => f(2) = $2^3 + 3(2)^2 - 4(2) - 12 = 8 + 12 - 8 - 12$ merits 7m in (i) + 4m in (ii);
while f(-2) = $(-2)^3 + 3(-2)^2 - 4(-2) - 12 = -8 + 12 + 8 - 12$ gets 10m in (i) + att 3m in (ii).

Blunders (–3)

B1 Error in algebraic division, to a max. of 2 blunders.
e.g. error in mult., division, addition, subtraction (e.g. signs), cancellations etc.
B2 Math. error in indices or brackets, e.g. $(-2)^3 = -6$, or $3(-2)^2 = -36$. Once per line if consistent.
B3 Incorrect root from factor, e.g. x + 2 => f(2) or divides by x – 2.
B4 Missing or incomplete step.
[Step could assume previous step, e.g. Step 2 assumes Step 1 is fine].

Slips (–1)

S1 Arithmetical slips in method I (but not sign errors).

Attempts (3 marks)

A1 First line of any correct method and stops.
A2 Effort to use Remainder Theorem, e.g. finds f(1) and stops.
A3 Says x = – 2 is a root and stops.

Worthless (0)

W1 Substitutes (x + 2), i.e. f(x + 2) = $(x + 2)^3 + 3(x + 2)^2 - 4(x + 2) - 12$ and continues.

(b) (ii) Roots 10 marks Att 3

Method I $x = -2$ or $f(-2)$ anywhere in (b) ...3m $$\begin{array}{r\|l} & x^2 + x - 6 \\ \hline x+2 & x^3 + 3x^2 - 4x - 12 \\ & \underline{x^3 + 2x^2} \\ & \quad x^2 - 4x \\ & \quad \underline{x^2 + 2x} \\ & \qquad -6x - 12 \\ & \qquad \underline{-6x - 12} \end{array}$$ $x^2 + x - 6 = 0$ ...4m $(x+3)(x-2) = 0$ ...7m $x = -3, \; 2$ ..10m	Method II $(x + 2)(x^2 + ax - 6) = x^3 + 3x^2 - 4x - 12$ ...3m $x^3 + ax^2 - 6x + 2x^2 + 2ax - 12 =$ $x^3 + 3x^2 - 4x - 12$ $a + 2 = 3$ or $-6 + 2a = -4$ $a = 1$ or $a = 1$ $(x + 2)(x^2 + x - 6) = 0$ ...4m $(x + 2)(x + 3)(x - 2) = 0$ ...7m $x = -2, \; -3, \; 2$ ..10m
III: $\boxed{-2}$ $\begin{array}{r\|rrr} 1 & 3 & -4 & -12 \\ & -2 & -2 & 12 \\ \hline & 1 & -6 & 0 \end{array}$ ...3m $(x + 2)(x^2 + x - 6) = 0$ ...4m $(x + 2)(x + 3)(x - 2) = 0$ ...7m $x = -2, \; -3, \; 2$ ..10m	IV: $f(-3) = (-3)^3 + 3(-3)^2 - 4(-3) - 12$ ...3m f(-3) and f(2) interchangeable $= -27 + 27 + 12 - 12$ ...4m $f(2) = (2)^3 + 3(2)^2 - 4(2) - 12$ $= 8 + 12 - 8 - 12$ ...7m $x = -3$, 2 and -2 see last line of note 1 in (b)(i) ..10m

* If (b)(ii) method I, II or III, as above, is given as an answer to both parts of (b): award 10 + 10 marks. But if ‘$x = -2$’, ‘root – 2’ or ‘$f(-2)$’ is not mentioned anywhere in (b)(ii) method I, II or III, then apply B(–3) and award 10m + 7m.

* If (b)(i) is not done and (b)(ii) method IV is given as the answer to (b), it merits 0m for (b)(i) + 10m for (b)(ii). If $x = -2$ is not stated in method IV: 0m for (b)(i) + 7m for (b)(ii).

* Draws a graph of the cubic and gets 3 roots from the graph: allow 10 + 10 marks for roots -2, -3 and 2. Mark on slips and blunder out of 10 att 3 + 10 att 3.

* Roots -2, -3, 2 without work: allow full marks (10m) for (b)(ii).

* Apply the closest method if methods are mixed: e.g. $f(-3) = 0$, divides by $x + 3$, and solves.

Blunders (–3)

B1 Error in algebraic division, to a max. of 2 blunders (Mult., div., addn., subtr., signs,etc).
B2 Mathematical error in indices, e.g. $(-3)^3 = -9$, or $3(-3)^2 = 81$. Once per line if consistent.
B3 Incorrect factors. Apply once.
B4 Incorrect or no roots from factors, or incorrect sign as in $f(-3) = 0 \Rightarrow$ root = 3. Once.
B5 Missing or incomplete step.
B6 Quadratic formula error solving the quadratic generated.
Max of 2 blunders (e.g. incorrect formula, substitution or simplification).

Attempts (3 marks)

A1 First line of any method and stops.
A2 Correct quadratic formula and stops. (If applied to cubic coefficients, still only Att 3).
A3 Effort to use Remainder Theorem, e.g. finds f(1) and stops.
A4 Differentiates the cubic. (Newton Raphson method relevant).

Worthless (0)

W1 Not all three roots correct without work. Compare note 3 above.

Part (c) **20 marks (10, 5, 5)** **Att 7 (3, 2, 2)**

(c) **(i)** Simplify $(x+\sqrt{a-x})(x-\sqrt{a-x})$, where $a-x \geq 0$.

(ii) Given that $x = 3$ is a solution of the equation

$$(x+\sqrt{a-x})(x-\sqrt{a-x}) = 0,$$

find the value of *a*.

(iii) Hence, find the other solution of the equation in part (ii), and verify your answer.

(c) (i) Simplify **10 marks** **Att 3**

$(x + y)(x - y) = x^2 - y^2$ …3m	$x^2 - x\sqrt{a-x} +$ …3m
$= x^2 - (\sqrt{a-x})^2$ …7m	$x\sqrt{a-x} - (\sqrt{a-x})^2$ …7m
$= x^2 - (a - x)$ <u>*or*</u> $x^2 - a + x$ ..10m	$= x^2 - (a - x)$ <u>*or*</u> $x^2 - a + x$ ..10m

* Candidate may multiply in traditional manner (term upon term): apply method II.

* Correct answer without work: 10 marks.

* Incorrect answer without work: If $x^2 + (a - x)$ *or* $x^2 - a - x$ *or* $x^2 - (a - x)^2$:award 7m. These only.
Otherwise, att 3m if x^2 is in the answer. If *neither* x^2 *nor* a – x is included, award 0m.

* Given expr. = [x + (a –x)][x – (a – x)] and continues, apply two blunders. If stops, 0m.
Given expr. = [x^2 + (a –x)][x^2 – (a – x)] and continues, apply two blunders. If stops, 0m.

Blunders (–3)

B1 Algebraic error, e.g. signs, indices, factors, central terms. Max of two blunders per line. If error is consistent, apply one blunder.

B2 Missing last step.

Misreadings (–1)

M1 $(x - \sqrt{a-x})(x - \sqrt{a-x})$, or $(x + \sqrt{a-x})(x + \sqrt{a-x})$.

Attempts (3 marks)

A1 Correct but partial multiplication of brackets and stops.

A2 Step correct, or part of a step correct.

Worthless (0)

W1 Incorrect answers, not containing x^2 or (a–x), with no work. See note 3 above.

(c) (ii) *a* = ? 5 marks Att 2

$x^2 - (a - x) = 0$ …2m $9 - (a - 3) = 0$ $9 - a + 3 = 0$ $a = 12$ …5m	$(3 + \sqrt{a-3})(3 - \sqrt{a-3}) = 0$ …2m $9 - 3\sqrt{a-3} + 3\sqrt{a-3} - (\sqrt{a-3})^2 = 0$ $9 - (a - 3) = 0$ $9 - a + 3 = 0$ $a = 12$ …5m

* Correct answer without work should be verified for full marks. If not, apply attempt marks.

Blunders (–3)
B1 Algebraic error.

Misreadings (–1)
M1 Misreading, if not serious.

Attempts (2 marks)
A1 First line or partial substitution of x = 3.
A2 Correct but unverified answer without work. (See Note 1 above)

Worthless (0m)
W1 Incorrect answer without work.

(c) (iii) Equation 5 marks Att 2

$x^2 + x - 12 = 0$ … 2m $(x + 4)(x - 3) = 0$ $x = -4, \ 3$ …4m Verify new root: $(-4 + \sqrt{12+4})(-4 - \sqrt{12-4})$ $= (-4 + 4)(-4 - 4) = (0)(8) = 0$ …5m	$x^2 + x - 12 = 0$ … 2m $(x + 4)(x - 3) = 0$ $x = -4, \ 3$ …4m Verify new root: $= (-4)^2 + (-4) - 12$ $= 16 - 4 - 12.$ …5m

* Accept answer from (c)(ii) for ‘a’ without further penalty.
* If quadratic formula is used: correct formula (2m), solved correctly (4m), verified (5m)

Blunders (–3)
B1 Incorrect factors. Apply once.
B2 Incorrect roots from factors. Apply once if consistent.
B3 Missing step or incomplete step.
B4 Error in quadratic formula or its application. Max two blunders.

Slips (– 1)
S1 New root not verified.

Attempts (2 marks)
A1 First line of either method.

Worthless (0)
W1 Incorrect answers without work.

QUESTION 4

Part (a)	**10 marks**	**Att 4**
Part (b)	**25 marks**	**Att 9**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

(a) Given that $i^2 = -1$, find the value of:

(i) i^8

(ii) i^7.

(a)(i) i^8 **5 marks** **Att 2**

$i^8 = +1$ or 1 …5m	$(i^2)^4$ …2m $= (-1)^4$ 2m still $= 1$ …5m	$(i^2).(i^2).(i^2).(i^2)$ …2m $= (-1)(-1)(-1)(-1)$ 2m still $= 1$ …5m

(a)(ii) i^7 **5 marks** **Att 2**

$i^7 = -i$ or $-1i$ …5m	$(i^2)^3.i$ …2m $= (-1)^3.i$ 2m still $= -i$ …5m	$(i^2).(i^2).(i^2).i$ …2m $= (-1)(-1)(-1).i$ 2m still $= -i$ …5m

* The same scheme applies separately to (a)(i) and (a)(ii).

* Be careful to distinguish carefully between 1 and *i* in candidate's work.

* If method I is used, or method I is preceded by some preparatory work, mark as follows -- consider the answer to have two elements, namely,

first element: the sign,
second element: the number (excluding the sign).

- For both elements correct, full marks (5m).
- For only one element correct, B(-3) and award 2 marks.
- For no element correct, <u>no marks</u> *<u>unless</u>* there is some correct statement meriting an attempt mark (see A1) *<u>or unless</u>* A2 applies. If A1 does not give the attempt marks, A2 might because it covers a different aspect.

For example, without work:
$i^8 = 8$ merits 2m (sign right), $i^7 = -7$ merits 2m (sign right), but $i^7 = 7$ merits 0 marks (both elements incorrect, no work as in A1, and A2 does not apply.)

Blunders (–3)

B1 $i^2 \neq -1$ in method II or III..

B2 Sign error in method II or III.

Attempts (2 marks each part)

A1 Line 1 or line 2 of method II or III, or $i^8 = i.i.i.i.i.i.i.i.$, or $i^7 = i^4 . i^3$, or any other correct relevant statement.

A2 In (a)(i), the answer has no *i* in it e.g. answer – 8.
In (a)(ii), the answer has an *i* in it, e.g. 7*i*

Worthless (0)

W1 Incorrect answer without work, i.e. unless A1 or A2 above allow attempt to be awarded.

Part (b) 25 marks (15, 5, 5) Att 9 (5, 2, 2)

(b) Let $z_1 = 2 + 3i$ and $z_2 = 5 - i$.

(i) Plot z_1 and z_2 and $z_1 + z_2$ on an Argand diagram.

(ii) Investigate whether $|z_1 + z_2| > |z_1 - z_2|$.

(b) (i) Plotting 15 marks Att 5

Plot z_1 …5m

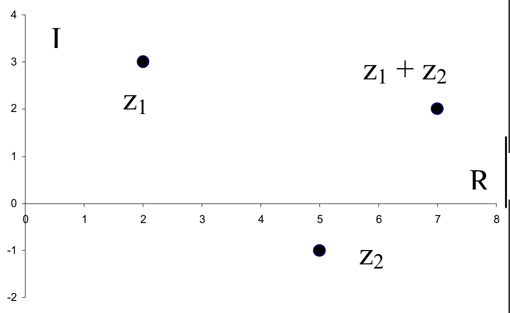
Plot z_2 ..10m

$\}$ Inter-changeable

$z_1 + z_2 = 2 + 3i + 5 - i$

$= 7 + 2i$ ..12m

Plot $z_1 + z_2$ …15m



- * If the axes are reversed they must be identified, or B1 applies
- * Unlabelled axes: assume horizontal axis is real, e.g.(3, 2) plotted on unlabelled axes is B1.
- * (3, 2) and (– 1, 5) plotted on unlabelled axes: apply B1 once i.e. penalise reversal of axes *once* in (b)(i) overall.
- * Axes drawn once (or twice), and then one or two points plotted incorrectly: award Att 5.
- * One unnamed point plotted: assume it is z_1

Blunders (–3)

B1 Incorrect or no plotting, e.g. plots 2 + 3*i* at 5*i*. Each time (if not penalising same error).

B2 Incorrect calculation of $z_1 + z_2$ by mixing up real and imaginary, or making sign error; Apply once in all of part (b). Note: treat 3*i*–*i* = 4*i* as slip in subtraction, not sign blunder.

Attempts (5 marks)

A1 A correct set of scaled axes (ticks sufficient).

A2 Any incorrect point plotted.

A3 Subst. for z_1 or z_2 in $z_1 + z_2$, and stops, e.g. 2+3*i*+z_2, <u>or</u> z_1+ 5–*i*, <u>or</u> 2+3*i*+ 5–*i*, or 7 + 2*i*.

(b) (ii) Mods 10 (5, 5) marks Att 4 (2, 2)

$z_1 + z_2 = 7 + 2i$ …2m

$|z_1 + z_2| = |7 + 2i| = \sqrt{49+4}$

$= \sqrt{53}$ …5m

$z_1 - z_2 = 2+3i - (5 - i)$ …2m

$= -3 + 4i$

$|z_1 - z_2| = \sqrt{9+16} = \sqrt{25}$ …4m

=> $|z_1 + z_2| > |z_1 - z_2|$ …5m

* Accept candidate's value for $z_1 + z_2$ from previous part.

* The two sections in (b)(ii) are interchangeable. Mark each part separately.

* May plot $z_1 - z_2$ and compare length with $z_1 + z_2$: allow full marks if conclusion, based on lengths, is stated; otherwise A5 applies.

* If not using $|z|^2 = z z$, thēn modulus confused with conjugate: att 2 + att 2 *max*. (See A4).

* Accept proof that $|z_1 + z_2|^2 > |z_1 - z_2|^2$, i.e. no square roots involved, 53 > 25.

* Accept correct Coordinate Geometry distance method; apply the usual slips and blunders.

* B1, B2, B3 and B4 apply once overall in (b)(ii) if consistent.

Blunders (–3) ... applying to each part of (b)(ii).

B1 Incorrect modulus formula. See notes above.

B2 Incorrect substitution into correct formula, e.g. $\sqrt{7^2 + 4i^2} = \sqrt{49-4} = \sqrt{45}$.

B3 Square root error, e.g. $\sqrt{49+4} = 7+2$.

B4 Adds real and imaginary parts.

B5 $|2 + 3i + 5 - i| = \sqrt{2^2 + 3^2 + 5^2 + 1^2 \text{ or } -1^2}$ or $4 + 9 + 25 + 1$.

B6 Sign error removing brackets, e.g. $-(5 - i) = -5 - i$.

B7 Modulus not found. Apply each time.

Slips (–1)

S1 Numerical slip, e.g. when adding real to real, or imaginary to imaginary.

S2 No final conclusion.

Attempts (2 marks for each part)

A1 Substitutes correctly for z_1 or z_2 in $z_1 + z_2$, or in $z_1 - z_2$, and stops.

A2 $\sqrt{a^2 + b^2}$ and stops, or coord. geometry distance formula correct and stops. Each time.

A3 $\sqrt{a^2 + b^2}$ *with* some correct substitution, or $a^2 + b^2$ *with* some correct substitution or distance formula with *one* error and *with* some correct substitution, and stops. Each time.

A4 $|z|^2 = z\bar{z}$ (or equivalent). Each time.

A5 Plots $z_1 - z_2$ and stops (no conclusion), i.e. Att 2m in section 2. (No marks in section 1).

Worthless (0)

W1 $\sqrt{a^2 - b^2}$ *without* substitution, or $a^2 + b^2$ *without* substitution.

W2 Other incorrect formula with/without substitution.

Part (c) 15 marks (5, 10) Att 5 (2, 3)

(c) Let $w = 1 + i$.

(i) Simplify $\frac{6}{w}$.

(ii) a and b are real numbers such that

$$a\left(\frac{6}{w}\right) - b(w+1) = 3(w+i).$$

Find the value of a and the value of b.

(c) (i) 6/w 5 marks Att 2

$\left(\frac{6}{1+i}\right)\left(\frac{1-i}{1-i}\right)$ …2m $= \frac{6-6i}{1-i^2}$ $= \frac{6-6i}{1+1}$ *or* $\frac{6-6i}{2}$. ..5m	$\left(\frac{6}{\text{w}}\right)\left(\frac{\overline{\text{w}}}{\overline{\text{w}}}\right)$ …2m $= \frac{6.\overline{\text{w}}}{\text{w}.\overline{\text{w}}}$ $= \frac{6-6i}{1+1}$ *or* $\frac{6-6i}{2}$. ..5m

* III: $6/(1+i) = a + ib$ (2m), cross mult, compare coeffs and solves a,b: mark on slip/blunder.

Blunders (–3)

B1 Incorrect conjugate.
B2 $i^2 \neq -1$. Apply once in (c)(i).
B3 Each omitted or incorrect term when multiplying out. Max of 2 (1 on num., 1 on denom.).
B4 Real and imaginary parts mixed up, e.g. when adding.
B5 Inverts in the last step, e.g. $2/(6 - 6i)$.
B6 Denominator not real after multiplication, or forgets to multiply denom. by conjugate.
B7 Multiplies out numerators and denominators and stops.

Slips (–1)

S1 Numerical slip when adding real to real, or imaginary to imaginary.

Misreadings (–1)

M1 $\text{w} = 1 - i$ used.

Attempts (2 marks)

A1 Substitutes for w and stops.
A2 Correct conjugate and stops.
A3 Any correct and relevant multiplication.

(c)(ii) a, b = ? **10 marks** **Att 3**

$a(3-3i) - b(1+i+1) = 3(1+i+i)$	…3m		
$\overset{R}{3a}\ \overset{I}{-3ai}\ \overset{R}{-2b}\ \overset{I}{-bi} = \overset{R}{3} + \overset{I}{6i}$	… 3m still		
$3a - 2b = 3$	…4m	….one equation correct	Order Inter-changeable
$-3a - b = 6$	…7m	…both equations	
$b = -3$	…9m	…one value correct	Order Inter-changeable
and $a = -1$	..10m	… both values correct	

- * Gets $3 - 3i = 1 - i$,or i: Allow 10 marks in (c)(i) but penalise with B(–3) if used in (c)(ii).
- * Correct values for *a* and *b* without work, but both tested: full marks.
- * Correct values for *a* and *b* without work, but both *not* tested: 0 marks.
- * May multiply across by *w* (att 3) and correctly solve resulting equation: mark using slips and blunders. See A4
- * (c)(i) may be done as part of (c)(ii): mark (c)(i) separately within (c)(ii).
- * May use answer from (c)(i), but if (c)(ii) oversimplified then Att 3m max in (c)(ii).

Blunders (–3)

B1 $(6 - 6i)/2 = 3 - 6i$ or $6 - 3i$, and continues; *or* $3 - 3i = 1 - i$, or $3 - 3i = i$, and continues.
B2 Sign errors, each time, e.g. $+bi$ in 2nd line, or when solving simultaneous equations.
B3 Error multiplying out brackets. Once, if consistent.
B4 Real and imaginary parts confused, when equating or adding, e.g. $R \neq R$, $I \neq I$.
B5 Error solving simultaneous equations (other than B2).

Slips (–1)

S1 Finds one value and stops.

Attempts (3 marks)

A1 Substitutes $1 + i$ for w and stops, or substitutes *answer (c)(i)* for $(6/w)$ and stops.
A2 Correct or partially correct multiplying out of brackets and stops, e.g. $6a/w$ and stops.
A3 Correct transposition and stops.
A4 Multiplies 2 or 3 terms by *w*, and stops, e.g. $a(6) - bw(w + 1) = 3(w + i)$ and stops.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

(a) The first term of a geometric sequence is 4 and the common ratio is 1.5.
Write down the next three terms of the sequence.

(a) T_2, T_3, T_4 **10 marks** **Att 3**

$Tn = ar^{n-1}$ $T_2 = 4(1.5) = 6$ ...3m $T_3 = 4(1.5)^2 = 4(2.25) = 9$ ...7m $T_4 = 4(1.5)^3 = 4(3.375) = 13.5$ ..10m	 $4(1.5) = 6$ ...3m $6(1.5) = 9$ ...7m $9(1.5) = 13.5$ ..10m

* No penalty for notation error, e.g. $Sn = ar^{n-1}$, i.e. ignore Sn.

* Correct terms, in correct order, with no work: full marks (10).

Blunders (–3)

B1 Incorrect squaring/cubing of 1.5, e.g. $(1.5)^2 = 3$. Once if consistent.
B2 Incorrect substitution into correct Tn formula. Once if consistent.
B3 Bracket or index error in Tn formula (but if both, see W1);
i.e. B(–3) for $Tn = (ar)^{n-1}$, $Tn = ar^n$ or $Tn = ar^{n+1}$
B4 $a \neq 4$, or $r \neq 1.5$. Each time except for B5.
B5 a and r interchanged.
B6 Each missing or incorrect term. [May write the *first,* instead of the *next,* three terms].
B7 Terms not evaluated, e.g. 4(1.5), $4(1.5)^2$, and $4(1.5)^3$ and stops. Apply once.

Attempts (3 marks)

A1 $a = 4$ or $r = 1.5$ or both, and stops.
A2 Second term = 6 and stops.
A3 Correct Tn of GP or correct Sn of GP formula and stops. [May use $S_n - S_{n-1}$ to find T_n.]
A4 Any correct ratio statement, e.g. $T_2 = T_1 \times r$, or $r = \frac{T_2}{T_1}$, *or* $r = \frac{\text{any term}}{\text{previous term}}$.
A5 4 + 1.5 = 5.5, 5.5 + 1.5 = 7, etc. (Reason: use of $a = 4$ implied).

Worthless (0)

W1 All terms incorrect with no work, e.g. 5.5, 7, 8.5 without work.
W2 An AP formula with no substitution and stops;
but if $a = 4$ or $r = 1.5$ is stated/used in applied AP formula, apply A1 (3 marks).
W3 4 followed by three incorrect numbers, without work.

Part (b) **20 marks (5, 5, 10)** **Att 7 (2, 2, 3)**

(b) The first two terms of a geometric series are $32 + 8 + \dots$

(i) What is the value of r, the common ratio?

(ii) Find an expression for S_n, the sum of the first n terms.

(iii) Find S_{10}, the sum of the first ten terms.
Give your answer correct to four decimal places.

(b) (i) r = ? **5 marks** **Att 2**

$r = T_2 \div T_1$ …2m

$r = 8 / 32$ …5m

* Correct answer without work: full marks. Incorrect answer without work, except B1: 0m.

Blunders (–3)
B1 $r = 4$ or 32 / 8. (Inverted).

Slips (–1)
S1 States the correct ratio but gives answer 32 / 8.

Attempts (2 marks)
A1 Correct Tn of GP formula and stops, or a correct ratio formula and stops.
A2 $a = 32$ or $ar = 8$ (or both) and stops.

Worthless (0)
W1 Formula for AP and stops, (but $a = 32$ mentioned merits attempt marks as in A2 above)
W2 $r = 8$ without work.
W3 $r = -24$ without work, or $r = T_2 - T_1$ and stops;
but for $r = 8 - 32 = -24$ or $r = 32 - 8 = 24$ allow att 2 (as $a = 32$ implied in both cases).

(b) (ii) Sn **5 marks** **Att 2**

$\text{Sn} = \dfrac{a(1-r^n)}{1-r}$	...2m	$\text{Sn} = \dfrac{a(r^n-1)}{r-1}$	...2m
$= \dfrac{32.[1-(0.25)^n]}{1-0.25}$	...5m	$= \dfrac{32.[(0.25)^n-1]}{0.25-1}$	...5m

Blunders (–3)
B1 $\text{Sn} = \frac{a(r^n+1)}{r+1}$ *used*; or $\text{Sn} = \frac{a(1+r^n)}{1+r}$ *used*; or $\text{Sn} = \frac{a(1-r)^n}{1-r}$ used; or $\text{Sn} = \frac{a(1-r)^n}{1-r}$ *used.*
These only. Others are worthless.

B2 Incorrect substitution into correct Sn of GP formula, including a and r interchanged.

Attempts (2 marks)

A1 Mention of $a = 32$ or $r = 0.25$ or candidate's value of r from (b)(i).

A2 Correct Sn of GP formula and stops, or $Sn = a + ar + ar^2 + \ldots$ and stops.

A3 $Sn = \frac{a(r^n + 1)}{r + 1}$, or $Sn = \frac{a(1 + r^n)}{1 + r}$, or $Sn = \frac{a(1 - r)^n}{1 - r}$, or $Sn = \frac{a(1 - r)^n}{1 - r}$.
These incorrect formulae (only) and *with some relevant substitution*; others worthless.

Worthless (0)

W1 Incorrect answers without work.

W2 An AP formula with no substitution; but a = 32 mentioned or subs., merits att marks.(A1)

(b) (iii) S_{10} 10 marks Att 3

$Sn = \frac{a(1 - r^n)}{1 - r}$ *or* $\frac{32.[1 - (0.25)^{10}]}{0.75}$ *or* Ans(b)(ii) subst. with n = 10 …3m	32 + 8 + 2 + …3m
$= \frac{32.[1 - 9.5367\text{x}10^{-7}]}{0.75}$ *or* $\frac{32.[1 - 0.000000953..]}{0.75}$ …7m	0.5 + 0.125 + 0.03125 + 0.0078125 + 0.001953125 + 0.00048828125 + 0.0001220703125 …7m
= 42.66662598 ..10m	= 42.6666259 ..10m

* No penalty for "calculator notation", e.g. 9.5367^{-07}, if subsequent blunder avoided.

* No need to round off in this case.

Blunders (–3)

B1 Decimal error, or error in fractions, in calculating each term, to a max of 2.

B2 Error in indices.

B3 Missing or incorrect term, if adding terms separately, to a max of 2.

Attempts (3 marks)

A1 Finds T_{10} and stops.

A2 S10 = 32 + 8 + 2 + … and stops. (First line of method II).

A3 Incorrect relevant S_n formula (see A3 of part (ii)) *with n = 10 substituted*, and stops.

A4 $S_{10} = T_1 + T_2 + T_3 + \ldots + T_{10}$, and stops.

A5 Correct Sn formula and stops.

Worthless (0)

W1 AP formula, but a = 32 mentioned merits Att 3 marks.

Part (c) **20 marks (10, 5, 5)** **Att 7 (3, 2, 2)**

> **(c)** The fifth term of an arithmetic series is 21 and the tenth term is 11.
> **(i)** Find the first term and the common difference.
> **(ii)** Find the sum of the first twenty terms.
> **(iii)** For what value of *n* > 0 is the sum of the first *n* terms equal to 0?

(c)(i) ***a, d* = ?** **10 marks** **Att 3**

$T_5 = a + 4d = 21$	…one equation …3m	Inter-	Gap = 10 or – 10 …3m
$T_{10} = \underline{a + 9d = 11}$	… second equation …4m	changeable	
$-5d = 10$			with nos. going down … 4m
$d = -2$	…one value found …7m	Inter-	.: d = – 2 …7m
.: $a + 4(-2) = 21 \Rightarrow a - 8 = 21$		changeable	Counting upward…
$\Rightarrow a = 29$	….2nd value found ..10m		=> a = 29 ..10m

- * Correct *a* and *d* without work: full marks. Correct *a* or *d* without work: 7m.
- * Incorrect Tn of AP formula used: 0 marks apart from exceptions in B1 and B2.
- * 31+29+27+… and stops. Apply 2 blunders (*a* incorrect, *d* not stated).
- * Accept plus signs, commas, or spaces between terms.

Blunders(–3)

B1 Tn = a + nd used.
B2 Tn = a + (n+1)d *used*, Tn = a – (n–1)d *used*, or Tn = a – (n+1)d *used*; <u>others 0 marks.</u>
B3 Transposition error when solving equations.
B4 Sign error when solving equations, or d = + 2 in method II.
B5 Only one correct value found, *a* or *d.*
B6 29, 27, 25, 23, 21, 19, 17, 15, 13, 11, without work, and stops; or 29+27+…and stops. i.e. award 7 marks.

Attempts (3 marks)

A1 Correct formula for Tn or Sn of AP, or T_5 or T_{10}, and stops, e.g. $T_5 = a + 4d$ and stops.
A2 a + 4d = 21, or a + 9d = 11, and stops; or correct Sn of AP formula and stops.
A3 21 – 11 = 10 or – 10; or 11 – 21 = – 10 or 10; or d = 2, and stops.

Worthless (0)

W1 GP formula.

(c)(ii) **S_{20}** **5 marks** **Att 2**

I: $S_n = (n/2)\{2a + (n-1)d\}$ …2m	II: $S_n = (n/2)\{a + T_n\}$ …2m
$S_{20} = (20/2)\{58 + 19(-2)\}$ … still 2m	$S_{20} = (20/2)\{29 + T_{20}\}$ …still 2m
	$T_{20} = a + 19d = 29 + 19(-2) = -9$
$= 200$ … 5m	$S_{20} = (20/2)\{29 - 9\} = 200$ …5m
III: 29 + 27 + 25 + 23 + 21 + 19 + 17 + 15 + 13 + 11 + 9 + 7 + 5 + 3 + 1 –1 – 3 – 5 – 7 – 9 …2m = 200 …5m	

* Candidate may use values of a and d found in (c)(i). Correct answer without work: 5m.

Blunders (–3)

B1 *Incorrect relevant* Sn of AP formula *used*. *Incorrect relevant* = one error only. See W2

B2 Incorrect substitution of *a, n* or *d* into correct formula, or *a* and *d* swapped. Apply once.

B3 Bracket error in simplifying.

B4 Error or omission in list (excluding knock-on errors).

Attempts (2 marks)

A1 Correct formula and stops, either Tn or Sn of AP.

A2 $T_2 = \underline{27}$ and stops, or $T_{20} = -9$ and stops.

A3 $S_{20} = 29 + \underline{27} + \ldots$ and stops.

Worthless (0)

W1 Incorrect answer without work.

W2 Incorrect Sn formula for AP with more than one error, and stops.

W3 Formula for GP and stops, but if *a* = 29 and/or '*r*' = –2 mentioned, then award att 2m.

(c)(iii) Sn = 0 **5 marks** **Att 2**

Sn = (n/2){2a + (n–1)d} = 0 … 2m

= (n/2){58 – 2(n–1)} = 0 …2m still

= (n/2){60 – 2n} = 0 => n{30 – n} = 0

=> n = 0 or 30. Ans 30. …5m

* Correct answer without work: full marks (5m). (Overlook the inclusion of n = 0).

Blunders (–3)

B1 Incorrect substitution of *a, n* or *d* into correct Sn formula.

B2 Error in factors in solving equation. Apply once.

B3 Transposition error when solving equation.

B4 Incorrect or no roots from factors.

B5 Error in quadratic formula, or its application.

B6 Missing or incorrect term if adding terms separately.

Attempts (2 marks)

A1 Correct Sn formula for AP and stops, or correct substitution of *a* and *d* into an incorrect but relevant Sn of AP formula.

A2 Unsuccessful trial and error for Sn.

Worthless (0)

W1 *n* = 0 without work, or incorrect *n* without work in this part.

W2 Formula for GP and stops, but if *a* = 29 and/or '*r* ' = –2 mentioned award Att 2m.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

(a) Let $g(x) = \frac{2x}{3} - 1$.

Find the value of x for which $g(x) = 5$.

(a) g(x) = 5 **10 marks** **Att 3**

$\frac{2x}{3} - 1 = 5$ ...7m $\frac{2x}{3} = 5+1$ (or6) $2x = 18$ $x = 9$ ..10m	$\frac{2x}{3} - 1 = 5$ ...7m $2x - 3 = 15$ $2x = 18$ $x = 9$ ..10m

* Correct answer without work: full marks (10)

* Successful trial and error if g(9) = 5 found: full marks (10).

Blunders (–3)

B1 Transposing error, each time.
B2 Failure to multiply the '–1' and/or the '5' by 3, in method II.
B3 Sign error, each time.
B4 (10/3) – 1 = 2 ⅓, or 7/3 , i.e. g(5) found, 7m. But for (10/3) – 1 and stops: award 4m.

Misreadings (–1)

M1 Solves (2x)/3 – 1 = 0.

Attempts (3 marks)

A1 First line of method I or II.
A2 Unsuccessful trial and error, e.g. g(1) = 2/3 – 1, or g(0) found, apart from g(5). See B4.

Worthless (0)

W1 5.[$\frac{2x}{3}$ – 1] and continues.
W2 Differentiates and stops/continues.

Part (b) **20 marks** **Att 7**

(b) Differentiate $x^2 - 2x$ with respect to x from first principles.

(b) 1^{st} principles **20 marks** **Att 7**

$y + \Delta y = (x + \Delta x)^2 - 2(x + \Delta x)$	7m	$f(x + h) = (x + h)^2 - 2(x + h)$
$y + \Delta y = x^2 + 2x\Delta x + \Delta x^2 - 2x - 2\Delta x$ $\underline{y = x^2 - 2x}$	@ 11m	$f(x + h) = x^2 + 2xh + h^2 - 2x - 2h$ $f(x) = x^2 - 2x$
$\Delta y = 2x\Delta x + \Delta x^2 - 2\Delta x$	@ 14m	$f(x + h) - f(x) = 2xh + h^2 - 2h$
$\frac{\Delta y}{\Delta x} = 2x + \Delta x - 2$	@ 17m	$\frac{f(x+h)-f(x)}{h} = 2x + h - 2$
$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x - 2$	@ 20m	$\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h} = 2x - 2$

* Overlook $\Delta x = 0$ or $h = 0$ in limit; and use of dy/dx instead of lim $\Delta y/\Delta x$.

* If first mention of LHS is in last line, then B4 and B5 apply; i.e. 14m for RHS correct.

* Perfectly correct RHS but _no_ LHS => B4 + B5 + B6 apply.

* After substit. and further work, B(–3) for each major step omitted; see steps @ above.

Blunders (–3)

B1 Error multiplying out $(x + \Delta x)^2$ or $(x + h)^2$. Apply once.

B2 Error multiplying out $-2(x + \Delta x)$, e.g. gets $-2x + 2\Delta x$. Apply once.

B3 $(\Delta x)^2 = \Delta^2.x^2$ (if it affects the solution); or $2x\Delta x = 2\Delta x^2$, but allow Δx^2 for $(\Delta x)^2$.

B4 Omits $y + \Delta y$, or Δy, or $f(x + h)$, or $f(x + h) - f(x)$ on LHS. Apply once.

B5 Omits $\Delta y/\Delta x$ or $\{ f(x + h) - f(x)\}/h$ on L.H.S.

B6 Omits limiting idea (word "lim" unnec.) or has other than $\Delta x \to 0$ or $h \to 0$ on L.H.S., i.e. should have "lim", or "$\Delta x \to 0$" (allow "$\Delta x = 0$" instead), or "dy/dx".

B7 Evaluates limit where Δx or h will not divide, e.g. no Δx on R.H.S. at that stage.

B8 Limit error, e.g. $\Delta y/\Delta x = 2x + \Delta x - 2$ but $\lim \Delta y/\Delta x \neq 2x - 2$.

B9 Differentiates from first principles x^2 or $x^2 - 2$.

Misreading (–1)

M1 Differentiates from first principles $x^2 + 2x$.

Slips (–1)

S1 Misreads own work. e.g. correct term such as $2x\Delta x$ subsequently "becomes" $2\Delta x$.

Attempts (7 marks)

A1 $y + \Delta y$ or $f(x + h)$ on LHS; or $x + \Delta x$, or $x + h$, substituted somewhere on RHS.

A2 Linear function differentiated from first principles: if correct, award Att 7.

Worthless (0)

W1 Answer $2x - 2$ without work (i.e. not from first principles).

Part (c) **20 marks (10, 5, 5)** **Att 8**

(c) Let $f(x) = 3 - 5x - 2x^2$, $x \in \mathbf{R}$.

(i) Find $f'(x)$, the derivative of $f(x)$, and hence find the co-ordinates of the local maximum point of the curve $y = f(x)$.

(ii) Solve the equation $f(x) = 0$.

(iii) Use your answers from parts **(i)** and **(ii)** to sketch the graph of $f : x \rightarrow 3 - 5x - 2x^2$, showing scaled and labelled axes.

(c) (i) f ′(x) **10 marks (5, 5)** **Att 4 (2, 2)**

$f'(x) = -5 - 4x$ … 5m (att 2)

$f'(x) = 0$ or $-5 - 4x = 0$ …2m

$-4x = 5 \Rightarrow x = -5/4$

$y = 3 - 5(-5/4) - 2(-5/4)^2$

$= 6\tfrac{1}{8}$ …5m

* Marks are non-transferrable from either (c)(ii) or (c)(iii) back to (c)(i).

* If f ′(x) is not used to find the max point, e.g. max point found from a graph: 0 marks.

Blunders (–3)

B1 Differentiation error, once per term. <u>Three</u> terms to check.
B2 Transposition error.
B3 Sign error, each time.
B4 y co-ordinate not found, or inappropriate multiplication by 4 when finding it.

Attempts (2 marks for each part)

A1 In 1st part of (c)(i), any term correctly differentiated, e.g. the constant 3.
A2 In 2nd part of (c)(i), states that f ′(x) = 0 and stops.

Worthless (0)

W1 No term differentiated correctly.

(c) (ii) f(x) = 0 **5 marks** **Att 2**

$3 - 5x - 2x^2 = 0$	$2x^2 + 5x - 3 = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	…2m
$(3 + x)(1 - 2x) = 0$ $3 + x = 0$ or $1 - 2x = 0$	$(x + 3)(2x - 1) = 0$ $x + 3 = 0$ or $2x - 1 = 0$	$= \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(3)}}{2(-2)}$	
$x = -3$ or $x = 1/2$	$x = -3$ or $x = 1/2$	$= \dfrac{5 \pm \sqrt{25 + 24}}{-4} = \dfrac{5 \pm 7}{-4} = -3, \dfrac{1}{2}$	…5m

* Any blunder in transposition, sign, factors, roots, formula, etc, reduces marks to 2 (max).

* <u>In (c)(ii)</u>, uses a graph to solve the quadr. correctly (x = –3, ½): award <u>5 marks in (c)(ii).</u>

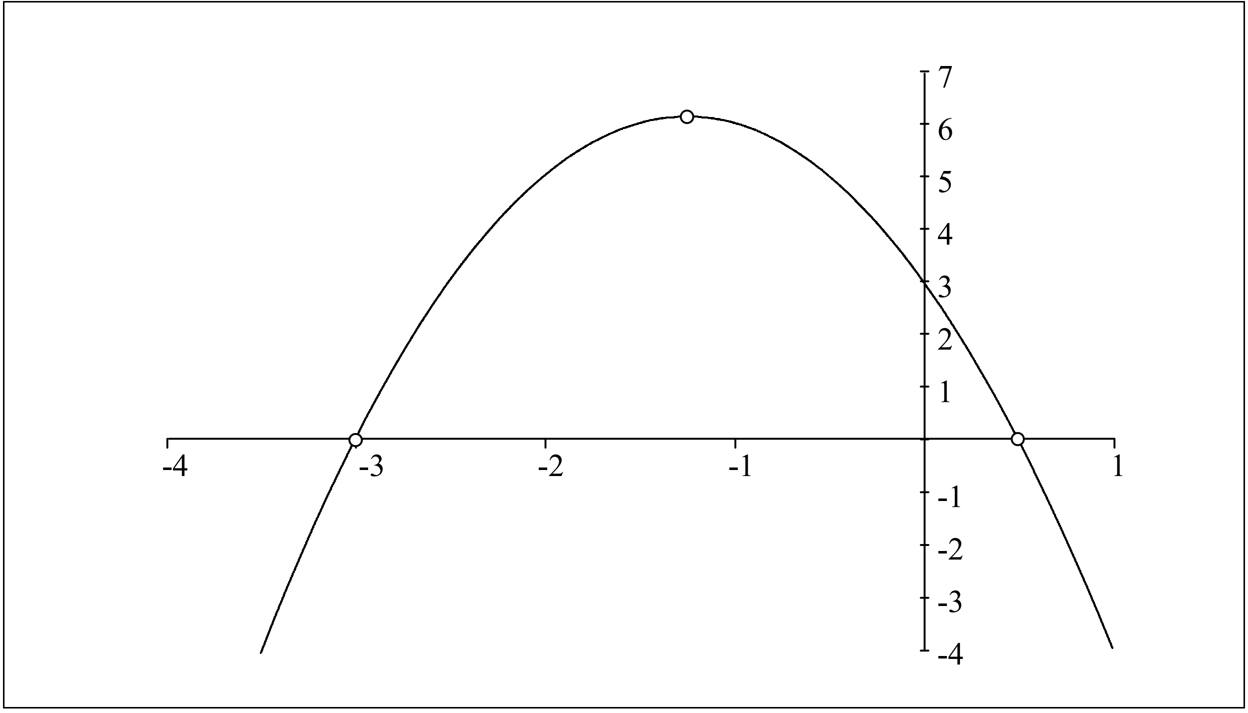
Blunders (–3)

B1 Error(s) in transposition, sign, or factors, or roots, quadratic formula.

Attempts (2 marks)
A1 Line 1 of any method above.
A2 An effort to use the Remainder Theorem to solve the quadratic, e.g. finds f(0).

Worthless (0)
W1 Incorrect transposition and stops.

(c) (iii) Sketch **5 marks** **Att 2**



* Accept candidate's answers to (i) and (ii), unless they oversimplify the graph (e.g. linear).
* Reasonable degree of accuracy required only.

Blunders (–3)
B1 Significant point omitted, i.e. max point, or intercept(s) on x-axis using candidate's data.
B2 Graph plotted upside-down, or in incorrect quadrant, using candidate's data.
B3 Finds and plots $f(-3) \neq 0$ and/or $f(½) \neq 0$.

Misreadings (–1)
M1 Graph drawn correctly from table -- but if incorrect, check B1 for significant points.

Slips (–1)
S1 The three significant points plotted correctly, but not joined.

Attempts (2 marks)
A1 Scaled axes drawn, and stops.
A2 Axes and max plotted, and stops.
A3 No graph, but mentions (– 3,0) and/or (½, 0).
A4 Effort to find f(–3) and/or f(½), or f(–1) and stops.

QUESTION 7

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks (5, 5)** **Att 4(2, 2)**

Differentiate with respect to x:

(i) x^3 **(ii)** $\dfrac{x^2 - x^4}{2}$.

(a)(i) Diff. **5 marks** **Att 2**

$3x^2$

(a)(ii) Diff. **5 marks** **Att 2**

$\dfrac{x^2 - x^4}{2} \Rightarrow \dfrac{2x - 4x^3}{2}$	$\dfrac{x^2}{2} - \dfrac{x^4}{2} \Rightarrow \dfrac{2x}{2} - \dfrac{4x^3}{2}$	$\dfrac{2.(2x - 4x^3) - (x^2 - x^4).0}{2^2}$

* Correct answer without work or notation: full marks.

* If done from first principles, ignore errors in procedure – just mark the answer.

* No need to simplify answer in (ii) to $x - 2x^3$.

* In (a)(i), for coefficient correct only *or* power correct only: allow 2m.

Blunders (–3) ...Applying to each part of (a).

B1 Differentiation error. Once per term. See Note 4 above for (a)(i).

B2 $\dfrac{dy}{dx} = \dfrac{2x - 4x^3}{0}$ or $\dfrac{2x}{0} - \dfrac{4x^3}{0}$ or $2x - 4x^3$.

B3 Each error in u/v formula.

Slips (–1)

S1 Differentiates $\frac{1}{2}x^2 - x^4$, or $x^2 - \frac{1}{2}x^4$, possibly without declaring it.

Attempts (2 marks for each part)

A1 In (a)(ii), $y = \frac{1}{2}x^2 - \frac{1}{2}x^4$ and stops.

A2 Any term (of the given function *or* the candidate's simplified function) differentiated correctly -- including the 2, i.e. d(2)/dx = 0.

A3 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., 'limit' mentioned, $\Delta x \to 0$, $f(x + h)$, etc.

A4 Writes down the notation *dy/dx* or $f'(x)$ and stops.

Worthless (0)

W1 No term differentiated correctly, but check attempts A1 to A4 first.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

(i)	Differentiate $(3x^3 - 2x^2 + 2)^4$ with respect to x .
(ii)	Given that $y = (5x^2 + 3)(4 - x^2)$, find $\dfrac{dy}{dx}$ when $x = 1$.

(b) (i) Chain rule **10 marks** **Att 3**

I:	II:
$4(3x^3 - 2x^2 + 2)^3.(9x^2 - 4x)$ ..10m	$u = 3x^3 - 2x^2 + 2$ and $du/dx = 9x^2 - 4x$ $y = u^4 \Rightarrow dy/du = 4u^3$ $dy/dx = dy/du \,.\, du/dx$ $= 4u^3.(9x^2 - 4x)$ <u>or</u> $4(3x^3 - 2x^2 + 2)^3.(9x^2 - 4x)$ ..10m

* Treat $4(3x^3 - 2x^2 + 2)^3$ and $(9x^2 - 4x)$ as two separate terms. See B1 and B2 below.

* No penalty for omission of brackets, as long as multiplication is implied.

* Answer $4(3x^3 - 2x^2 + 2)^3$: treat as B2: <u>award 7m</u>.
Answer $4(9x^2 - 4x)^3$: B1+B2: <u>award 4m</u>. Answer $9x^2 - 4x$: apply B1, B3: <u>award 4m</u>.

* If candidate tries to multiply out first, mark using slips and blunders.

Blunders (–3)
B1 Calculus error(s) in $4(3x^3 - 2x^2 + 2)^3$ part of derivative. Apply once.
B2 Calculus error(s) in $(9x^2 - 4x)$ part of derivative. Apply once.
B3 The two parts not multiplied, but *implied multiplication* is acceptable
e.g. $4(3x^3 - 2x^2 + 2)^3\ 9x^2 - 4x$.
B4 In method II, each error applying the chain rule.

Attempts (3 marks)
A1 Any correct relevant derivative, e.g. the “0” in the second term, i.e. $d2/dx = 0$.
A2 $u = 3x^3 - 2x^2 + 2$ and stops.
A3 Some correct element of *chain rule*, e.g. coefficient 4, or power 3.
A4 $y = 12x^3 - 8x^2 + 8 \Rightarrow dy/dx = 36x^2 - 16x$. Oversimplified.
A5 Some correct effort to multiply out given expression.

(b) (ii) u.v **10 marks** **Att 3**

	$y = 20x^2 + 12 - 5x^4 - 3x^2$ <u>or</u> $17x^2 + 12 - 5x^4$ ...3m
$(5x^2 + 3)(-2x) + (4 - x^2)(10x)$ ...7m	$dy/dx = 40x - 20x^3 - 6x$ <u>or</u> $34x - 20x^3$ ...7m
$= 14$ ..10m	$= 14$ . .10m

* In method I, no penalty for omission of brackets as long as multiplication is implied.

* If u/v used, apply B2 twice (central sign, division by v^2). There may be other errors.

* $dy/dx = (10x)(-2x) = (10)(-2) = -20$ merits 4 marks, i.e. 10 – B1 – B1.

* Error(s) simplifying dy/dx before evaluation at x = 1: penalise from the final 3m.

Blunders (–3)
B1 Differentiation error. Once per term. (Two terms to check in I, three or four terms in II).
B2 Error in u.v formula, e.g. central sign.
B3 Derivatives switched, i.e.$dy/dx = (5x^2 + 3)(10x) + (4 - x^2)(-2x)$. Apply once.
B4 In II, each omitted or incorrect term in the expansion (line 1) to a max of 2 blunders.
B5 Doesn’t evaluate derivative at x = 1, or puts dy/dx = 1, i.e. loses last three marks.
B6 Mathematical error in last step, e.g. $34 + 1 - 20 + 1 = 16$. See S1 below.

Slips (–1)

S1 Numerical error totting numbers in last step, e.g. 40 – 20 – 6 = 16.

Attempts (3 marks)

A1 Any correct derivative, eg. an implied “0”, or dy/dx = (10x)(–2x) and *stops*. See note 3.
A2 $u = 5x^2 + 3$ or $v = 4 - x^2$, or vice versa, and stops.
A3 One or more terms multiplied correctly in method II.

Worthless (0)

W1 u.v or u/v rule (from Tables) and stops.
W2 Substitutes into original (y) function - - no differentiation.

Part (c) 20 marks (5, 5; 5, 5) Att 8 (2, 2; 2, 2)

A train is travelling along a track. Suddenly, the brakes are applied. From the time the brakes are applied ($t = 0$ secs.), the distance travelled by the train, in metres, is given by $s = 30t - \frac{1}{4}t^2$.

(i) What is the speed of the train at the moment the brakes are applied?
(ii) How many seconds does it take for the train to come to rest?
(iii) How far does the train travel in that time?

(c)(i) ds/dt 10 marks (5, 5) Att 4 (2, 2)

ds/dt = …2m (dy/dx acceptable for ds/dt)
$30 - \frac{1}{2}t$ … 5m

= $30 - \frac{1}{2}(0)$ …2m
= 30 … 5m

* Marks are non-transferable between parts of (c), i.e. no retrospective marking allowed.
* No differentiation (to find ds/dt): award 0 marks from first five.
* t = 0 substituted into distance (s) equation: award 0 + att 2 overall.
* In (c)(i), for $30 - \frac{1}{2}t = 0 \Rightarrow t = 60$ award 5m + 0m. Do not assume it is (c)(ii).
* Misreads, $30 - 1/4t^2$, or $30 - 4t^2$: oversimplified: max is (4 + att2) + Att 2 + Att 2. See M1.
* If the parts of (c) are unlabelled, and context doesn’t identify which part is which, assume the questions were answered in sequence from (c)(i) 1st part, 2nd part, to (c)(ii) to (c)(iii).
* 30 or ds/dt = 30 without work: award 5m + 5m.
* For $30 - \frac{1}{2} = 29\frac{1}{2}$, or, $30 - \frac{1}{2}(0) = 29\frac{1}{2}$, award 5 + 2m.

Blunders(–3)

B1 Differentiation error, once per term. (Two terms to check).
B2 Incorrect value for *t* substituted into speed (ds/dt) equation.
B3 States speed = d^2s/dt^2 and continues. May get answer of 30.

Misreadings (–1)

M1 Misread as $30t + 1/4t^2$, or $30t - 4t^2$: not oversimplified, apply M(-1). Needn’t lose more.

Attempts (2 marks for each part)

A1 ds/dt mentioned in the first section, i.e. apply once in (c)(i).
A2 t = 0 substituted into any incorrect equation, in second section. (e.g. see note 3).

Worthless (0)
W1 Both terms incorrect, but ds/dt mentioned in first section merits att marks.(See A2)
W2 Incorrect answer to second part without work.
W3 States speed = d^2s/dt^2 and stops.

(c)(ii) **5 marks** **Att 2**

ds/dt = 0 <u>or</u> 30 – ½ t = 0 … 2m
t = 60 …5m

* Accept candidate's ds/dt from (c)(i).
* No retrospective award of marks for (c)(i) from (c)(ii).

Blunders (–3)
B1 ds/dt $\neq$ 0, or t = 0 substituted into speed (ds/dt) equation getting 30 sec.
B2 Distance s = 0 correctly solved, i.e. t = 0, 120. If incorrectly solved (e.g. one value), then worthless, 0 marks.
B3 Transposition error.

Attempts (2 marks)
A1 ds/dt = 0 and stops, or answer (c)(i) = 0 and stops.

(c)(iii) s **5 marks** **Att 2**

t = 60 => s = $30(60) - ¼\,(60)^2$ …2m
= 1800 – ¼(3600)
= 1800 – 900
= 900 …5m

* Accept the substitution of candidate's answer from (c)(ii), *apart from exception W1 below.*

Blunders (–3)
B1 Incorrect t substituted, i.e. $t \neq$ ans (c)(ii).
B2 Incorrect equation substituted, e.g. ds/dt substituted. [B1 and B2 may both occur].
B3 Distance $s = 30t - ¼\,t^2 = 0$ correctly solved for t. (t = 0, 120)
B4 Distance s = ans.(c)(ii) correctly solved for t.
B5 Mathematical errors, e.g. $¼(60)^2 = 15^2$.

Attempts (2 marks)
A1 Some mention or attempted use of s in this part.

Slips (–1)
S1 Numerical slips

Worthless (0)
W1 Solved s = 0 in (c)(ii) and now substitutes the answers back into s in this part.
W2 Solved ds/dt = 0 in (c)(ii) and now substitutes the answer back into ds/dt here.
W3 Solves ds/dt = 0 or any other value.
W4 Ans (c)(iii) = Ans (c)(i) – Ans (c)(ii).

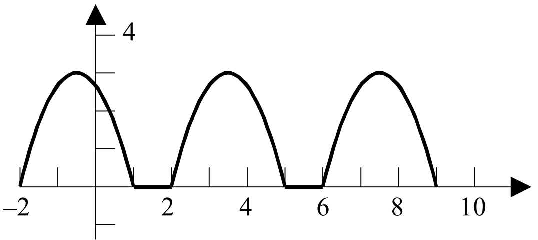
QUESTION 8

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks (5, 5)** **Att 4(2, 2)**

Part of the graph of a periodic function is shown.

Write down the period and range of the function.



(a) Period, Range **10 marks (5, 5)** **Att 4 (2, 2)**

Period = 4 … 5m (att 2)

Range = [0,3] … 5m (att 2)

- * If answers are unidentified, assume the first one is the period.
- * Be lenient as regard notation, for example:
  "Period = 4" may be given as 0 → 4, or 1 → 5, or (0 , 4), or $0 < n < 4$.
- * and "range = [0, 3]" may be given as range 3, or [3, 0], or (3, 0) or $0 < n < 3$.
- * Incorrect answers without work, *apart from those mentioned below*: no marks.

Blunders (–3)

B1 Period = 3, or Period = 5.

B2 Confusing period and range: 'period = 3, range = [0,4]', or 'period = 3, range = 4'. Once.

B3 Range = 1,2,3.

Slips (–1)

S1 Range = 0,1,2,3.

Part (b) **20 marks (10; 5, 5)** **Att 7(3, 2, 2)**

(i) The function g is defined for natural numbers by the rule:

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even.} \end{cases}$$

Find $g(13) + g(14) + g(15)$.

(ii) Given that $h(x) = x^2$, write down $h(x + 3)$.

Hence, find the value of x for which $h(x) = h(x + 3)$.

(b)(i) Function **10 marks** **Att 3**

$1 + 0 + 1$ …9m i.e. 3m each on a Hit/Miss basis.

$= 2$ …1m

* Correct answer without work: full marks.

* 13 + 14 + 15 = 42 earns 0 + 0 + 0 + 1 marks (resp.) = 1 mark, so award Att 3 instead. Likewise, 13g + 14g+15g = 42g earns 0+0+0+1 marks, and so merits Att 3m. Also '42g'.

* 1(13)+0(14)+1(15) = 28 earns 0+3+0+1 marks (resp.) = 4 marks. And 1g+0g+1g= 2g: 4m

* g(13) = g(1), g(14) = g(0), g(15) = g(1) and stops: award 6 marks i.e.10– B(–3) – S(–1).

* Treat 0 + 1 + 0 = 1 as a misreading: award 9 marks.

Blunders (–3)

B1 Each incorrect g(x) value.

Slips (–1)

S1 Numerical error when adding, or values not added.

(b)(ii) h(x + 3) **10 marks (5, 5)** **Att 4 (2,2)**

$h(x + 3) = (x + 3)^2$ *or* $x^2 + 6x + 9$ …5m (att 2)

Hence $x^2 = (x + 3)^2$ or $x^2 + 6x + 9$ …2m i.e. h(x) = previous answer.

$x = -1.5$ …5m

* If $h(x+3) = x^2 + 9$ (B1or B2), then $x^2 = x^2 + 9$ merits *att 2* in *second* part, i.e. 2m + 2m.

* Incorrect answers without work: no marks, apart from $x^2 + 9$, $x^2 + 3$, $x^2 + 3x$ and $x^2 + 6x$.

* If h(x + 3) = h(an algebraic expression), treat the algebraic expression as the answer.

Blunders (–3) ... applying to both parts of (b)(ii).

B1 $h(x + 3) = x^2 + 9$, i.e. award 2m in the *first* part. See notes above.

B2 $h(x+3) = h(x) + h(3) = x^2 + (3)^2 = x^2 + 9$. Award att 2 in *first* part. See Notes if continued.

B3 $h(x +3) = x^2 + 3$, $x^2 + 3x$ or $x^2 + 6x$ i.e. 2m in *first* part. If continued, mark as in Note 1.

B4 Sign error, or transposition error, e.g. $x = -6/9$.

Attempts (2 marks for each part)

A1 An effort to solve a quadratic generated in first section, i.e. Att 2 in *second* section.

Part (c) **20 marks (10, 5, 5)** **Att 7 (3, 2, 2)**

Let $f(x) = x^3 + 2x^2 - 1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) L is the tangent to the curve $y = f(x)$ at $-2/3$.
Find the slope of L.

(iii) Find the two values of x at which the tangents to the curve $y = f(x)$ are perpendicular to L.

(c)(i) f ′(x) **10 marks** **Att 3**

$f'(x) = 3x^2 + 4x$ …10m (Att 3)

Blunders (–3)

B1 Differentiation error, once per term. <u>Three</u> terms to check.

Attempts (3 marks)

A1 f ′(x) = dy/dx and stops, or an effort at first principles (e.g. y + Δy mentioned, etc).

Worthless (0)

W1 No term differentiated correctly.

(c)(ii) Slope **5 marks** **Att 2**

$3.\left(\frac{-2}{3}\right)^2 + 4.\left(\frac{-2}{3}\right)$ ...2m	$3\,(-0.66)^2 + 4\,(-0.66)$ …2m
$= 3.\left(\frac{4}{9}\right) - \frac{8}{3} = \frac{4}{3} - \frac{8}{3}$	$= 3\,(0.4356) - 2.64$
$= \frac{-4}{3}$ ...5m	$= -1.33$ …5m

* Allow use of candidate's f ′(x) from (c)(i) unless oversimplified (when Att would apply)

* Allow reasonable accuracy and rounding if using decimals, otherwise a slip (–1).

Blunders (–3)

B1 $3x^2 + 4x = -2/3$.

B2 Incorrect relevant equation correctly substituted, e.g. f (–2/3) found, (= –11/27).

Attempts (2 marks)

A1 Slope = f ′(x), or slope = dy/dx, and stops, or slope = $3x^2 + 4x$ and stops.

Worthless (0)

W1 $3x^2 + 4x = 0$ and continues, i.e. solves equation but no substitution of –2/3.

(c) (iii) ⊥ slope 5 marks Att 2

⊥ slope = ¾ <u>or</u> $3x^2 + 4x = ¾$ …2m	⊥ slope = ¾ <u>or</u> $3x^2 + 4x = ¾$ …2m
$12x^2 + 16x = 3$	$12x^2 + 16x = 3$
$12x^2 + 16x - 3 = 0$	$12x^2 + 16x - 3 = 0$
$(2x + 3)(6x - 1) = 0$	$x = \dfrac{-16 \pm \sqrt{16^2 - 4(12)(-3)}}{2(12)}$
	$= \dfrac{-16 \pm \sqrt{400}}{24} = \dfrac{-16+20}{24}$ and $\dfrac{-16-20}{24}$
$x = -3/2,\ 1/6.$ …5m	$= -3/2$ and $1/6$ …5m

- * Allow use of the candidate's $f'(x)$ from previous part; however, if it oversimplifies the question then the max mark attainable is Att 2.
- * Ignore efforts to find y values.
- * Quadratic formula could be applied to $3x^2 + 4x - ¾ = 0$: mark similar to method II above.

Blunders (–3)

B1 Incorrect perp slope formed from candidate's slope of L.
B2 Transposition or sign error.
B3 Incorrect factors. Apply once.
B4 Incorrect roots from factors. Apply once.
B5 Quadratic formula errors in formula, substitution, and simplification. Apply at most two blunders.

Attempts (2 marks)

A1 Perp slope = ¾ and stops
A2 States $m_1.m_2 = -1$ and stops, or $m_2 = -1/m_1$ and stops, or equivalent.
A3 $x^3 + 2x^2 - 1 = ¾$, the slope ¾ meriting the attempt marks here.
A4 Substitutes ¾ into $3x^2 + 4x$, the slope ¾ meriting the attempt marks here.
A5 Quadratic formula correct, and stops.

Worthless (0)

W1 Incorrect answer without work.
W2 $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ used.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2003

MATHEMATICS

ORDINARY LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
    - Blunders - mathematical errors/omissions (-3)
    - Slips - numerical errors (-1)
    - Misreadings (provided task is not oversimplified) (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
    - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
    - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

7. The phrase "and stops" means that no more work is shown by the candidate.

8. In parts of questions where there are 5 marks available, one blunder will bring the candidate down to the attempt mark. Hence, these are all listed as attempts in the scheme, rather than as a separate lists of blunders and attempts. *(This note applies to the paper 2 scheme only.)*

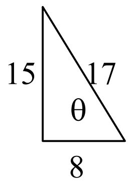
QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

(a) A right-angled triangle has sides of length 8 cm, 15 cm and 17 cm.
Find its area.

Part (a) **10 marks** **Att 3**

I	II	III
 Area = ½ base × h	2s = 8 + 15 + 17 or s = 20	$\sin\theta = \frac{15}{17}$
= ½ × 8 × 15	$A = \sqrt{20(20-8)(20-15)(20-17)}$	A = ½ × 8 × 17 × $^{15}/_{17}$
= 60	= $\sqrt{3600}$ or 60	= 60

* Accept correct answer without work.

Blunders (-3)

B1 Hypotenuse ≠ 17, e.g. ½ × 8 × 17 = 68.

B2 Omits ½, i.e. 8 × 15 (4 m) = 120 (7 m).

B3 Mathematical error, e.g. blunder in manipulating fractions or adds instead of multiplies.

B4 Incorrect relevant formula, e.g. ½ × 8 × 15 × 17 = 1020, if task is not simplified.
[Note: 8 × 15 × 17 = 2040 is B4 and B2]

B5 Wrong function in method III,or $\frac{1}{2}\times 8\times 15\times \sin 17$ calculated correctly.
[$\frac{1}{2}\times 8\times 15\times \sin 17$ and stops is 4 marks]

B6 Each arbitrary dimension, subject to attempt mark.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 Obvious slip in reading tables or calculator.

Misreadings (-1)

M1 Each obvious misreading of question if task is not simplified or changed.

Attempts (3 marks)

A1 Recognises or indicates 17 as hypotenuse.

A2 8 × 17 or 15 × 17 or 8 × 15 × 17 or 8 + 15 + 17 and stops, or s = 20 and stops.

A3 Some correct substitution in reasonable formula, e.g. ½ × 8 × h and stops.

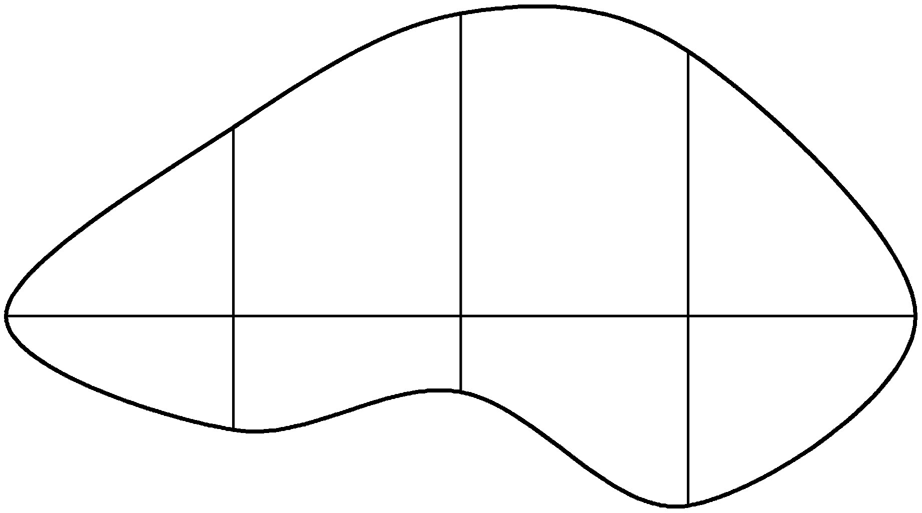
A4 Formula from tables with some substitution.

A5 Correct relevant formula which is not transcribed from the tables.

A6 Some relevant work, e.g. defines sin, cos or tan.

Part (b) **20 marks (10,10)** **Att 6 (3,3)**

In order to estimate the area of the irregular shape below, a horizontal line is drawn across widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.

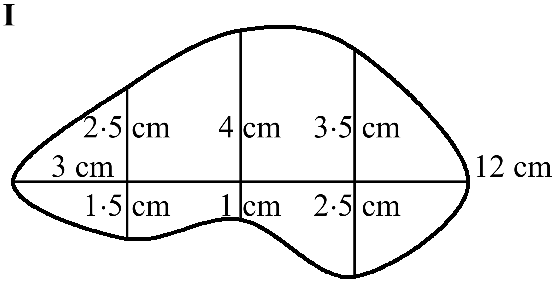


(i) Measure the horizontal line and the offsets as accurately as you can.
Make a rough sketch of the shape in your answerbook and record the measurements on it.

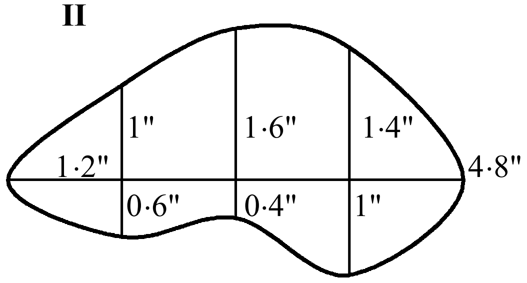
(ii) Use Simpson's Rule with these measurements to estimate the area of the shape.

Part (b)(i) **10 marks** **Att 3**

I



II



or 4 cm 5 cm 6 cm 1·6" 2" 2·4"

or Other units in the ratio h : y_2 : y_3 : y_4 = 3 : 4 : 5 : 6 (Total offsets y_2, y_3, y_4)

- * Allow tolerance ± 0·2 cm or inches.
- * Accept page of question with measurements written.
- * Award 2 marks for sketch of boundary plus 2 marks (hit or miss) for each of 4 measurements, i.e. 1 mark for each top and 1 mark for each bottom measurement, if taken separately, subject to attempt mark of 3.
- * Note: Lengths may be given in Part (ii). This is acceptable for 8 marks here.
- * Accept lengths of offsets as shown on diagram or added e.g. 25,15 or 40.
- * Note: Each correct length and the diagram gets 2 mark. [subject to the attempt mark]
- * 30 for horizontal or 120 is acceptable.
- * Accept correct lengths in ratio 3:4:5:6 [Note: All 4 lengths must be in correct ratio, otherwise award attempt 3 marks]

Attempts (3 marks)
A1 Some relevant step, e.g. one length within tolerance, or 1st = 0.
A2 A rough sketch of boundary or a vertical or horizontal line segment.

Worthless (0)
W1 No diagram and no accurate length measurement

Part (b) (ii) **10 marks** **Att 3**

I Area = $^{h}/_{3}\{F + L + 2(\text{odds}) + 4(\text{evens})\}$ 3 m	**II** $^{h}/_{3}\{F + L + TOFE\}$
= $^{3}/_{3}\{0 + 0 + 2(4) + 4(2{\cdot}5 + 3{\cdot}5)\}$+ $^{3}/_{3}\{0 + 0 + 2(1) + 4(1{\cdot}5 + 2{\cdot}5)\}$	F/L O E
or = $^{3}/_{3}\{0 + 0 + 2(5) + 4(6 + 4)\}$ 7 m	0 4 6
= $^{3}/_{3}\{0 + 0 + 10 + 40\}$	1 4
= 50 cm^2	$^{3}/_{3}\{ \quad \times 2 \quad \times 4\}$

* Candidate must not lose more than 3 marks for calculations or 4 marks for substitutions.
* Accept candidate's values explicitly written, if part (i) is not answered.
* If part (i) is not answered, candidate may obtain 18 marks for part (b) if he/she uses correct values correctly in part (ii).
* Allow $\frac{h}{3} = \{F + L + TOFE\}$ and penalise in calculations, if used.

Blunders (-3)
B1 Incorrect $\frac{h}{3}$ (once).
B2 Incorrect F and/or L or extra term with F and L (once).
B3 Incorrect TOFE (once).
B4 E **or** O omitted (once).
B5 Mathematical blunder, e.g. distribution error (once).
B6 Finds area of top or bottom only. [Top = 32 : Bottom = 18]

Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)
A1 Identifies F and/or L or odds or evens and stops.
A2 Statement of Simpson's Rule not transcribed from tables.
A3 E **and** O omitted (candidate may be awarded attempt mark at most).
A4 Some correct relevant calculation only.
A5 Completes one rectangle or area of one rectangle.
A6 Completes all rectangles, even if areas are added.
A7 Correct or consistent answer without showing work.
A8 Some relevant step.

Worthless (0 marks)
W1 Incorrect answer without work.
W2 Formula transcribed from tables and stops.

Part (c) **20 marks (5,5,5,5)** **Att 8 (2,2,2,2)**

A wax crayon is in the shape of a cylinder of diameter 10 mm,

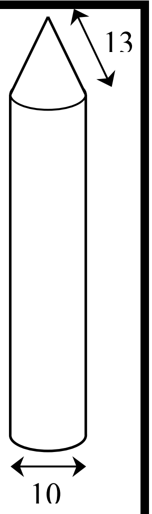
surmounted by a cone of slant height 13 mm.

(i) Show that the vertical height of the cone is 12 mm.

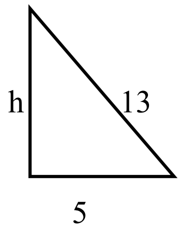
(ii) Show that the volume of the cone is 100π mm^3.

(iii) Given that the volume of the cylinder is 15 times the volume of the cone, find the volume of the crayon, in cm^3, correct to two decimal places.

(iv) How many complete crayons like this one can be made from 1 kg of wax, given that each cm^3 of wax weighs 0.75 grammes.



(c)(i) **5 marks** **Att 2**

I	II
 $h^2 + 5^2 = 13^2$ $h = \sqrt{144}$ or 12	$13^2 = 12^2 + 5^2$ $169 = 144 + 25$ $169 = 169$

- * Accept Pythagorean triple 5, 12, 13 explicitly written.
- * Must use 13 to merit full marks.
- * Note: One Blunder results in attempt mark of 2, subject to marks already secured.
- * Accept correct trigonometric method.

Attempts (2 marks)

A1 Pythagoras' Theorem used incorrectly. [e.g. 13 – 10 = 3]

A2 Mathematical blunder, e.g. $5^2 = 10$ or transposing error.

A3 $r \neq 5$, or $\frac{1}{3}(3\cdot14)(10)^2(12)$.

A4 Uses volume of cone = 100π (and not Pythagoras) to find h.

A5 Some relevant step, e.g. mentions Pythagoras or 13^2 or $r = 5$ or draws the perpendicular height of a triangle.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Worthless (0)

W1 An answer without work.

W2 Diagram reproduced and no further work.

W3 $h = \frac{1}{3}(13 + 13 + 10) = 12$

(c) (ii) **5 Marks** **Att 2**

Volume = $^1/_3\ \pi\ r^2\ h$	$= {}^1/_3 \times \pi \times 5^2 \times 12$	$= 100\pi$ or 314 or similar
	2 marks	5 marks

* Allow candidate's h and r from part (i).

* Note: One Blunder results in attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 Incorrect relevant cone formula, e.g. πrl, πr^2h, $^1/_3\ \pi rh$, $^1/_3\ r^2h$, with some correct substitution.

A2 Incorrect and inconsistent substitution into a correct formula.

A3 Correctly fills in formula and stops.

A4 Mathematical blunder, e.g. $^1/_3 \times 25 = 75$.

A5 Identifies $r = 5$ and / or $h = 12$, in this part.

A6 Some correct step.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Worthless (0 marks)

W1 $\pi = 3{\cdot}14$ or similar and stops.

(c)(iii) **5 marks** **Att 2**

Vol of cylinder = 1500π ==> Vol of crayon = $1600\pi = 5026{\cdot}5$ mm^3	$= 5{\cdot}026$ cm^3	$= 5{\cdot}03$ cm^3
2 marks	4 marks	5 marks

* Accept candidate's values consistent with previous parts.

* Allow answer consistent with $3{\cdot}1 \leq \pi \leq {}^{22}/_7$ ($4{\cdot}96 \leq \text{vol} \leq 5{\cdot}03$) with or without work.

* Note: one Blunder results in attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 1 cm$^3 \neq 1000$ mm^3.

A2 Volume of crayon = 1500π or volume of crayon = 1600.

A3 Mathematical blunder, e.g. multiplies instead of divides by 1000.

A4 Takes π outside tolerance.

A5 Some relevant step, e.g. Volume of cylinder = 1500π or $100\pi = 314$ or similar.

A6 1 cm^3 = 1000 mm^3 or volume of cylinder = $\pi\ 5^2\ h$ and stops.

Slips (- 1)

S1 Failure to round off or rounds off too soon, if it affects the answer.

(c) (iv) **5 marks** **Att 2**

(1 kg = 1000 gms) ➔ $\frac{1000}{0.75}$ cm^3 or 1333·3… cm^3 (2 marks)

Number of crayons = $\frac{1000}{0 \cdot 75} \div 5.026... = 265 \cdot 25$ or $\frac{1000}{0 \cdot 75} \div 5 \cdot 03 = 265.07$ (4 marks)

$= 265$ (5 marks)

* Accept candidate's values from previous parts. (Correct answer arises from $\frac{1000}{0 \cdot 75 \times answer\, to\, c(iii)}$)

* Note: One Blunder results in attempt mark of 2,subject to marks already secured.

Attempts (2 marks)

A1 1 kg ≠ 1000 gms and continues.
A2 Mathematical blunder, e.g. divides by wrong number.
A3 Some correct step, e.g. $\frac{1}{0 \cdot 75}$ in line 1 or answer in c(iii) $\times$ $0 \cdot 75$ and stops.
A4 Divides some number by candidate's answer from part (iii).
A5 Correct (acceptable or consistent) answer without showing work.

Slips (- 1)

S1 Failure to find integral part of number.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Find the distance between the two points (3, 2) and (8, 14).

(a) **10 marks** **Att 3**

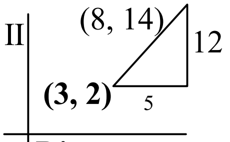
I **Distance** = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **3m**

= $\sqrt{(8-3)^2 + (14-2)^2}$ **7m**

= $\sqrt{5^2 + 12^2} = \sqrt{25+144}$

= $\sqrt{169}$ **or 13** **10m**

II



3m

Distance = $\sqrt{5^2 + 12^2}$ **7m**

= $\sqrt{169}$ **or 13** **10m**

* If a formula for distance is not written, any sign error is at least a blunder, e.g.

Distance = $\sqrt{(8-3)^2 - (14-2)^2} = \sqrt{-119}$	One blunder
Distance = $\sqrt{8+3)^2 + (14+2)^2} = \sqrt{377}$	One blunder
Distance = $\sqrt{(8-3)^2 + (14+2)^2} = \sqrt{281}$	One blunder
Distance = $\sqrt{(3+2)^2 + (8+14)^2} = \sqrt{509}$	Two blunders, B1 & B2

Blunders (-3)

B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or switches x and y (once).

B2 Incorrect relevant formula, e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ or $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$.

B3 Two or more incorrect substitutions.

B4 Mathematical error, e.g. $5^2 = 10$.

B5 Incorrect use of √, e.g. distance = 169.

B6 Last step omitted.

Slips (-1)

S1 <u>One</u> incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$ part of formula.

S2 <u>One</u> incorrect substitution, if formula is written.

S3 <u>Obvious</u> misreading of co-ordinate.

S4 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Some correct step, e.g. (3, 2) with x_1 and y_1 identified.

A2 Correct relevant formula and stops.

A3 Formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.

A4 Oversimplifies, e.g. $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$ with some correct substitution, even if completed.

A5 One or both points plotted reasonably well.

A6 States Pythagoras' Theorem or $\sqrt{a^2 + b^2}$.

A7 √169 or 13 without showing work.

A8 Uses translation, e.g. (5, 12) and stops.

Worthless (0)

W1 Irrelevant formula, even if completed, i.e. midpoint or $\sqrt{x_2y_1 - y_2x_1}$ or similar, subject to A1.

Part (b) **20 marks (5, 10, 5)** **Att 7 (2, 3, 2)**

$a(-2, 2)$, $b(4, 6)$ and $c(0, -4)$ are three points.
p is the midpoint of $[ab]$ and q is the midpoint of $[ac]$.

(i) Find the co-ordinates of p and the co-ordinates of q.

(ii) Plot a, b, c, p and q on a co-ordinate diagram on graph paper.
Show the line segments $[bc]$ and $[pq]$ on your diagram.

(iii) Using slopes, or otherwise, prove that pq is parallel to bc.

(b) (i) **5 marks** **Att 2**

p, midpoint of $[ab]$ = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{2+6}{2}\right)$ 2 m

$= (1, 4)$ 4 m

q, midpoint of $[ac]$ = $\left(\frac{-2+0}{2}, \frac{2-4}{2}\right)$ $= (-1, -1)$ 5 m

* Award 4 marks for one correct midpoint and 5 marks for both.
* Accept (1, 4) and (- 1, - 1) without work.
* One blunder results in an attempt mark of 2, subject to marks already secured.

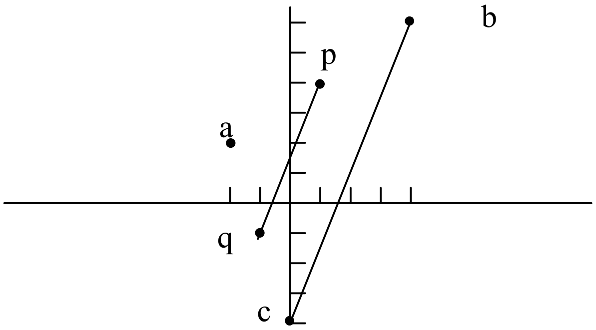
Attempts (2 marks)

A1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or switches x and y (once).
A2 Incorrect relevant formula, e.g.
$\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right) or \left(\frac{y_1+y_2}{2}, \frac{x_1+x_2}{2}\right) or \left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$, with some correct substitution.
A3 Two or more signs incorrect in substitution, if formula is written.
A4 Mathematical error, e.g. - 2 + 4 = - 2 or blunder in use of fractions.
A5 Some relevant step, e.g. (- 2, 2) with x_1 and y_1 identified.
A6 Correct relevant formula and stops.
A7 Diagram with correct midpoint indicated but co-ordinates not written.

Slips (- 1)
S1 One incorrect sign or substitution, if formula is written.
S2 Each numerical slip to a maximum of 3.
S3 Treats *p* or *q* as the midpoint of [*bc*].

Worthless (0 marks)
W1 Irrelevant formula with no substitution, e.g. slope formula.

(b) (ii) **10 marks** **Att 3**



* Accept candidate's points from (i).
* Award 3 marks for the axes plus one mark for each point and line (segment) correct.
* Accept a vertical x-axis and horizontal y-axis.
* Intervals must be indicated or implied

Blunders (- 3)
B1 Switches x and y co-ordinates, e.g. (4, 6) plotted as (6, 4). Penalise once, subject to 2nd asterisk.
B2 Scale unreasonably inconsistent. Penalise once subject to 2nd asterisk.

Attempts (3 marks)
A1 Some correct step, e.g. any line segment [*pq*] drawn.

(b) (iii) **5 marks** **Att 2**

I Slope *pq* $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{-1-1}$	2 marks	**II** 2-point formula for line
$= {}^{-5}/_{-2}$ or ${}^{5}/_{2}$	3 marks	*pq*: 5x - 2y + 3 = 0
Slope *bc* $= \frac{-4-6}{0-4} = {}^{-10}/_{-4}$ or similar	4 marks	*bc*: 5x - 2y - 8 = 0
Slopes are equal	5 marks	Slopes are equal

* Accept a correct geometrical proof.
* Note: One blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or switches x and y (once).

A2 Incorrect relevant formula, e.g. $\frac{y_2 + y_1}{x_2 + x_1} \, or \, \frac{y_2 - y_1}{x_1 - x_2}$, with some correct substitution.

A3 Two or more incorrect substitutions, if formula is written.

A4 Mathematical error, e.g. - 1 - 4 = - 3.

A5 Some relevant step, e.g. construction line for geometrical proof.

A6 Correct relevant formula or relevant statement and stops.

A7 Any formula with (x_2 - x_1) and/or (y_2 - y_1) and some correct substitution.

Slips (- 1)

S1 **<u>One</u>** incorrect sign in (x_2 - x_1) or (y_2 - y_1) part of formula.

S2 **<u>One</u>** incorrect substitution, if formula is written.

S3 Each numerical slip to a maximum of 3.

Part (c) 20 marks (10, 10) Att 6 (3, 3)

L is the line $3x + 2y + 12 = 0$.
K is the line that passes through the point (7, 3) and is perpendicular to *L*.

Find the equation of *K* and hence find the point of intersection of *K* and *L*.

Equation of K 10 marks Att 3

I Slope of L = – $^3/_2$	**II** Slope of L = – $^3/_2$	**III** Slope of L = – $^3/_2$ (3m)
Slope of K = $^2/_3$	K: 2x - 3y = c or 2(7) - 3(3) = c	Slope K = $^2/_3$ or K: y = $^2/_3$x + c
		or 3 = $^2/_3$(7) + c (7m)
K: y - 3 = $^2/_3$(x - 7)	c = 5 or K: 2x - 3y = 5	c = – $^5/_3$ or y = $^2/_3$x – $^5/_3$ (10m)

* Step 2 presupposes step 1 in each of above methods.

* Errors in simplifying the equation of K to be penalised in next part, if used.

* Incorrect slope of L warrants penalty of - 3 at most. Allow candidate to use it correctly.

* Answers without work: y - 3 = $^2/_3$(x - 7) or any correct variation Award full marks
y - 3 = – $^3/_2$ (x - 7) or 2x - 3y = c, c ≠ 5 or y = $^2/_3$x + c, c ≠ – $^5/_3$ …Award 7 marks
y - 3 = *m*(x - 7), *m* not relevant ...Award 4 marks
3x - 2y + c = 0, i.e. equation of line with neither slope nor point correct. Award 3 marks

Blunders (-3)

B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or switches x and y (once).

B2 Incorrect relevant formula (once each type), e.g. $\frac{y_2 + y_1}{x_2 + x_1} \, or \, \frac{y_2 - y_1}{x_1 - x_2}$, with correct substitution and y + y_1 = m(x + x_1) or blunder in slope of L.

B3 Two or more incorrect substitutions.

B4 Mathematical error, e.g. transposing error in method III.

B5 $m_1 \times m_2 \neq - 1$ or blunder in slope of K.

B6 Uses an arbitrary point.

Slips (-1)

S1 <u>One</u> incorrect sign in (x - x_1) or (y - y_1) part of formula.

S2 <u>One</u> incorrect sign in substitution, if formula is written.

S3 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Some correct step.
A2 Correct formula and stops, e.g. $m_1 \times m_2 = -1$ or $-{}^{a}/_{b}$.
A3 Formula with $(x_2 - x_1)$ and/or $(y_2 - y_1)$ and some correct substitution.
A4 Draws a pair of perpendicular lines with (7, 3) as the point of intersection.

K ∩ L **10 marks** **Att 3**

I L: 3x + 2y = - 12 => 9x + 6y = - 36	**II** 3x + 2y = - 12 => $y = \frac{-3x-12}{2}$	(3m)
K: 2x - 3y = 5 => $\underline{4x - 6y = 10}$	$2x - 3y = 5 \Rightarrow 2x - 3\left(\frac{-3x-12}{2}\right) = 5$	
13x = - 26	4x + 9x + 36 = 10	
x = - 2	x = - 2	(7m)
2(- 2) - 3y = 5 => y = - 3	3(- 2) + 2y = - 12 => y = - 3	(10m)

- * Accept correct point verified in both equations for full marks.
- * Award 7 marks for one co-ordinate found correctly.
- * Accept candidate's equation of K from 1st part of (c), if he/she has already been penalised. [Note: in simplifying K apply only a maximum of 1 Blunder or a maximum of 3 Slips.]

Blunders (- 3)

B1 Fails to find the 2nd variable.
B2 Transposing error, e.g. 13x = -26 => $x = -{}^{13}/_{26}$ or - 4 - 3y = 5 => - 3y = 5 - 4.
B3 Fails to multiply both sides in method I (once).
B4 Mathematical blunder, e.g. deals incorrectly with fractions in method II.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Some correct step, e.g. some correct multiplication in method I.
A2 Verifies correct point in one equation.
A3 Verifies an irrelevant point in L and/or K or finds a point in L and/or K.
A4 Graphical solution, written or indicated [must be (-2, -3) or consistent with L ∩ K].
A5 Correct answer without work.

Worthless (0 marks)

W1 Incorrect answer without work or inconsistent graphical solution.

QUESTION 3

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 marks**	**Att 7**
Part (c)	**15 marks**	**Att 6**

Part (a) **15 marks (10, 5)** **Att 5 (3,2)**

The circle C has equation $x^2 + y^2 = 25$.

(i) Verify that the point (–4, 3) is on the circle C.

(ii) Write down the co-ordinates of a point that lies outside C and give a reason for your answer.

(a) (i) **10 marks** **Att 3**

I $(-4)^2 + (3)^2$	**II** $\sqrt{(0+4)^2+(0-3)^2} = \sqrt{25}$ or 5	**III** $(0+4)^2 + (0-3)^2$ (7m)
$= 25$	$r = \sqrt{25}$ or 5	$= 25$ (10m)

* Accept "Distance from (- 4, 3) to (0, 0) is 5 which is length of radius".

Blunders(-3)

B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2), e.g. $\sqrt{(-4-3)^2+(0-0)^2}$.

B2 Incorrect relevant formula, e.g. $\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}\,or\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}$, with correct substitution.

B3 Two or more signs incorrect in substitution, if formula is written.

B4 Mathematical error, e.g. $(-4)^2 = 8$.

B5 Incorrect centre.

B6 Incorrect radius, if used.

B7 Uses √ incorrectly.

Attempts (3 marks)

A1 Some relevant step, e.g. mentions (0, 0) or 5 and/or plots (- 4, 3).

A2 Correct relevant formula and stops, e.g. $x^2 + y^2 = r^2$.

A3 Any formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.

A4 States Pythagoras' Theorem or $(-4)^2$ or similar.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.

(a) (ii) **5 marks** **Att 2**

I	Point (- 4, 5) or similar	$(-4)^2 + (5)^2 = 41$ or > 25
II	Point (- 4, 5) or similar	$\sqrt{(0+4)^2+(0-5)^2} = \sqrt{41}$ or > 5

* Award 5 marks for a correct point and correct reason.

* Having identified the point, accept 41 or > 25 **or** √41 or > 5 without work.

* Accept candidate's centre and radius from (i).

* Apply the scheme for part (i) where appropriate.

* Apply relevant attempt marks in both parts (i) and (ii) if the attempt is made in both parts.

* One blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 Some relevant step, e.g. draws a circle and a point outside it.

A5 Some correct substitution, using a point inside or on the circle [not (-4, 3) again].

A6 Writes the co-ordinates of a point that is outside the circle and stops.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.

Part (b) **20 marks (15, 5)** **Att 7 (5, 2)**

> The line $x - 2y + 5 = 0$ intersects the circle $x^2 + y^2 = 10$ at the points *a* and *b*.
>
> **(i)** Find the co-ordinates of *a* and the co-ordinates of *b*.
>
> **(ii)** Draw a co-ordinate diagram on graph paper, showing the line, the circle and the points of intersection.

(b) (i) **15 marks** **Att 5**

I $x = 2y - 5$	**II** $y = \dfrac{x+5}{2}$	(5m)
$(2y - 5)^2 + y^2 = 10$	$x^2 + \left(\dfrac{x+5}{2}\right)^2 = 10$	(6m)
$4y^2 - 20y + 25 + y^2 = 10$	$x^2 + \dfrac{x^2 + 10x + 25}{4} = 10$	(9m)
$5y^2 - 20y + 15 = 0$	$4x^2 + x^2 + 10x + 25 = 40$	
$y^2 - 4y + 3 = 0$	$x^2 + 2x - 3 = 0$	
$(y - 3)(y - 1) = 0$ or $y = \dfrac{4 \pm \sqrt{16-12}}{2}$	$(x + 3)(x - 1) = 0$ or $x = \dfrac{-2 \pm \sqrt{4+12}}{2}$	
$y = 3$	$x = -3$	(11m)
or $y = 1$	or $x = 1$	(12m)
$y = 3 \Rightarrow x = 2(3) - 5 = 1$	$x = -3 \Rightarrow y = {}^{(-3+5)}/_2 = 1$	(14m)
$y = 1 \Rightarrow x = 2(1) - 5 = -3$	$x = 1 \Rightarrow y = {}^{(1+5)}/_2 = 3$	(15m)

* Accept two correct points verified correctly in both line and circle.

* Case A: $y = 3 \Rightarrow x^2 + (3)^2 = 10 \Rightarrow x = 1$ and $y = 1 \Rightarrow x^2 + (1)^2 = 10 \Rightarrow x = 3$: apply B5 (-3).

* Case B:

$$\begin{array}{l} x - 2y = -5 \Rightarrow x^2 + 4y^2 = 25 \\ \underline{x^2 + \ y^2 = 10} \\ 3y^2 = 15 \text{ and continues} \end{array}$$

merits A1 for $x - 2y = -5$.

Blunders (- 3)

B1 Transposing error (once).

B2 Blunder in squaring (2y - 5) or $\dfrac{x+5}{2}$.

B3 Error in quadratic formula or application, e.g. $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$.

B4 Error in factorising.

B5 Substitutes into circle for both 2nd variables and fails to find correct points.

B6 Mathematical blunder.

B7 One correct point verified correctly in both line and circle.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.

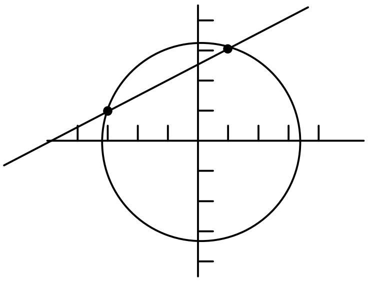
Attempts (5 marks)

A1 Some relevant step, e.g. x - 2y = - 5 or correct relevant formula.
A2 Correct substitution of any value of x or y into the equation of the line or circle.
A3 Accurate graphical solution [graph may merit more marks in part (ii)].
A4 (- 3, 1) and/or ((1, 3) without showing work.
A5 Incorrectly avoids quadratic equation. Award attempt mark at most.
A6 Centre (0, 0) or radius = $\sqrt{10}$.

Worthless (0 marks)

W1 Only one correct ordinate without showing work. (e.g. x = 1 and stops)

(b) (ii) **5 marks** **Att 2**



Axes	**2 m**
Line or circle	**4 m**
Line an d circle	**5 m**

* Accept candidate's points from (i).
* Accept horizontal y-axis and vertical x-axis.
* Scale must be indicated or implied for full marks.
* One blunder results in attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 Confuses x and y co-ordinates, e.g. (- 3, 1) incorrectly plotted as (1, - 3).
A2 Scale unreasonably inconsistent.
A3 Incorrect centre or radius, with some relevant work and stops. e.g. draws a circle and/or line.
A4 Some relevant step, e.g. effort at finding a point on the line or circle.
A5 Axes drawn and stops.
A6 Centre (0, 0) or radius = √10 and stops.

Part (c) 15 marks (5, 5, 5) Att 6(2, 2, 2)

The circle K has equation $(x+2)^2 + (y-3)^2 = 25$.
p and q are the endpoints of a diameter of K and pq is horizontal.
(i) Find the co-ordinates of p and the co-ordinates of q.
(ii) Hence, or otherwise, write down the equations of the two vertical tangents to K.
(iii) Another circle also has these two vertical lines as tangents.
The centre of this circle is on the x-axis.
Find the equation of this circle.

(c) (i) 5 marks Att 2

I [$x^2 + y^2 + 4x - 6y - 12 = 0$]	**II**
Centre (- 2, 3)	Centre (- 2, 3) (2 m)
	Equation of pq is y = 3
Radius = 5	$(x + 2)^2 + (3 - 3)^2 = 25$ (3m)
p is (- 7, 3)	x = - 7 (4 m)
q is (3, 3)	x = 3 (5 m)

* Note: One Blunder results in an attempt mark of 2,subject to marks already secured.
* Marks for centre and radius in method I are interchangeable.

*Attempts (*2 marks*)*
A1 Blunder in expanding $(x + 2)^2$ or $(y - 3)^2$, if used.
A2 Incorrect and inconsistent centre.
A3 Incorrect and inconsistent radius.
A4 x = 3 or y = - 2 in method II.
A5 Mathematical blunder, e.g. transposing error.
A6 Some relevant step, e.g. draws a horizontal line or some correct expansion, e.g. x^2.
A7 Centre (- 2, 3) **or** radius = 5 and stops.
A8 Correct answer without showing work.
A9 y = k, k arbitrary.
A10 Shows some knowledge of diameter.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.

Misreadings(-1)
M1 Reads horizontal as vertical. (once)

(c) (ii) **5 marks** **Att 2**

I	II	
$x = k$		
	$x = k$	(2 m)
	$(k + 2)^2 + (y - 3)^2 = 25 \Rightarrow y^2 - 6y + k^2 + 4k - 12 = 0$	
	$b^2 - 4ac = 0 \Rightarrow 36 - 4(k^2 + 4k - 12) = 0$ or $k^2 + 4k - 21 = 0$	
$x = - 7$	$x = - 7$	(4 m)
$x = 3$	$x = 3$	(5 m)

- * Accept distance from $(- 2, 3)$ to $x - k = 0$ is $5 \Rightarrow |- 2 - k| = 5 \Rightarrow x = - 7$ or 3.
- * One blunder results in an attempt mark of 2,subject to marks already secured.
- * Accept candidate's values from (i).
- * Accept correct or consistent answers without showing work.

Attempts (2 marks)

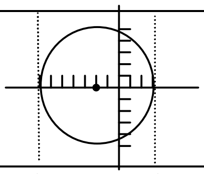
A1 $y = - 7$ and $y = 3$.
A2 Mathematical blunder in method II.
A3 Some relevant step, e.g. draws a vertical line or $x = k$, k arbitrary.
A4 Correct relevant formula, e.g. $y - y_1 = m(x - x_1)$ or distance from a point to a line.
A5 $m_1m_2 = - 1$ or slope of horizontal is 0 or similar.
A6 Centre $= (- 2, 3)$ or radius $= 5$ written for this part.
A7 $y - 3 = - {}^{1}/_{0}\,(x + 7)$ and $y - 3 = - {}^{1}/_{0}\,(x - 3)$.

Slips (- 1)

S1 $x = - 7$ or $x = 3$ but not both.

(c) (iii) **5 marks** **Att 2**

Centre $(- 2, 0)$	(2 m)
Radius $= 5$	
$(x + 2)^2 + (y - 0)^2 = 5^2$ or 25	(5 m)



- * One blunder results in an attempt mark of 2,subject to marks already secured.
- * Accept candidate's values from previous parts.
- * Accept correct or consistent answer without work.
- * Accept $(h - x)^2 + (k - y)^2 = r^2$ and continues or $(h + 2)^2 + (k - 0)^2 = 25$.

Attempts (2 marks)

A1 Confuses x and y, e.g. $(0, - 2)$.
A2 Incorrect relevant formula, e.g. $(x + h)^2 \pm (y + k)^2 = r^2$ or $(x - h)^2 - (y - k)^2 = r^2$.
A3 Two or more errors in substitution.
A4 Incorrect and inconsistent centre or radius.
A5 Some relevant step, e.g. $y = 0$.
A6 Correct relevant formula written for this part. e.g. writes $x^2 + y^2 = r^2$ and stops.
A7 Centre $(- 2, 0)$ and/or $r = 5$ written for this part.
A8 Draws a circle with centre on the x-axis.

Slips (- 1)

S1 One incorrect sign or substitution in $(x - h)$ or $(y - k)$ part.

QUESTION 4

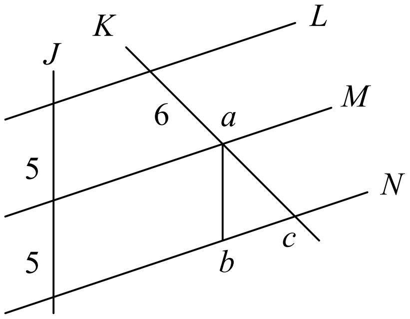
Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a)	**10 marks (5, 5)**	**Att 4 (2, 2)**

In the diagram, L, M and N are parallel lines. They make intercepts of the indicated lengths on J and K. ab is parallel to J.

(i) Write down the length of $[ab]$.

(ii) Write down the length of $[ac]$.



(a) (i)	**5 marks**	**Att 2**

$|ab| = 5$

* Note: One blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 $|ab| = 6$.
A2 Some relevant step, e.g. something relevant added to diagram.
A3 Mentions *parallelogram* or similar.
A4 States some relevant theorem.

Slips (-1)

S1 States "Opposite sides of a parallelogram are equal in measure" or similar indication.

Worthless (0)

W1 Measures $|ab|$ with a ruler ≈ 13 mm.

(a) (ii)	**5 marks**	**Att 2**

$|ac| = 6$

* One Blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2 marks)

A1 $|ac| = 5$.
A2 Some relevant step, e.g. completes a parallelogram.
A3 Mentions congruent triangles or similar.
A4 States some relevant theorem.

Slips (-1)

S1 States "Intercepts of equal length on K" or similar indication.

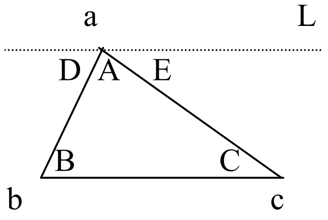
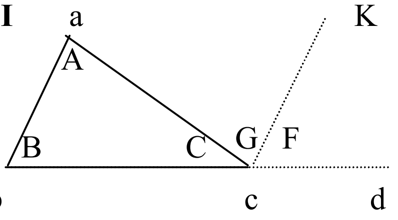
Worthless(0)

W1 Measures $|ac|$ with a ruler ≈ 13 to 14 mm.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

> **(i)** Prove that the sum of the degree-measures of the angles of a triangle is 180°.
> **(ii)** Deduce that the degree-measure of an exterior angle of a triangle is equal to the sum of the degree-measures of the two remote interior angles.

(b) (i) **10 marks** **Att 3**

I  Construction: Through a draw L // bc. Proof: $|\angle B| = |\angle D|$ $|\angle C| = |\angle E|$ $|\angle A| + |\angle B| + |\angle C| = |\angle A| + |\angle D| + |\angle E|$ $= 180^o$	**II**  Construction: Through c draw K//ab. (3 m) Proof: $|\angle B| = |\angle F|$ (2 m H or M) $|\angle A| = |\angle G|$ (2 m H or M) $|\angle A| + |\angle B| + |\angle C| = |\angle G| + |\angle F| + |\angle C|$ (2 m H/ M) $= 180^o$ (1 m H or M)

* If candidate's work is not worthless, he/she must be awarded at least attempt 3 marks.
* Accept steps clearly indicated on a diagram.
* Method II merits 10 marks for part (i) plus 7 marks for part (ii).
* Deduct 3 marks from the overall mark if the steps are not written in a logical order, subject to marks already secured.

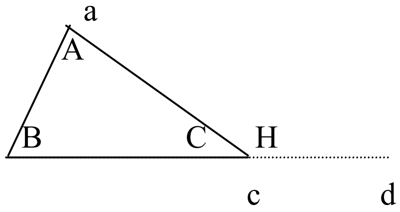
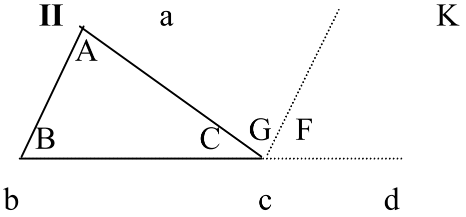
Attempts (3 marks)

A1 Some relevant step, e.g. "alternate angles are equal in measure".
A2 Construction and stops.
A3 A triangle drawn and stops.
A4 Special case, e.g. $50^o + 60^o + 70^o = 180^o$.
A5 No necessary construction merits, at most, an attempt mark.
A6 Straight angle = 180^o
A7 Uses part (ii) to prove part (i)

Worthless (0 marks)

W1 An irrelevant theorem merits 0 marks, subject to the attempt mark.

(b) (ii) **10 marks** **Att 3**

I  Construction: Produce [bc] to d Proof: $|\angle A| + |\angle B| + |\angle C| = 180^o$ $|\angle H| + |\angle C| = 180^o$ $|\angle H| = |\angle A| + |\angle B|$	**II**  Construction: Produce [bc] to d Through c draw K//ab (3 m) Proof: $|\angle G| = |\angle A|$ (2 m H or M) $|\angle F| = |\angle B|$ (2 m H or M) $|\angle G| + |\angle F| = |\angle A| + |\angle B|$ (3 m H or M)

* If candidate's work is not worthless, he/she must be awarded at least attempt 3 marks.
* Accept steps clearly indicated on a diagram.
* Award 0 for an incorrect step or a step that is omitted and not indicated.
* Deduct 3 marks from the overall mark if the steps are not written in a logical order, subject to marks already secured.

Attempts (3 marks)

A1 Some relevant step, e.g. straight angle = 180^o or angle sum of $\Delta = 180^o$, written for this part.

A2 Triangle (for this part) or construction and stops or a special case.

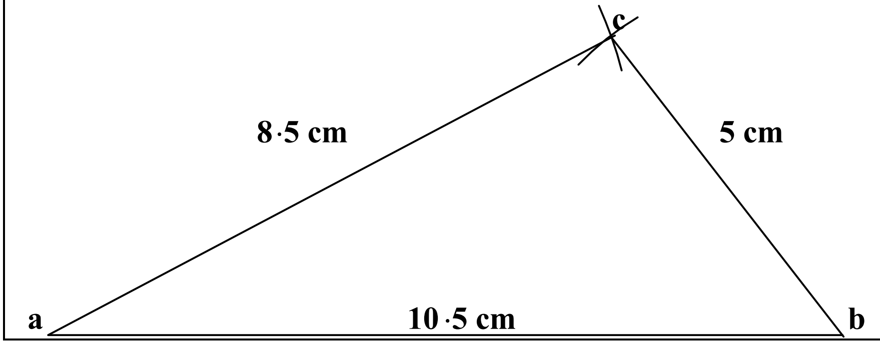
Part (c) **20 marks (10, 5, 5)** **Att 7 (3, 2, 2)**

(i) Construct a triangle *abc* in which $|ab| = 10.5$ cm, $|bc| = 5$ cm and $|ac| = 8.5$ cm.

(ii) Choose any point *p* that is *outside* the triangle and construct the image of *abc* under the enlargement of scale factor 0.4 and centre *p*.

(iii) Given that the area of this image triangle is 3.36 cm^2, calculate the area of the original triangle *abc*.

(c) (i) **10 marks** **Att 3**



1 side => 3 marks
2 sides => 7 marks
3 sides => 10 marks

- * Accept sides within ± 5 mm.
- * Do not demand labels.

Blunders (-3)

B1 Measure of side outside tolerance, each time, subject to attempt and B2.

B2 Draws triangle equiangular to the correct one, i.e. not accurate but to scale (once).

Slips (-1)

S1 Measurement within tolerance but side not straight, no straight edge used. (once)

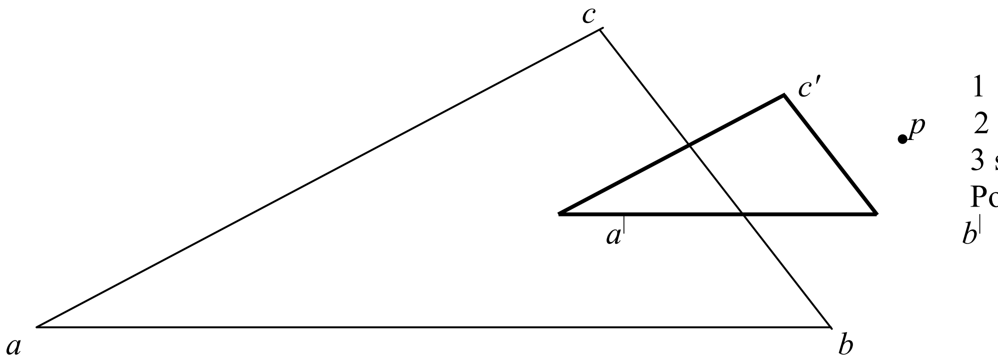
S2 Points *a, b* and *c* not joined (once).

Attempts (3 marks)

A1 Some relevant step, e.g. 1 side of any length.

A2 A triangle which is not similar and no side within tolerance.

(c) (ii) **5 marks** **Att 2**



1 side (2 m)
2 sides (3 m)
3 sides (4 m)
Point p (5 m)

* Accept sides of image within ± 2 mm [10·5 → 4·2, 8·5 → 3·4 and 5 → 2 cms].
* One blunder results in an attempt mark, subject to marks already secured.

Attempts (2 marks)
A1 Some relevant step, e.g. some point *p* drawn and stops.
A2 Any triangle which does not merit more than an attempt mark.

Slips (- 1)
S1 Point p omitted or not outside triangle and each image side omitted, subject to attempt.
S2 Scale factor 1· 4 [10·5 → 14·7, 8·5 → 11·9 and 5 → 7 cms].
S3 Vertices of image triangle not joined (once).

(c) (iii) **5 marks** **Att 2**

Given *Area* $\Delta a'b'c'$ = 3·36		
I Area $\Delta abc = \dfrac{3.36}{(0.4)^2}$	**II** 2s = 10·5 + 8·5 + 5 = 24	**III** $\Delta a'b'c'$ = ½(4·2)h' = 3·36
	Area $\Delta abc = \sqrt{12(15)(35)(7)}$	=> h' = 1·6 => h = $^{1\cdot 6}/_{0\cdot 4}$ = 4
= 21 cm^2	= 21	Area Δabc = ½(10·5)(4) = 21

* One blunder results in an attempt mark of 2, subject to marks already secured.
* Accept candidate's values from previous parts, if already penalised.
* Allow candidate to find angles and area using trigonometry.

Attempts (2 marks)
A1 3·36 ÷ 0·4 or 3·36 × 0·4 or3·36 × $(0{\cdot}4)^2$.
A2 0·4 ÷ 3·36 or $(0{\cdot}4)^2$ ÷ 3·36.
A3 Misplaced decimal point.
A4 Some relevant step, e.g. $(0{\cdot}4)^2$ or some operation with 3·36 and another number.
A5 Area formula from tables with some substitution.
A6 Relevant formula not transcribed from tables.
A7 ½ × 10·5 × 5, even if completed.
A8 ½ × 10·5 × h where h is measured.
A9 Correct or consistent answer without work.

Slips (- 1)
S1 Numerical slips to a maximum of 3.

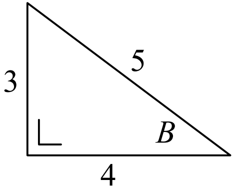
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a)	**10 marks**	**Att 3**

The lengths of the sides of a right-angled triangle are shown in the diagram and B is the angle indicated.

Find the value of $\sin B \cos B$, as a fraction.



(a)	**10 marks**	**Att 3**

$\sin B = \frac{3}{5}$	3 marks
$\cos B = \frac{4}{5}$	7 marks
$\sin B \cos B = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$	10 marks

* Accept correct answer without work.
* Allow use of decimals, provided final answer converted to a fraction.
* Rad or Grad acceptable, if used correctly. **[** e.g. B = $0 \cdot 643$**....** ; $40 \cdot 966$**..]**

Blunders (-3)

B1 Incorrect ratio, e.g. $\sin B = \frac{5}{3}$ (once, if consistent error)

B2 Confuses sin with cos, e.g. $\sin B = \frac{4}{5}$ and $\cos B = \frac{3}{5}$ (once).

B3 Mathematical error, e.g. error manipulating fractions.

Slips (-1)

S1 Gives the answer as a decimal ($0 \cdot 48$). or $B \approx 36{\cdot}8^{\circ}$, $\sin B \cos B = ½ \sin 2B = ½ \sin 73{\cdot}73\ldots = 0{\cdot}48$.

S2 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Any trigonometrical function defined, e.g. $\tan B = \frac{opp}{adj}$.

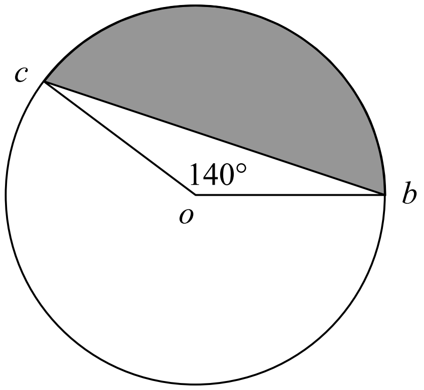
A2 Some fractions using 3, 4, or 5 without reference to sin or cos and not obviously correct.

A3 Some substitution into sine or cosine formula.

A4 Some relevant statement, e.g. $\sin B \cos B = ½ \sin 2B$ or $B \approx 36{\cdot}896..^{\circ}$. or $5^2 = 3^2 + 4^2$

Part (b) 20 marks (10, 5, 5) Att 7 (3, 2, 2)

A circle has centre *o* and radius 7 cm.
The two points *b* and *c* are on the circle and $|\angle boc| = 140°$.



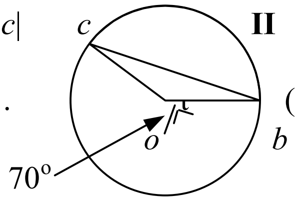
(i) Find the area of the triangle *obc*, correct to the nearest cm^2.

(ii) Find the area of the sector *obc*, correct to the nearest cm^2.

(iii) Taking the areas correct to the nearest cm^2, express the area of the shaded region as a fraction of the total area enclosed by the circle.
Give your answer as a fraction in its simplest form.

(b) (i) 10 marks Att 3

I Area Δ *obc* = ½ × r × r × sin |∠*boc*| (3m)
= ½ × 7 × 7 × 0·6427…
= 15·748…
≈ 16 cm^2



II Area Δ*obc* = |*ct*|.|*ot*|
(Using Trig.)= 6·57×2·39 (7m)
= 15·7… (9m)
≈ 16 cm^2 (10m)

* May use $\sqrt{s(s-a)(s-b)(s-c)}$.
* No more than 3 marks may be lost for calculations.
* If only 1st step and answer are written:

1st line plus area = 16	Award 10 marks
1st line plus area = 15·something	Award 9 marks
1st line plus area = neither, but note B7	Award 3 marks

Blunders (- 3)
B1 $r^2 = 2r$, i.e. $7^2 = 14$.
B2 Mathematical error in dealing with fractions.
B3 Treats *ab* as a line segment in ½ ab sinC, e.g. area = ½ × 7 × sin140°.
B4 Incorrect substituting, if not an obvious misreading.
B5 Misplaced decimal point.
B6 Incorrect function read.
B7 Incorrectly uses rad or grad. $[24\cdot015;\ 19\cdot82]$

Slips (- 1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Obvious slip in reading the tables or calculator.
S3 Each numerical slip to a maximum of 3.

Attempts (3 marks)
A1 15·something or 16, or $24\cdot015$ or $19\cdot82$ without work
A2 Some correct substitution into a reasonable formula, e.g. 7 × 7 or sin140°.

A3 Sine or cosine formula with some substitution.
A4 $|\angle ocb| = 20^{o}$ and stops.
A5 *ot* drawn perpendicular to *bc*.
A6 Some relevant step, e.g. defines some trigonometric function for this part.

(b) (ii) 5 marks Att 2

I Area sector *obc* = $^{140}/_{360}\ (\pi r^2)$	**II** Area sector = $\frac{1}{2} r^2 \theta$	
or $^{140}/_{360} \times \pi \times 7 \times 7$	$\frac{1}{2} \times 7 \times 7 \times {}^{140\pi}/_{180}$	2 marks
= 59·8647…	= 59·8647…	4 marks
≈ 60 cm^2	≈ 60 cm^2	5 marks

* One blunder results in an attempt mark of 2, subject to marks already secured.
* Accept reasonable approximation for π, i.e. $3{\cdot}1 \le \pi \le {}^{22}/_{7}$.
* Accept area of reflex sector *obc* (i.e. 94 cm^2) for this part.

Attempts (2 marks)
A1 Fails to convert to radians in method II, and continues.
A2 Mathematical error in dealing with fractions.
A3 Incorrect relevant formula, e.g. length of arc with some correct substitution.
A4 $r^2 = 2r$.
A5 Incorrect substitution (e.g. value of r) unless an obvious slip.
A6 Some relevant step, e.g. $^{140}/_{360}$ or $^{140}/_{180}$.
A7 59·something or 60 without work.
A8 Misplaced decimal point(once).

Slips (- 1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Obvious slip in reading the tables or calculator.
S3 Each numerical slip to a maximum of 3.
S4 Answer left as $19 \cdot 0\dot{5}\pi$ or similar.

(b) (iii) 5 marks Att 2

Shaded area = 60 - 16 or 44	2 marks
Area enclosed by circle = πr^2 = 153·9… ≈ 154	3 marks
Fraction = $\dfrac{44}{154}$	4 marks
Simplest form = $\dfrac{2}{7}$	5 marks

* Steps one and two are interchangeable. First step merits 2 marks and others one mark each.
* If a candidate's fraction = $\frac{a}{b}$, not in simplest form, do not award 1 mark for last step.
* Accept values from previous parts, subject to 2^{nd} asterisk.
* If a candidate secures marks at any stage, those marks cannot be reduced for subsequent errors.
* One blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2 marks)
A1 Correct answer without work.
A2 Some relevant step, e.g. statement, *sector - triangle* or work for (i) or (ii) repeated here.
A3 π × 7 × 7 and stops.

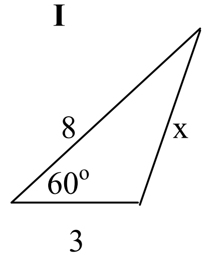
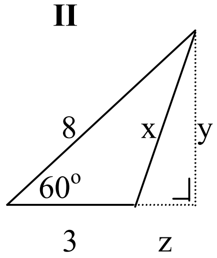
Slips (-1)

S1 Each step incorrect, omitted or incomplete (if not completed later).

Part (c) **20 marks (10, 10)** **Att 6 (3, 3)**

> One side of a triangle has length 8 cm and another has length 3 cm.
> The angle between these two sides measures 60°.
> **(i)** Find the length of the third side.
> **(ii)** Find the measures of the two remaining angles, correct to the nearest degree.

(c) (i) **10 marks** **Att 3**

I		**II**
 $x^2 = b^2 + c^2 - 2bc\cos60^o$	(3 m)	 $^y/_8 = \sin60^o$, $y = 4\sqrt{3}$ $^{(3+z)}/_8 = \sin30^o$, $z = 1$
$x^2 = 3^2 + 8^2 - 2(3)(8)\cos60^o$	(7 m)	$x^2 = y^2 + z^2 = 48 + 1$
$x^2 = 49$ $x = \sqrt{49}$ or 7	(10 m)	$x = \sqrt{49}$ or 7

* 7 marks for correct formulae with correct substitution. Calculation errors incur -3 at most.

Blunders (- 3)

B1 Error in cosine formula (once).

B2 Incorrect substitution and continues.

B3 Incorrect function read, e.g. sin instead of cos or vice versa.

B4 Uses radian (or Grad) mode incorrectly
[$x = \sqrt{118}$ or$10 \cdot 89$**...**or $\sqrt{44 \cdot 786}$ *or* $6 \cdot 69$.....](once in each part).

B5 $73 - 48\cos60^o = 25\cos60^o$.

B6 Mathematical error, e.g. transposing error in method II.

B7 Angle not between two sides and continues correctly. Mark as per (c)(ii) for Sine rule and (c)(i) for Cosine rule.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.

S2 Obvious slip in reading tables or calculator.

Attempts (3 marks)

A1 Some relevant step, e.g. right-angled triangle completed in II.

A2 Correct answer without work.

A3 Correct formula with some but not all substitutions.

A4 Some correct use or definition of sin, cos or tan.

A5 $x^2 = 8^2 \pm 3^2 = 64 \pm 9$ and continues.

A6 Attempt at a scaled diagram.

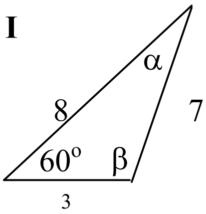
Worthless (0 marks)

W1 Incorrect answer without work, but $\sqrt{118}$ or$10 \cdot 89$**...**or $\sqrt{44 \cdot 786}$ or $6 \cdot 69$**..** is an attempt.

W2 Formula transcribed from tables and stops.

(c) (ii) 10 marks Att 3

I



$\frac{\sin\alpha}{3} = \frac{\sin 60}{7}$

$\sin\alpha = 3\times\frac{0\cdot866}{7}$

$= 0\cdot3711..$

$\alpha = 21\cdot78..^{\circ}$

$\approx 22^{\circ}$

$\beta = 180^{\circ} - (60^{\circ} + 21\cdot78..^{\circ})$

$\beta = 98\cdot21..^{\circ}$

$\beta \approx 98^{\circ}$

II

$3^2 = 8^2 + 7^2 - 2(8)(7)\cos\alpha$ (3 m)

$9 = 64 + 49 - 112\cos\alpha$

$\cos\alpha = \frac{104}{112}$ or $0\cdot928..$

$\alpha = 21\cdot78..^{\circ}$

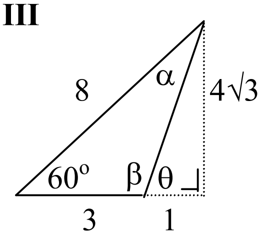
$\approx 22^{\circ}$ (7 m)

$\beta = 180^{\circ} - (60^{\circ} + 21\cdot78..^{\circ})$

$\beta = 98\cdot21..^{\circ}$

$\beta \approx 98^{\circ}$ (10 m)

III



$\tan\theta = \frac{4\sqrt{3}}{1}$ or $6\cdot928..$ (3 m)

$\theta = 81\cdot78^{\circ}.$

$\beta = 98\cdot21..^{\circ}$

$\approx 98^{\circ}$ (7 m)

$\alpha = 180 - (60^{\circ} + 98\cdot21..^{\circ})$ or $81\cdot78..^{\circ} - 60^{\circ}$

$\alpha = 21\cdot78^{\circ}.$

$\alpha \approx 22^{\circ}$ (10 m)

* Note that $8^2 > 3^2 + 7^2 \Longrightarrow \beta$ is obtuse and α is acute.

* Accept incorrect values from (i), if already penalised.

Blunders (- 3)

B1 Error in sine or cosine rule or defining a trigonometrical function (once).
B2 Incorrect substitution (once).
B3 Blunder in using or reading a trigonometrical function.
B4 Radians (or Grad) used incorrectly (once in this part).
B5 Mathematical blunder, e.g. distributes incorrectly.
B6 Misplaced decimal point (once).
B7 Sum of angles in a $\Delta \neq 180^{\circ}$.
B8 Finds only one of remaining angles correctly to the nearest degree.
B9 Uses $\frac{\sin\beta}{8} = \frac{\sin 60^{\circ}}{7} \Rightarrow \sin\beta = 0\cdot989\ldots \rightarrow \beta = 81\cdot786..^{\circ} \approx 82$ and $\alpha \approx 38^{\circ}$.

Slips (- 1)

S1 Each numerical slip to a maximum of 3.
S2 Failure to round off or rounds off too early if it affects the answer (once).

Attempts (3 marks)

A1 Some relevant step, e.g. sum of angles in a $\Delta = 180^{\circ}$.
A2 $\alpha = 21\cdot7..^{\circ}$ or 22° and/or $\beta = 98\cdot2..^{\circ}$ or 98° without work.
A3 Relevant formula with some or all correct substitutions.
A4 Some correct definition or use of sin, cos or tan.
A5 Scale drawing and stops or reads angle from scaled diagram.

QUESTION 6

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks(5,5)** **Att 4(2,2)**

I write down today's date as 09062003 and then select one of the digits at random.

(i) What is the probability that I select the 9?

(ii) What is the probability that I select an odd digit?

Part (a)(i) **5 marks** **Att 2**

$\frac{1}{8}$ or $0 \cdot 125$ or $1:8$ or $1 - \frac{7}{8}$ or $1 - 0 \cdot 875$ or

$\frac{1}{\binom{8}{1}}$ or 1 in 8 or $12\frac{1}{2}\%$.

* Accept correct answer without work.

* Note: One Blunder results in attempt mark of 2, subject to marks already secured.

* Accept $\left(\frac{1}{8}\right)$ *but* $\binom{1}{8}$ is an attempt.

Attempts (2)

A1 $\binom{8}{1}$ or ${}^{8}C_{1}$ and stops.

A2 Inverted fraction: $\frac{8}{1}$ or $\frac{8}{7}$, or $8:1$.

A3 Writing down any relevant digit, 1,7,8,5. e.g. ① 9 4

A4 9 circled in list with no further work.

A5 One correct step, e.g. any incorrect ratio in the form $\frac{a}{b}$ *or* $a:b$ without work.

A6 Any definition of probability or statement of probability theorem. eg. $\frac{\#\text{E}}{\#\text{S}}$.

Slips (-1)

S1 Numerical errors to a maximum of 3. (e.g. a miscount of digits, if obvious)

Misreadings(-1)

M1 Counts the zero's as a unit, so that total number of digits in list is 5 $\Rightarrow$ an answer of $\frac{1}{5}$

Worthless (0)

W1 Writes down original list and no further work..

W2 Adds the digits to total 20 and no further work. But if $\bar{x} = 2 \cdot 5$ is found award attempt mark, since this implies 8 digits.

Part (a)(ii) **5 marks** **Att 2**

$\frac{2}{8}$ or $\frac{1}{4}$ or $0\cdot25$ or 1 : 4 or $\frac{\binom{2}{1}}{\binom{8}{1}}$ or $1 - \frac{6}{8}$ or $1 - \frac{3}{4}$

or $1 - 0\cdot75$ or $\frac{1}{8}+\frac{1}{8}$ or 2 in 8 or 1 in 4 or 25%.

or Sample space:

Odds	Evens
2	6

or

Odds	Evens
9 3	0 0 0 0 6 2

and continues

- * Accept correct answer without work.
- * Note: One Blunder results in attempt mark of 2, subject to marks already secured.
- * Accept an answer consistent with candidate's answer from part (i).
- * Note: twice candidate's answer from part (i) gets full marks here.
- * Note M1: the special case.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Attempts (2)

A1 $\binom{2}{1}$ *or* $\binom{8}{1}$ or similar and stops.

A2 Inverted fraction, e.g. $\frac{8}{2}$, or 4 : 1.

A3 Writes down any relevant digit from acceptable solutions above.

A4 3 and 9 identified as odd in a list or stated as being odd.

A5 One correct step, e.g. any incorrect ratio in the form $\frac{a}{b}$ *or* $a : b$ without work.

A6 Any definition of probability or statement of probability theorem. eg. $\frac{\#\text{E}}{\#\text{S}}$.

A7 Incomplete Sample Space.

A8 $\frac{1}{8}\times\frac{1}{8}$

Misreadings (-1)

M1 Special case: Takes the zero's as odd, i.e. 2 and 6 are even ⇒ rest are odd. This gives rise to common incorrect answer of $\frac{6}{8}$ *or* $\frac{3}{4}$, which merits 4 marks.

Worthless (0)

W1 Writes down original list and no further work.

Part (b) **20 marks (5,10,5)** **Att 7 (2,3,2)**

Two women, Ann and Bríd, and two men, Con and Declan, sit in a row for a photograph.
(i) How many different arrangements of the four people are possible?
(ii) Write out the four possible arrangements that have the two women in the middle.
(iii) If the arrangement of the four people is chosen at random from all of the possible arrangements, what is the probability that the two women will be in the middle?

Part (b)(i) **5 marks** **Att 2**

$4\times3\times2\times1$ or 24 or 4! or List or $^4P_4$

* Accept correct answer without work.
* Note: one Blunder results in an attempt mark of 2, subject to marks already secured.
* In solution $4\times3\times2\times1$**,** multiplication must be clearly indicated, but allow omission of 1.

Attempts (2 marks)
A1 Multiplication not clearly indicated, e.g.

4	3	2	1

A2 Each incorrect or omitted box, with the exception of 1, but penalise if a number other than 1 appears instead of 1.
A3 Addition instead of multiplication.
A4 One correct step, (e.g. partial list).
A5 One or more boxes drawn.
A6 Writes any permutation or factorial or combination symbol and stops.
A7 Any correct relevant integer written down, e.g. 4 or 3 or 2 or 1.

Slips (-1)
S1 Numerical errors to a maximum of 3.
S2 Each omitted entry in list method, subject to the attempt mark, i.e. to a maximum of 3 omissions.

Part (b)(ii) **10 marks** **Att 3**

CABD CBAD DABC DBAC

* If a fully correct list is written out in part (i), this merits 7 marks here.
* Marking: 4 correct $\Rightarrow$ 10 marks (Note: This incl. a list from (i) with the 4 correct arrangements clearly marked).
3 correct $\Rightarrow$ 7 marks
2 correct $\Rightarrow$ 4 marks
1 correct $\Rightarrow$ 3 marks

Blunders (-3)
B1 Each arrangement omitted, subject to the attempt mark.
B2 Only 3 names used in list [DAC,CAD,DBC,CBD]
B3 List in b(i) without 4 correct arrangements marked clearly.

Attempts (3)
A1 Only 1 correct arrangement.
A2 Any 4 incorrect arrangements, but NOTE misreadings below.
A3 Any relevant step.

Misreadings (-1)
M1 List correct but with both men in the middle.
M2 List which keeps both women together at either end, i.e. ABCD, BACD, ABDC, BADC or CDAB, CDBA, DCAB, DCBA. Note: 2 or 1 correct here merits Att 3.

Part (b)(iii) **5 marks** **Att 2**

$\frac{4}{24}$ or $\frac{1}{6}$ or $0\cdot1\dot{6}$ or $1 - \frac{20}{24}$ or $1 - \frac{5}{6}$ or $\frac{2\times1\times1\times2}{4!}$

or Sample space and continues. or 4 in 24 or $16\frac{2}{3}\%$ or 1 : 6

- * Note: One blunder results in attempt mark of 2, subject to marks already secured.
- * Accept correct answer and no work shown.
- * Note: Any list from Part (b)(i) or b(ii) gets at least attempt mark here.
- * Accept $\frac{\#\,\textbf{(}\text{Part(ii)}\textbf{)}}{\#\textbf{(}\text{Part(i)}\textbf{)}}$

Attempts (2)

A1 $\begin{pmatrix}6\\1\end{pmatrix}$ or similar and stops.

A2 Inverted fraction, e.g. $\frac{24}{4}$.

A3 Writes down any relevant number from acceptable solutions above.

A4 4! or similar and stops.

A5 One correct step, e.g. any incorrect ratio in the form $\frac{a}{b}$ *or* $a : b$ without work.

A6 Any definition of probability or statement of probability theorem. eg. $\frac{\#\text{E}}{\#\text{S}}$. [An earlier statement does not suffice here]

A7 Incomplete Sample Space.

NOTE

* General note on Part 6 (b): Note: a fully correct list made at 6 (b) (i) with the 4 correct arrangements in part (b) (ii) correctly and exclusively marked out merits (5 + 10 + 2) marks for part (b).

Part (c) **20 marks (5,5,5,5)** **Att 8 (2,2,2,2)**

In a certain school the examination subjects for senior students are grouped as follows:

Compulsory Subjects	Block A	Block B	Block C
Irish English mathematics	French German	biology home economics construction studies accounting	business organisation history physics

As well as taking all three of the compulsory subjects, each student must choose *one* subject from Block A, *two* from Block B and *one* from Block C.

(i) In choosing two subjects from Block B, how many different selections are possible?

(ii) In choosing the full range of subjects, how many different selections are possible?

(iii) One student has already decided to do German and construction studies.
How many different selections of the remaining subjects are possible for this student?

(iv) If the student referred to in part (iii) selects her remaining subjects at random, what is the probability that she will select both biology and physics?

Part(c)(i) **5 marks** **Att 2**

$\binom{4}{2}$ or $\frac{4\times 3}{2\times 1}$ or 6 or $\frac{4!}{2!\times 2!}$ or $\frac{4\times 3\times 2\times 1}{2\times 1\times 2\times 1}$

or $\frac{{}^4P_2}{2!}$ or List [BH,BC,BA,HC,HA,CA]

* Accept correct answer and no work shown.

* No penalty for $\left(\frac{4}{2}\right)$, but $\frac{4}{2}$ is a Blunder.

* No penalty for omitting 1, where it occurs in variations above.

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2)

A1 Inverted, e.g. $\binom{2}{4}$.

A2 Addition instead of multiplication, where method requires it.

A3 Missing value e.g. $\frac{4}{2}$

A4 One relevant step, e.g. 4!.

A5 Any integer from the above variations1,2,3,4.

A6 Writes any combination, permutation, factorial symbol and stops., e.g. !, P, C.

A7 Writes 36 as the answer to part (i)

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 Each choice omitted in list method, subject to the attempt mark.

Misreadings(-1)

M1 Any obvious misreading which does not oversimplify or change the task.

Worthless (0)

W1 Incorrect answer and no work, subject to above attempts.

W2 Table reproduced and no further work.

Part (c) (ii) **5 marks** **Att 2**

$1\times 2\times 6\times 3$ or 36 or $1\times\binom{2}{1}\times\binom{4}{2}\times\binom{3}{1}$ or List

* Accept correct answer and no work shown.

* Accept candidate's answer for $\binom{4}{2}$ and then used in 3rd variation.

* The 1 can be omitted, where it appears in variations above.

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

* Note: For attempt mark here answers must be clear and distinct from any attempt in 6(c)(i).

* Note: first or third variation above merits FULL marks for part(i) and part(ii).

* 6 × (ans (c)(i)) merits full marks for this part.

Attempts (2)

A1 Inverted, e.g. $\binom{2}{4}$

A2 Addition instead of multiplication, or necessary multiplication not indicated.

A3 One relevant step, e.g. 4! or 2! or an attempt to combine 2 or more subjects.

A4 Any integer from the above variations 1,2,3,4, 6.

A5 Writes down any combination, permutation or factorial symbol.

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 Each selection omitted in list method, subject to attempt 2.

Misreadings (-1)

M1 Any obvious misreading which does not oversimplify or change the task.

Worthless (0)

W1 Incorrect answer and no work, subject to above attempts.

Part (c) (iii) **5 Marks** **Att 2**

$1\times 1\times 3\times 3$ or 9 or $1\times 1\times\binom{3}{1}\times\binom{3}{1}$ or List

* Accept correct answer and no work shown.

* No penalty if 1 omitted in any of above.

* One Blunder results in an attempt mark of 2, subject to marks already secured.

* Accept $\frac{1}{4}\times$ candidate's answer in c (ii).

Attempts (2)

A1 Inverted, e.g. $\binom{1}{3}$.

A2 Addition instead of multiplication or necessary multiplication not indicated.

A3 One relevant step, e.g. 3!.

A4 Any number from above variations, 1,3.

A5 Writes down any combination, permutation or factorial symbol.

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 Each selection omitted in list method, subject to the attempt mark.

Worthless (0)

W1 Incorrect answer and no work, subject to attempts above.

Part(c) (iv) **5 marks** **Att 2**

$\frac{1}{9}$ or $1:9$ or $\frac{1}{3}\times\frac{1}{3}$ or $0.1\dot{1}$ or $1-\frac{8}{9}$ or

$1-0\cdot 8\dot{8}$ or 1 in 9

or Sample space:

B, BO	B, H	*B, P*
HE, BO	HE, H	HE, P
A, BO	A, H	A, P

and continues

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

* Note: Accept the reciprocal of the candidates answer to part (iii) for full marks.

* Note: Special case misreading.

* Note: Full or partial sample space here merits an attempt for part (iii).

Attempts (2 marks)

A1 Inverted fraction, e.g. $\frac{9}{1}$.

A2 Addition rather than multiplication, e.g. $\frac{1}{3}+\frac{1}{3}$

A3 Writes down any relevant number, which appears in solutions, or any $\binom{n}{r}$ or ${}^{n}P_{r}$ or !.

A4 One correct step, e.g. any incorrect ratio in the form $\frac{a}{b}$ *or* $a:b$, without work.

A5 Any definition of probability or statement of probability theorem. eg. $\frac{\#E}{\#S}$. [an earlier statement does not suffice here]

A6 Any relevant step.

Misreadings (-1)

M1 Special case: includes Construction Studies and writes $\frac{1}{4}\times\frac{1}{3}$, also sample space above with B and P, clearly indicated, without continuing.

Slips (-1)

S1 Numerical errors to a maximum of 3.

S2 Rounding $0\cdot 8\dot{8}$ to $0\cdot 88$.

S3 Each incorrect entry to Sample space to a maximum of 3.

Worthless (0)

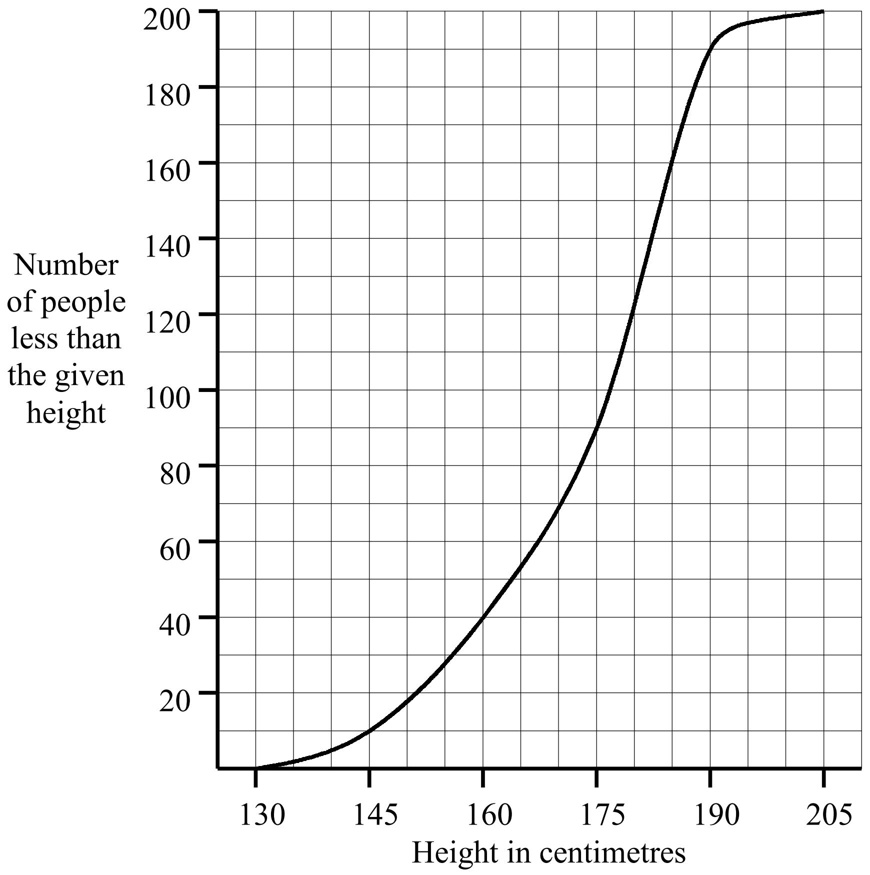
W1 Incorrect answer and no work, subject to attempts above.

QUESTION 7

Part (a)	**30 marks**	**Att 10**
Part (b)	**20 marks**	**Att 7**

Part (a)	**30 marks (10,5,10,5)**	**Att 10(3,2,3,2)**

The heights of 200 people are recorded to the nearest centimetre. The results are represented by the ogive below.



(i) Copy the cumulative frequency table below and use the ogive to complete it.

Height	<130	<145	<160	<175	<190	<205
Number of people	0					

(ii) Hence, copy and complete the following grouped frequency table:

Height	130 - 144	145 - 159	160 - 174	175 - 189	190 - 204
Number of people					

(iii) Using your grouped frequency table, and taking mid-interval values, find an estimate of the mean height.

(iv) Use the ogive to estimate the number of people who are taller than the mean.

Part (a)(i)	**10 Marks**	**Att 3**

Height	<130	<145	<160	<175	<190	<205
Number of people	0	10	40	90	190	200

* Note: Each value correct gets 2 marks subject to the attempt mark of 3.

* Accept values written down, not an a table.

Blunders(-3)
B1 Makes out a grouped frequency table or a cumulative cumulative table for this part.

Attempts (3 marks)
A1 Any one value read correctly from the ogive.
A2 Copy of table and no further work.

Part (a)(ii) **5 marks** **Att 2**

Height	130 - 144	145 - 159	160 - 174	175 - 189	190 - 204
Number of people	10	30	50	100	10

* Note: One Blunder results in attempt mark of 2, subject to marks already secured.
* Accept a table consistent with candidate's table from part (i)

Attempts (2)
A1 Adds instead of subtracts i.e. cumulating cumulatives.
A2 Any correct entry.
A3 Copies the table and stops.

Slips (-1)
S1 Numerical errors to a maximum of 3.
S2 0, 10, 30, 50, 100.

Part (a) (iii) **10 marks** **Att 3**

I

x	f	fx
137	10	1370
152	30	4560
167	50	8350
182	100	18200
197	10	1970
	200	34450

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{34450}{200} = 172 \cdot 25$$

II Mid-Intervals 137, 152, 167, 182, 197

$$\frac{(137\times10)+(152\times30)+(167\times50)+(182\times100)+(197\times10)}{10+30+50+100+10}$$

or $\frac{1370+4560+8350+18200+1970}{200}=\frac{34450}{200}$ **(**or $172\cdot25$**)**

* Accept correct or consistent answer and no work i.e. uses calculator.
* All answers (except those in B1) must be consistent with **written** mid-interval and frequency values, otherwise incorrect answer without work merits zero.
* Accept answer consistent with candidate's answer from part (ii).
* Leaves answer as $\frac{34450}{200}$ is acceptable for full marks.

Blunders(-3)

B1 Mid-interval values not used. [Taking the lower endpoint of each interval gives $\bar{x} = 165 \cdot 25$; the upper one gives: $179 \cdot 25$.]

B2 Multiplies instead of adds in denominator in (II). $[\frac{34450}{15000000}$ or $0 \cdot 023]$

B3 Adds instead of multiplies in (II). $[\frac{1035}{200}$ or $5 \cdot 175]$

B4 Uses 5 as denominator $[\frac{34450}{5}$ or $6890]$

B5 Inverts, i.e. $\frac{200}{34450}$ or $0 \cdot 0058$

B6 No frequencies, i.e. $\frac{137 + 152 + 167 + 182 + 197}{200} = \frac{835}{200}$ or $4 \cdot 175$.

B7 Omits a class, if not already penalised.

B2 + B3 $\Rightarrow \frac{1035}{15000000}$ or $0 \cdot 000069$

B4 + B6 $\Rightarrow \frac{137 + 152 + 167 + 182 + 197}{5} = \frac{835}{5}$ or 167

B1 + B6 $\Rightarrow$ e.g. $\frac{130 + 145 + 160 + 175 + 190}{200} = \frac{800}{200}$ or 4 or similar.

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 Each incorrect mid-interval value to a maximum of 3.

Attempts (3)

A1 Mean = $\frac{\sum fx}{\sum f}$ or $\frac{\sum x}{n}$ and stops.

A2 A correct mid-interval value and stops.

A3 A correct relevant multiplication and stops.

A4 $\sum f = 200$ and stops.

A5 Some relevant step, e.g finds the median or modal class.

Part (a)(iv) **5 marks** **Att 2**

80 were less tall than the mean ⇒ 120 are taller than the mean

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

* Allow a tolerance of ± 5 people. [Allow answers from 115 → 125]

Attempts (2).

A1 Indication on drawn ogive but no value stated.

A2 Reading from incorrect axis.

A3 Gets median or upper or lower quartile(s), in this part, or an answer equal to 110 or 160 without work

Slips (-1)

S1 Does not subtract from 200

Part (b) **20 marks (10, 5, 5)** **Att 7(3, 2, 2)**

(i) The mean of the following five numbers is 10. Find the standard deviation of the numbers.

7, 9, 10, 11, 13.

(ii) The mean of the following five numbers is also 10. Find the standard deviation of these numbers.

5, 7, 9, 13, 16.

(iii) What does comparing the two standard deviations tell you about the two sets of numbers?

Part (b)(i) **10 marks** **Att 3**

X	$d = \mid x - \bar{x} \mid$	d^2
7	3	9
9	1	1
10	0	0
11	1	1
13	3	9
		20

$$\sigma_1 = \sqrt{\frac{d^2}{n}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$$

* Note: Candidates may assign a frequency of k (likely 1) to **each** X-value and use $\sqrt{\frac{\sum fd^2}{\sum f}}$ .This is acceptable.

* Award the greater mark to whichever part, (i) or (ii), has the better attempt.

* Apply same scheme to parts (i) and (ii).

* Accept correct answer without work. (may have used a calculator)

Blunders (-3)

B1 No d^2 column $\Rightarrow \sqrt{\frac{8}{5}} = \sqrt{1\cdot 6}$ or $1\cdot 26$

B2 No $\sqrt{\ }$ used $\Rightarrow \frac{20}{5} = 4$ [Note: Mean deviation: $\frac{8}{5} = 1\cdot 6 \Rightarrow$ apply B1 and B2.]

B3 Denominator $\neq 5$, e.g. $\sqrt{20} = 4\cdot 47$.

B4 Mathematical error, e.g. $3^2 = 6$ *or* $(-3)^2 = -9$ (Apply once only)

B5 Inconsistent k values for frequencies, if used, i.e. k values not same for each X value.

Misreadings (-1)

M1 Any obvious misreading which does not oversimplify or change the task.

M2 Uses a mean other than 10.

Attempts (3 marks)

A1 Correct relevant formula and stops, e.g. $\sqrt{\frac{\sum fd^2}{\sum f}}$ *or* $\sqrt{\frac{d^2}{n}}$ *or* $\frac{\sum x}{n}$ *or* $\frac{\sum fx}{\sum f}$

A2 Writes d or finds one deviation and stops.

A3 Attempts to find mean of the 5 numbers, even though this was given.[must be work]

Worthless (0)

W1 Writes down 5 given numbers and no further work

Part (b)(ii) **5 marks** **Att 2**

X	$d = \lvert x-\bar{x} \rvert$	d^2
5	5	25
7	3	9
9	1	1
13	3	9
16	6	36
		20

$$\sigma_2 = \sqrt{\frac{d^2}{n}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$$

* Apply same scheme as part **(b) (i)**

* Note: If this part gets better mark [out of 10] then award that mark and give the attempt mark to the part (i), if there is one step in the right direction.

Part (b) (iii) **5 marks** **Att 2**

The values in the 2nd set are more widely dispersed about the mean than is the case in the 1st set.

* Note one blunder results in attempt mark of 2, subject to marks already secured.

* Watch for words like “Scattered” or “Dispersed” or “ spread out” or “deviation“ or “distance” or “ bunched”

* Sentences similar to “The range in the 2nd set is more spread out.” or “1st set of numbers is bunched closer than the 2nd.” merit full marks.

Attempts (2)

A1 Compares the two σ values, but says nothing about dispersion e.g. “$\sigma_2 > \sigma_1$”

A2 $\sigma_2 = 2\times \sigma_1$ and stops.

Slips (-1)

S1 Phrase reversed.

Worthless (0)

W1 Sentence similar to “The numbers in the 2nd set are bigger than those in the 1st”.

QUESTION 8

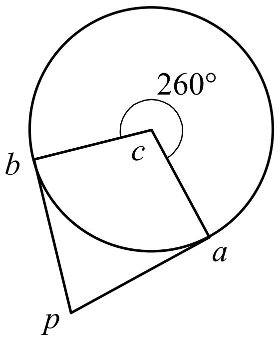
Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

In the diagram, the lines *pa* and *pb* are tangents to the circle at *a* and *b* respectively *c* is the centre of the circle.
Find

(i) $|\angle bca|$

(ii) $|\angle apb|$.



Part (a) **10 marks** **Att 3**

(i) $|\angle bca| = 360° - 260° = 100°$

(ii) $|\angle apb| = 360° - (100° + 90° + 90°) = 80°$

- * Note: One angle correct or consistent gets 7 marks, both correct gets 10 marks.
- * Angle notation or degree notation not required.
- * Accept correct answers and no work.
- * Accept angles clearly marked on a diagram.
- * Note: $|\angle bca| = 260^\circ$ merits 7 marks.

Blunders (-3)

B1 Finds the measure of only one of the required angles.

B2 $|\angle \text{apb}| = 50°$ i.e. ½ $|\angle bca|$ or 130°, i.e. ½ $|reflex\ \angle bca|$

B3 Uses an arbitrary angle, say 100°, for $|\angle pac|$ and proceeds to get $|\angle apb| = 360° - (100° + 100° + 100°) = 60°$.

B4 Angle at a point ≠ 360° and continues.

B5 Sum of angles in a quadrilateral ≠ 360° and continues.

B6 Misses one of the 90° angles to give $|\angle apb| = 360° - (100° + 90°) = 170°$.

B7 $|\angle apb| = 360° - (100° + 90° + 90°)$ and stops.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Misreadings (-1)

M1 Any obvious misreading which does not oversimplify or change the task.

Attempts (3 marks)

A1 Angle at a point = 360° and stops

A2 Sum of angles in a quadrilateral = 360° or angle sum of $\Delta = 180°$ and stops.

A3 $|\angle pac| = 90°$ and stops.

A4 Anything added to the diagram.

Worthless (0)
W1 Incorrect answer and no work shown, but note the Blunders above.

Part (b) **20 marks** **Att 7**

Prove that the degree-measure of an angle subtended at the centre of a circle by a chord is equal to twice the degree-measure of an angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

Part (b) **20 marks** **Att 7**

I

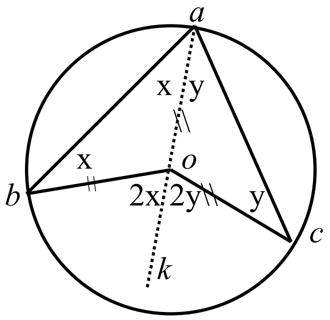


Diagram with construction	….Step i	7 marks
$\|oa\| = \|ob\|$ {= $\|oc\|$}	….Step ii (May be indicated on diagram)	8 marks
x = x or { y = y}	….Step iii	11 marks
$\|\angle bok\| = 2x$	….Step iv	14 marks
$\|\angle koc\| = 2y$	….Step v	17 marks
$\|\angle boc\| = 2\|\angle bac\|$	….Step vi	20 marks

II

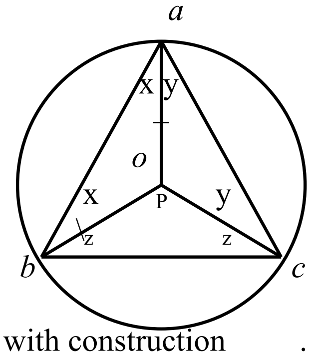


Diagram with construction	….Step i	7 marks
$\|oa\| = \|ob\|$ {= $\|oc\|$}	….Step ii (May be indicated on diagram)	8 marks
x = x or { y = y, z = z}	….Step iii	11 marks
$2x + 2y + 2z = 180°$	….Step iv	14 marks
$P = 180 - 2z$	….Step v	17 marks
$2(x + y) = 180° - 2z$ i.e. $\|\angle boc\| = 2\|\angle bac\|$	… Step vi	20 marks

- * If candidate's work is not worthless, he/she must get at least attempt 7.
- * Method I: clear diagrams without steps written (order of steps not clear), apply B3.
- * Method II: clear diagram without steps written, award 11 marks.

Blunders (-3)
B1 Incorrect step or part of step or a step omitted.
B2 All angles marked the same. Penalise once.
B3 Steps in incorrect order.

Attempts (7marks)
A1 Outline diagram and stops. (circle with angle)
A2 Only one step or part of a step and stops.
A3 Special case using degrees or proves angle in a semi-circle, avoiding exterior angles.
A4 A memorised proof, without a diagram, is an attempt if it can be reconciled with a diagram.

Part (c) 20 marks (10,5,5,) Att 7 (3,2,2,)

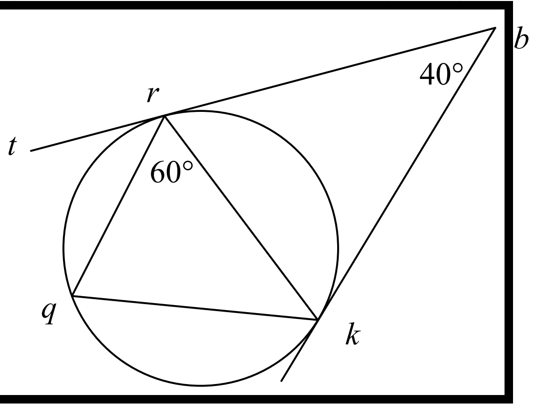
In the diagram, *br* and *bk* are tangents to the circle at *r* and *k* respectively. *q* is a point on the circle and *t* is a point on *br*, as shown.

Find

(i) $|\angle bkr|$

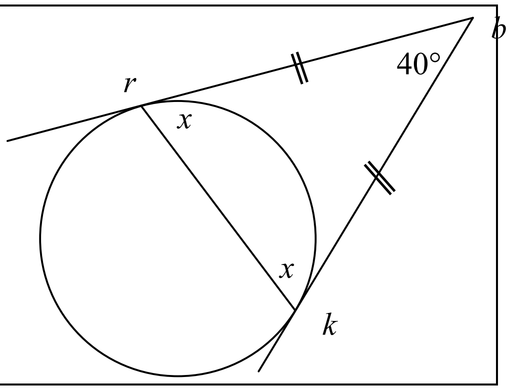
(ii) $|\angle qrt|$

(iii) $|\angle qkr|$



(c)(i) 10 marks Att 3

$\|br\| = \|bk\|$	…step i	3 marks
$\Rightarrow \|\angle bkr\| = \|\angle brk\|$	…step ii	4 marks
$2x + 40° = 180°$	…step iii	7 marks
$2x = 140°$		
$x = 70°$	…step iv	10 marks
$\|\angle bkr\| = 70°$		



* Accept angles correctly marked on diagram.
* Accept correct answer and no work.

Blunders (-3)
B1 Incorrect step or part of a step or step omitted.
B2 Leaves as 2x + 40° = 180° or 2x = 140°
B3 Incorrect transposition.

Slips (-1)
S1 Each numerical error to a maximum of 3.

Misreadings (-1)
M1 Any obvious misreading which does not oversimplify or change the task.

Attempts (3 marks)

A1 $|\angle bkr| = 90°$, i.e. assumes [*rk*] is diameter.

A2 States $\triangle$ *brk* is isosceles and stops.

A3 Some relevant step.

Worthless (0)

W1 Diagram reproduced without modification.

W2 Incorrect answer and no work

(c) (ii) **5 marks** **Att 2**

$|\angle qrt| + 60° + 70° = 180° \Rightarrow |\angle qrt| = 180° - 130° \Rightarrow |\angle qrt| = 50°$

* Note: One blunder results in an attempt mark of 2, subject to marks already secured.

* Accept angles clearly marked on a diagram.

* Accept correct or consistent answer and no work.

Attempts (2 marks)

A1 Transposition error as part of meaningful work.

A2 Leaves as $|\angle qrt| + 60° + 70° = 180°$ or $|\angle qrt| = 180° - 130°$

A3 Straight line angle ≠ 180°.

A4 Any relevant step.

Misreadings (-1)

M1 Obvious misreading which does not oversimplify or change the task.

Worthless (0)

W1 Incorrect answer and no work.

W2 Angle measured with a protractor.

(c)(iii) **5 marks** **Att 2**

I $|\angle qkr| = 50°$… Angle in alternate segment.

II $|\angle \text{kqr}| = 70° \Rightarrow |\angle \text{qkr}| = 180° - (60° + 70°) = 50°$

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

* Accept any answer = (c)(ii) without work

Attempts(2 marks*)*

A1 Transposition error in method II, as part of meaningful work.

A2 Straight line angle ≠ 180° in method II.

A3 Mentions angle in alternate segment.

A4 Draws diagram with angles marked equal, but measure not given.

A5 Any mention of straight angle or 180°.

A6 $|\angle \text{kqr}| = 70°$ and stops.

Slips (-1)

S1 Each numerical error to a maximum of 3.

Worthless (0)

W1 Incorrect answer and no work.

QUESTION 9

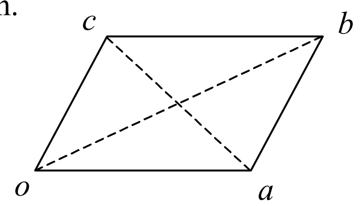
Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10(5,5) marks** **Att 4(2,2)**

The diagram shows a parallelogram *oabc*, where *o* is the origin.

(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\overrightarrow{ac}$ in terms of $\vec{a}$ and $\vec{c}$



Part (a)(i) **5 marks** **Att 2**

$\vec{b} = \overrightarrow{oa} + \overrightarrow{ab} = \vec{a} + \vec{c}$ or $\vec{b} = \overrightarrow{oc} + \overrightarrow{cb} = \vec{c} + \vec{a}$

* Accept correct answer without work.

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

* Note: Allow $\overrightarrow{oa}$ for $\vec{a}$ and $\overrightarrow{oc}$ for $\vec{c}$ in parts (i) and (ii)

* Note: Arrows not required.

Attempts (2)

A1 Incorrect direction.

A2 Error in using triangle or parallelogram law e.g. $\vec{a}$ - $\vec{c}$

A3 Does not simplify to $\vec{a}$ and $\vec{c}$.

A4 Correct relevant step e.g. relevant arrow added to given diagram

A5 Correct relevant application of vectors, e.g. $\overrightarrow{ab} = \vec{c}$.

A6 Relevant statement e.g. "opposite sides of parallelogram are equal in measure".

Misreadings (-1)

M1 Any obvious misread which does not oversimplify or change the task.

Worthless (0)

W1 Diagram reproduced without modifications.

Part (a)(ii) **5 marks** **Att 2**

$\overrightarrow{ac} = \overrightarrow{oc} - \overrightarrow{oa} = \vec{c} - \vec{a}$ or $\overrightarrow{ac} = \overrightarrow{ab} + \overrightarrow{bc} = \vec{c} - \vec{a}$

* Accept correct answer and no work shown.

* Note: One Blunder results in an attempt mark of 2, subject to marks already secured.

Attempts (2)

A1 Incorrect direction.

A2 Error in using triangle or parallelogram law e.g. $\vec{a}$ - $\vec{c}$

A3 Does not simplify to $\vec{a}$ and $\vec{c}$.

A4 Correct relevant step e.g. relevant arrow added to given diagram

A5 Correct relevant application of vectors, e.g. $\vec{ab} = \vec{c}$.

A6 Relevant statement e.g. “opposite sides of parallelogram are equal in measure”.

Slips (-1)

S1 $\vec{ac} = \vec{ao} + \vec{c}$

Worthless (0)

W1 Diagram reproduced with no modifications.

Part (b) (i) 20 marks (10, 5, 5) Att 7 (3, 2, 2)

Let $\vec{p} = \vec{i} - 3\vec{j}$ and $\vec{q} = -2\vec{i} + 4\vec{j}$.

(i) Express $3\vec{q} - 2\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Calculate $\left|3\vec{q} - 2\vec{p}\right|$, correct to one decimal place.

(iii) Express $\vec{qp}$ in terms of $\vec{i}$ and $\vec{j}$.

Part (b) (i) 10 marks Att 3

$3\vec{q} - 2\vec{p} = \mathbf{3}(-2\vec{i} + 4\vec{j}) - \mathbf{2}(\vec{i} - 3\vec{j}) = (-6\vec{i} + 12\vec{j}) - (2\vec{i} - 6\vec{j}) = -8\vec{i} + 18\vec{j}$

4 marks **7 marks** **10 marks**

Blunders(-3)

B1 Mixes up $\vec{i}$’s and $\vec{j}$’s.

B2 Algebraic error, e.g. $-8\vec{i}^{\,2} + 18\vec{j}^{\,2}$ or a sign error.

B3 Distributive law error, e.g. $3(-2\vec{i} + 4\vec{j}) = -6\vec{i} + 4\vec{j}$.

Misreading (-1)

M1 Reads as $3\vec{p} - 2\vec{q}$ and continues. (Answer: $(7\vec{i} - 17\vec{j})$)

Attempts (3)

A1 $-8\vec{i}$ or $18\vec{j}$ and stops.

A2 $-6\vec{i} + 12\vec{j}$ or $-2\vec{i} + 6\vec{j}$ and stops.

A3 Plots one or more relevant vectors

A4 Some relevant step.

Slips(-1)

S1 Numerical errors to a maximum of 3.

Part (b)(ii) **5 marks** **Att2**

$$\left|3\vec{q}-2\vec{p}\right| = \sqrt{x^2+y^2} = \sqrt{(-8)^2+(18)^2} = \sqrt{64+324} = \sqrt{388} = 19\cdot 69 = 19\cdot 7$$

2 m **4 m** **5 m**

- * Accept candidate's answer from (b) (i), without work.
- * Accept correct use of Pythagoras or the coordinate geometry length formula.
- * Note: One Blunder results in an attempt mark of 2, subject to marks already secured.
- * Apply coordinate geometry principles if candidate uses length of line segment formula.

Slips (-1)

S1 Leaves answer as $19\cdot 69$ or $\sqrt{388}$

S2 Each numerical error to a maximum of 3.

Attempts(2 marks)

A1 Incorrect relevant formula e.g. $\sqrt{x^2-y^2}$ $or\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$.

A2 Mathematical error, e.g. $(-8)^2 = -64$ or $(18)^2 = 36$

A3 Leaves answer as $\sqrt{(-8)^2+(18)^2}$.

A4 Correct formula and stops

A5 $(-8)^2 = 64$ or $(18)^2 = 324$ and stops.

A6 Some relevant step.

Misreadings (-1)

M1 Any obvious misreading which does not oversimplify or change the task.

Part(b) (iii) **5 marks** **Att 2**

$$\vec{qp} = \vec{p} - \vec{q} = (\vec{i}-3\vec{j})-(-2\vec{i}+4\vec{j}) = \vec{i}-3\vec{j} + 2\vec{i}-4\vec{j} = 3\vec{i}-7\vec{j}$$

- * Accept correct answer without work
- * Note: One Blunder results in attempt mark of 2, subject to marks already secured.

Attempts (2)

A1 Mixes up $\vec{i}'s$ and $\vec{j}'s$.

A2 Distributive law error, e.g. $-(-2\vec{i} + 4\vec{j}) = -2\vec{i} + 4\vec{j}$.

A3 $\vec{qp} = \vec{p} + \vec{q}$ or $\vec{q} - \vec{p}$

A4 Leaves answer as $\vec{i}-3\vec{j} + 2\vec{i}-4\vec{j}$

A5 $3\vec{i}$ or $-7\vec{j}$ and stops.

A6 Plots $\vec{p}$ and/or $\vec{q}$.

A7 Treats as $\vec{q}\,.\,\vec{p}$ with some correct work.

Misreadings (-1)

M1 $\vec{pq}$ and continues

M2 Any obvious misreading which does not oversimplify or change the task.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Part(c) **20 marks(10,10)** **Att 6(3,3)**

Let $\vec{x} = 8\vec{i} + 6\vec{j}$ and $\vec{y} = 8\vec{i} - 15\vec{j}$.

(i) Find $\vec{x}.\vec{y}$, the dot product of $\vec{x}$ and $\vec{y}$.

(ii) Hence, find the measure of the angle between $\vec{x}$ and $\vec{y}$, correct to the nearest degree.

Part (c)(i) **10 marks** **Att 3**

I	II
$\vec{x}.\vec{y} = (8\vec{i}+6\vec{j}).\,(8\vec{i}-15\vec{j})$	Using Cosine formula $\cos\theta = -\frac{26}{170}$
$= 64\vec{i}^{\,2}$ - $72\,\vec{i}.\vec{j} - 90\,\vec{j}^{\,2}$	$\vec{x}.\vec{y} = \lvert\vec{x}\rvert\,\lvert\vec{y}\rvert\cos\theta$
$= 64 - 90$	$= (10)(17)\left(-\frac{26}{170}\right)$
$= -26$	$= -26$

* Allow correct answer without work.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\vec{x}.\vec{y} = \lvert\vec{x}\rvert\lvert\vec{y}\rvert\sin\theta$ *or* $\lvert\vec{x}\rvert = \sqrt{a^2 - b^2}$

B2 $\vec{i}^{\,2} \neq 1$ *and / or* $\vec{j}^{\,2} \neq 1$

B3 $\vec{i}\,.\,\vec{j} \neq 0$

B4 Leaves as $(10)(17)\left(-\frac{26}{170}\right)$.

Slips (-1)

S1 Each numerical slip to a maximum of 3

Attempts (3 marks)

A1 Correct formula and stops

A2 Some correct work in multiplication e.g. $64\,\vec{i}^{\,2}$ *or* $90\,\vec{j}^{\,2}$

A3 $\vec{i}^{\,2} = 1$ *and / or* $\vec{j}^{\,2} = 1$ *and / or* $\vec{i}\,.\,\vec{j} = 0$ and stops

A4 $\lvert\vec{x}\rvert = 10$ and / or $\lvert\vec{y}\rvert = 17$ and stops.

Part(c) (ii) 10 marks Att 3

I	II
$\vec{x}.\vec{y} = \lvert\vec{x}\rvert\lvert\vec{y}\rvert\cos\theta$ $-26 = 10 \times 17 \cos\theta$ $\theta = \cos^{-1}\left(\frac{-26}{170}\right) = 99^\circ$	$\cos\theta = \frac{\vec{x}.\vec{y}}{\lvert\vec{x}\rvert\lvert\vec{y}\rvert}$ $\cos\theta = \frac{-26}{(10)(17)} = \frac{-26}{170} \Rightarrow \theta = \cos^{-1}\left(\frac{-26}{170}\right) = 99^\circ$

Blunders (-3)

B1 Incorrect relevant formula e.g. $\vec{x}.\vec{y} = |\vec{x}||\vec{y}|\sin\theta$ *or* $|\vec{x}| = \sqrt{a^2 - b^2}$, and continues.
B2 Uses trigonometric method or coordinate geometry incorrectly and continues.
B3 Transposing error
B4 Leaves as $\cos\theta = -0\cdot1529$
B5 Incorrect function read e.g. $\cos\theta = 0\cdot1529$

Slips (-1)
S1 Each numerical slip to a maximum of 3
S2 Failure to round off to the nearest degree.

Attempts (3 marks)
A1 Correct relevant formula and stops
A2 $|\vec{x}| = 10$ and / or $|\vec{y}| = 17$ and stops.
A3 $\vec{x}.\vec{y} = |\vec{x}||\vec{y}|$.
A4 Slope of ox and/or oy
A5 Plots $\vec{x}$ and / or $\vec{y}$
A6 Correct answer without work.

Worthless (0 marks)
W1 Incorrect trigonometric or coordinate geometry method to find θ

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Expand $(1+x)^6$ fully.

Part (a) **10 marks** **Att 3**

$$(1+x)^6 = \binom{6}{0} + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6$$ **7 m**

$$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$ **10 m**

* Accept long multiplication or Pascal's triangle.
* Accept $(x+1)^6$ expanded correctly.
* Accept answer without work.
* Note: 1 or 2 terms correct → 3 marks: 3 to 6 correct → 7 marks: 7 correct → 10 marks.

Blunders (-3)
B1 Error in powers (once only).
B2 Error in working out binomial coefficients (apply once), subject to attempt.
B3 Puts powers of x as denominators e.g. $\binom{6}{2}\left(\frac{x}{2}\right)$ or $\frac{6}{2}x^2$ (apply once).
B4 Puts a + sign between coefficient and power of x e.g. $\binom{6}{2} + x^2$ (apply once).
B5 Does not work out binomial coefficients, (once).

Slips (-1)
S1 Numerical errors to a maximum of 3.

Misreadings (-1)
M1 $(1-x)^6$ or $(x-1)^6$ and continues correctly.

Attempts (3 marks)
A1 Any term written correctly or part of Pascal's Triangle or coefficients only.
A2 Any step towards getting a binomial coefficient e.g. $\binom{6}{2}$.
A3 Any correct step towards long multiplication.

Part (b) **20 marks (15,5)** **Att 7(5,2)**

(i) Find the sum to infinity of the geometric series:

$$\frac{2}{5} + \frac{2}{50} + \frac{2}{500} + \ldots$$

(ii) Use your result from part **(i)** to express the recurring decimal 1.444…
as a fraction, (that is, in the form $\frac{a}{b}$, where $a, b \in \mathbf{N}$).

Part (b)(i) **15 marks** **Att 5**

I	II	
$a = \frac{2}{5},\ r = \frac{1}{10}$	Let $x = 0\cdot4 + 0\cdot04 + 0\cdot004...$	5 marks
$S_\infty = \frac{a}{1-r}$	$x = 0\cdot4444...$	6 marks
$= \frac{\frac{2}{5}}{1 - \frac{1}{10}}$	$10x = 4\cdot4444...$	9 marks
$= \frac{\frac{2}{5}}{\frac{9}{10}}$	$9x = 4$	12 marks
$= \frac{4}{9}$	$x = \frac{4}{9}$	15 marks

* Accept finding S_n and then $Lim_{n\to\infty}\ S_n$.
* Apply scheme for I if $\frac{2}{5}\left(1 + \frac{1}{10} + \frac{1}{100} + ...\right)$ is used
* Note: Candidate may use decimal values. $0\cdot4 + 0\cdot04 + 0\cdot004 + ...$ to give $\frac{0\cdot4}{1-0\cdot1}$ etc.

Blunders (-3)

B1 Incorrect a, unless an obvious slip.
B2 Incorrect r, unless an obvious slip.
B3 Incorrect Relevant formula e.g. $\frac{a}{1+r}$ or a correct relevant formula misused.
B4 Misplaced decimal point.
B5 Omits last step.
B6 Mathematical error in dealing with fractions.
B7 Transposition error.
B(-3) × 2 Calculates S_n and stops.

Slips (-1)

S1 Each numerical error to a max of 3.

Attempts (5 marks)

A1 Correct relevant formula and stops.

A2 Correct a and/or r and stops.

A3 $T_n = a\,r^{n-1}$ *or* $\frac{2}{5}\left(\frac{1}{10}\right)^{n-1}$

A4 Adds 2 or more given terms e.g. $\frac{2}{5} + \frac{2}{50} + \frac{2}{500} = \frac{111}{250}$

A5 One correct step in adding given fractions.

A6 Treats as an Arithmetic series, with further work.

A7 Attempt at expressing $1 \cdot 444$ *as fractions e.g.* $\frac{4}{10}$ *or* $\frac{4}{100}$

A8 Correct answer without work.

A9 Some relevant step.

Worthless (0)

W1 Formula for AP and stops.

W2 $\frac{2}{5} + \frac{2}{50} + \frac{2}{500} = \frac{6}{555}$

Part(b) (ii) **5 marks** **Att 2**

$$1\cdot\dot{4} = 1 + \frac{4}{10} + \frac{4}{100} + \ldots = 1 + \frac{4}{9} = \frac{13}{9}$$

- * Accept correct answer and no work.
- * Accept candidate's answer from part (i).
- * Note: One Blunder results in an attempt mark of 2, subject to marks already secured.
- * Apply blunders as in previous part, if not already applied.
- * Use of method II in part (b) (i) merits attempt mark here.
- * Accept 1 + answer from b(i) completed, with or without work for full marks.

Attempts (2*)*

A1 Writes $1 + 0 \cdot 444...$

A2 Any relevant step.

Part (c)	**20 marks (10,10)**	**Att 6 (3,3)**

(i) €500 is invested at 4% per annum compound interest.
Find the value of the investment after ten years.

(ii) A person invests €500 in an account at the beginning of each year for ten consecutive years. Compound interest is added to the account at the rate of 4% per annum. By treating the values of the ten investments as terms in a geometric series, find the total amount in the account at the end of the tenth year.

Part(c)(i)	**10 marks**	**Att 3**

I	*II*	
$A = P\left(1+\frac{r}{100}\right)^n$	$A_1 = 500(1.04) = €520$	3 *marks*
$A = 500\left(1+\frac{4}{100}\right)^{10}$	$A_{10} = 500(1.04)^{10}$	7 *marks*
$A = €740\cdot12$	$A_{10} = €740.12$	10 *marks*

* Accept long method of working from year to year.

* Note: Long method here and terms not added merits 10 m here and 7 m in (c) (ii)

Blunders (-3)

B1 Incorrect *r*.

B2 Decimal error.

B3 $€500(1\cdot04)^{10}$ and stops.

B4 Serious numerical error e.g. $(1\cdot04)^{10} = 10\cdot4$

B5 Subtracts in long method.

B6 Sign error in formula

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 No rounding to nearest cent at end.

S3 Premature rounding that affects the final amount.

S4 Each year omitted in long method, subject to the attempt mark.

Misreadings(-1)

M1 Any obvious misreading which does not oversimplify or change the task.

Attempts (3 marks)

A1 Mention of $1\cdot04$ *or* $\frac{4}{100}$.

A2 4 % of €500 = €20 and stops.

A3 Correct Compound Interest formula or S_n for a GP and stops

A4 Correct answer without work.

Worthless (0)

W1 $\frac{500}{4} = 125$

Part(c) (ii) **10 marks** **Att 3**

$$500(1\cdot04)+500(1\cdot04)^2+500(1\cdot04)^3+.....+500(1\cdot04)^{10}$$

$$500\left(1\cdot04+1\cdot04^2+1\cdot04^3+...+1\cdot04^{10}\right)$$

$$a=1\cdot04,\quad r=1\cdot04,\quad n=10$$

$$S_n=\left(\frac{a\left(r^n-1\right)}{r-1}\right)$$ 3 marks

$$=500\left(\frac{a\left(r^n-1\right)}{r-1}\right)$$ 4 marks

$$=500\left(\frac{1\cdot04\left(1\cdot04^{10}-1\right)}{1\cdot04-1}\right)$$ 7 marks

$= €6243.18$ 10 marks

- * Accept correct long method
- * If long method (year by year) is used in part (i), it gets 7 marks here also, if terms not added. i.e.B5

Blunders (-3)

B1 Incorrect a, unless an obvious slip.
B2 Incorrect r, unless an obvious slip.
B3 Error in relevant formula. S_n for GP or Compound interest formula.
B4 Index error.
B5 Failure to add terms in long method.
B6 Decimal error
B7 Serious mathematical error e.g. $(1\cdot04)^{10}=10\cdot4$.

Misreadings (-1)

M1 Obvious misreading which does not oversimplify or change the task.

Attempts (3 marks)

A1 Correct or relevant formula and stops.
A2 Mention of $1\cdot04,\ or\ \ 0\cdot04\ \ or\ \ \frac{4}{100}$
A3 Any relevant step.

QUESTION 11

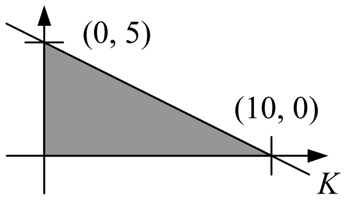
Part (a)	**15 marks**	**Att 5**
Part (b)	**35 marks**	**Att 14**

Part (a) **15 marks (10, 5)** **Att 5(3,2)**

The line *K* cuts the *x*-axis at (10, 0) and the *y*-axis at (0, 5).

(i) Find the equation of *K*.

(ii) Write down the three inequalities that together define the region enclosed by *K*, the *x*-axis and the *y*-axis.



Part (a)(i) **10 marks** **Att 3**

I

Slope of K = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{0-10} = -\frac{5}{10} = -\frac{1}{2}$ 3m

$y - y_1 = m(x - x_1)$ 7m

$y - 0 = -\frac{1}{2}(x - 10)$ or $y - 5 = -\frac{1}{2}(x - 0)$ or $x + 2y = 10$ 10m

II

Slope of K = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{0-10} = -\frac{5}{10} = -\frac{1}{2}$ 3m

$y = mx + c$ 7m

$5 = -\frac{1}{2}(0) + c \Rightarrow c = 5$ 10m

* Accept y - 0 = -½(x –10) without work.

* Apply scheme for Q2 ,Q3 where relevant.

Blunders (– 3)

B1 Incorrect relevant formula and continues.

B2 Mixes up x's and y's.

B3 Mathematical blunder, e.g. 5-0 / 0-10 = ½

B4 y – 0 = m(x – 10) where m is not equal to - ½ without work.

B5 $y - y_1 =$ - ½(x – x_1) where (x_1, y_1) is not (0,5) or (10,0) without work.

B6 Transposing error especially in method II. Penalise once.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Attempts (3 marks)

A1 Any correct, relevant formula and stops, or any incorrect line with correct slope without work.

Part (a)(ii) **5 marks** **Att 2**

$x \geq 0$:	$y \geq 0$:	$x + 2y - 10 \leq 0$ or equivalent
2 m	4 m	5 m

- * Note: One blunder results in attempt mark of 2, subject to marks already secured.
- * Accept correct inequalities without work.
- * Accept use of < and/or >. [i.e. omits " = " sign.] Refers solely to omission of " = " sign. Incorrect direction of inequality should be penalised. See A1, subject to marks already secured.
- * Accept inequality consistent with candidate's equation for K, if already penalised.
- * Note: One incorrect or not stated is a slip.

Attempts (2 marks)

A1 Incorrect direction of inequality sign.
A2 Mathematical error in testing a point (e.g. sign error)
A3 Incorrect or no conclusion , e.g. $x + 2y - 10 = 0 \Rightarrow 0 + 2(0) - 10 = 0$
A4 Transposing error.
A5 One correct inequality written down and stops.
A6 Substitutes any point and stops.
A7 Some correct work at simplifying K.
A8 Some relevant step.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Part (b) **35 marks (20,10,5)** **Att 14(8,4,2)**

A developer is planning a scheme of holiday homes, consisting of large and small bungalows. Each large bungalow will accommodate 8 people and each small bungalow will accommodate 6 people. The development is not permitted to accommodate more than 216 people. The floor area of each large bungalow is 200 m^2 and the floor area of each small bungalow is 100 m^2. The total floor area of all the bungalows must not exceed 4000 m^2.

(i) Taking x as the number of large bungalows and y as the number of small bungalows, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The expected net annual income from each large bungalow is €14 000 and from each small bungalow is €8000. How many of each type should be built in order to maximise the total expected net annual income?

(iii) The developer decides to build as indicated in part **(ii)**. The cost of building each large bungalow is €110 000 and the cost of building each small bungalow is €85 000. The total cost of the development is equal to the building costs plus €1.58 million. How many years will it take to recoup the total cost of the development?

Part (b)(i) Inequalities **10 marks (5,5)** **Att 4(2,2)**

I

People: $8x + 6y \leq 216$

Floor Area: $200x + 100y \leq 4000$

II

	Large	Small	Maximum
People	$8x$	$6y$	216
Floor Area	$200x$	$100y$	4000

- * Accept correct multiples / fractions of inequalities or different letters.
- * Do not penalise here for incorrect or no inequality sign. Penalise in graph if used.
- * Case : 8 6 216 | Award 10m , but penalise in graph if linkup is incorrect.
  200 100 4000 |

Blunders

B1 Mixes up x's and y's (once if consistent error).

B2 Confuses rows and columns, e.g. $8x + 200y \leq 216$ (once if consistent)

B3 Misplaced decimal point, e.g. $2x + y \leq 4$

Attempts (2 marks for each inequality)

A1 Incomplete relevant data in table and stops (each inequality)

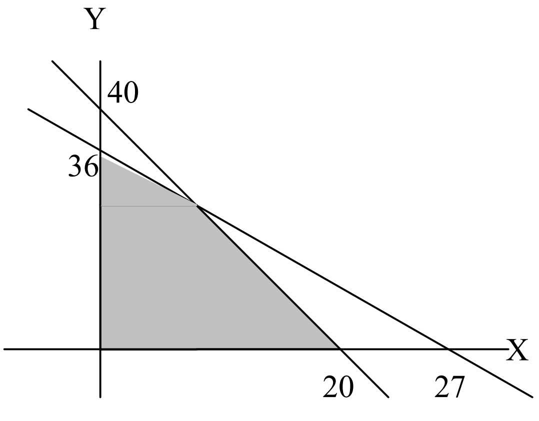
A2 Any other correct inequality, e.g. $x \geq 0$, $y \geq 0$, (each time).

A3 Some variable ≤ 216 or ≤ 4000 (each time).

A4 $8x$ and / or $6y$ and stops (1 x Att 2).

A5 $200x$ and / or $100y$ and stops (1 x Att 2).

Part (b)(i) graph **10 marks (5,5)** **Att 4 (2,2)**



- * Each half-plane merits 5 marks, attempt 2 marks.
- * Points or scales required.
- * Half-planes required but no penalty for not indicating intersection if half-planes indicated. If half-planes are indicated correctly, do not penalise for incorrect shading.
- * Accept correct shading of intersection for half-planes, but candidates may shade out areas that are not required and leave intersection blank.
- * Correct shading over-rules arrows.

* Cases: Two lines drawn **and no shading**, only one of the following applies.

Case 1:	Both sets of arrows in expected direction	10 marks
Case 2:	Both sets of arrows in unexpected direction	10 marks
Case 3:	One set of arrows " correct" and the other "incorrect"	7 marks (5 + Att 2)
Case 4:	One line with and the other without arrows	7 marks (5 + att 2)
Case 5:	No arrows	4 marks (Att 2, Att 2)

Blunders (-3)

B1 No half-plane indicated (each time)

B2 Blunder in plotting a line or calculations (each line)

B3 Incorrect shading (once) , e.g. one or both of the small triangles shaded.

Attempts (2 marks each half-plane)

A1 Some relevant work towards a point on a line, i.e. 2 m for each line attempted

A2 Draws axes or axes and one line (award 1 x Att 2 m).

A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2).

(b) (ii) Intersection of lines **5 marks** **Att 2**

8x + 6y = 216	or	4x + 3y = 108	y = 28
200x + 100y = 4000		4x + 2y = 80	x = 6

* Accept candidate's own equations from previous parts.

* If *y* is calculated, accept consistent value for *x* without further work and vice versa .

* Note : One blunder results in attempt mark of 2, subject to marks already secured.

Attempts (2)

A1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.

A2 Sign error.

A3 *x* or *y* value only.

A4 Transposing error.

A5 Correct or consistent answer without work or from a graph.
[Should get same values from graph as if they had been found algebraically]

A6 Any relevant step towards solving equations.

Slips (-1)

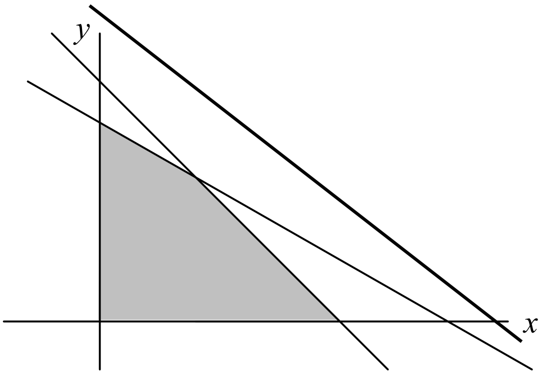
S1 Each arithmetical slip to a maximum of 3.

Worthless (0 marks)

W1 Incorrect answer without work and inconsistent with graph.

(b)(ii) Income **5 marks** **Att 2**

I



(6, 28) (⇒ 84000 + 224000 = 308000)
⇒ 6 large and 28 small bungalows

II

Step 1 Vertices	14000x + 8000 y	Income
Step 2 (0,36)	0 + 288000	288000
Step 3 (6,28)	84000 + 224000	308000
Step 4 (20,0)	280000 + 0	280000

Step 5: 6 large and 28 small bungalows

* Accept point of intersection from previous part.
* Information does not have to be in table form.
* Award 1 mark for each consistent step, subject to the attempt mark
* Accept only vertices consistent with previously accepted work, not arbitrary ones. If (0,40) is tested and result is used to give max. income, apply -1. Otherwise ignore
* Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
* Accept any correct multiple or fraction of 14 000x + 8 000 y in this part of (b) (ii).
* If no marks have been awarded for intersection of lines and this point is written here, award Att 2 for the previous part and also reward it here if the step is correct.
* Answer must be explicit, e.g. award 4 marks if step 3 is indicated but step 5 not written.
* Testing **only** (6,28) to get 308000 merits Att 2 for this part of (ii) even if the candidate writes 6 large and 28 small bungalows. [method II]
* No comparison means the attempt mark , at most.

Slips (-1)
S1 Each arithmetic slip to a maximum of 3.
S2 Each step of the solution omitted, subject to the attempt mark
[Step 1 may be implied].

Attempts (2 marks)
A1 Any relevant work involving x or y and / or 14 000, 8 000 or similar.
A2 Any attempt at substituting coordinates into some expression.
A3 States 6 large and / or 28 small bungalows with no other work.

Part(b) (iii) **5 marks** **Att 2**

$$\text{Total cost} = 6 \times €110000 + 28 \times €85000 + €1580000 = €4620000$$

$$\text{Number of Years} = \frac{4620000}{308000} = 15$$

* Accept correct answer without work if consistent with previous work.

* Note: One Blunder results in Attempt mark of 2, subject to marks already secured.

Slips (-1)

S1 Numerical slips to a maximum of 3

Attempts(2 marks)

A1 Mathematical error e.g. 3040000 - 1580000 instead of adding
A2 Fails to add in 1580000
A3 Decimal error
A4 Inverts the final fraction e.g. 308000 / 4620 000
A5 Any work with 110 000 or 85 000 and / or 6 , 28
A6 Some work in division trying to find the no. of years
A7 Writes down 1 580 000
A8 Some relevant step.

Worthless (0)

W1 Simply writing down € 110 000 or € 85 000 or € 1.58 million and no other work.