M 28

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LEAVING CERTIFICATE EXAMINATION, 2002

MATHEMATICS — ORDINARY LEVEL

PAPER 2 (300 marks)

MONDAY, 10 JUNE — MORNING, 9.30 to 12.00

Attempt **FIVE** questions from **Section A** and **ONE** question from Section B.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

SECTION A

Attempt FIVE questions from this section.

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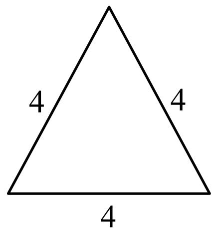
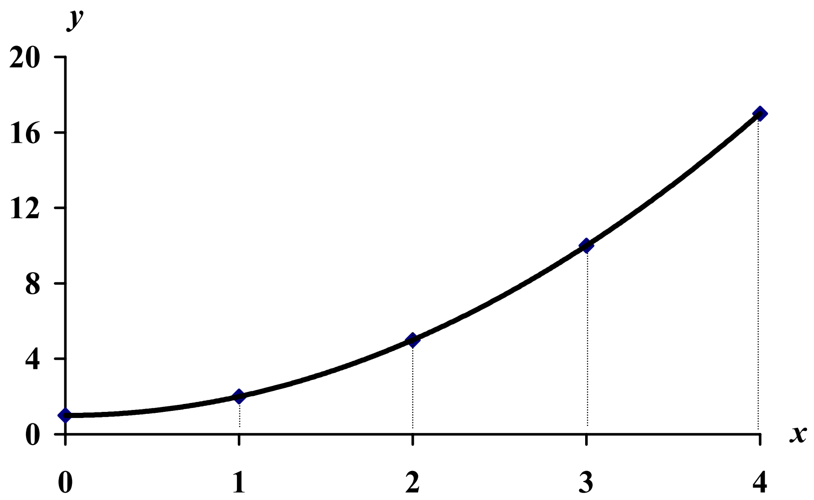
1. **(a)** Each side of an equilateral triangle measures 4 units.
Calculate the area of the triangle, giving your answer in surd form.

Note: Area of a triangle = $\frac{1}{2}ab\sin C$.



(b) The diagram shows the curve $y = x^2 + 1$ in the domain $0 \le x \le 4$.

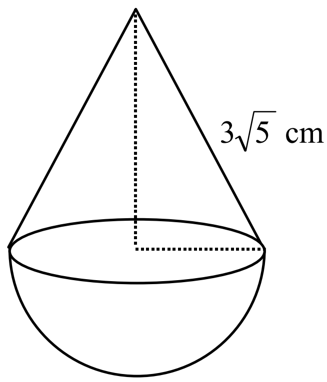


(i) Copy the following table. Then, complete it using the equation of the curve:

x	0	1	2	3	4
y					

(ii) Hence, use Simpson's Rule to estimate the area between the curve and the x-axis.

(c) A solid is in the shape of a hemisphere surmounted by a cone, as in the diagram.



(i) The volume of the hemisphere is 18π cm^3.
Find the radius of the hemisphere.

(ii) The slant height of the cone is $3\sqrt{5}$ cm.
Show that the vertical height of the cone is 6 cm.

(iii) Show that the volume of the cone equals the volume of the hemisphere.

(iv) This solid is melted down and recast in the shape of a solid cylinder. The height of the cylinder is 9 cm. Calculate its radius.

2. **(a)** Find the co-ordinates of the point of intersection of the line $4x + y = 5$ and the line $3x - 2y = 12$.

(b) The line L has equation $4x - 5y = -40$.
$a(0,8)$ and $b(-10,0)$ are two points.

(i) Verify that a and b lie on L.

(ii) What is the slope of L?

(iii) The line K is perpendicular to L and it contains b. Find the equation of K.

(iv) K intersects the y-axis at the point c. Find the co-ordinates of c.

(v) d is another point such that $abcd$ is a rectangle. Calculate the area of $abcd$.

(vi) Find the co-ordinates of d.

3. **(a)** Write down the co-ordinates of any three points that lie on the circle with equation $x^2 + y^2 = 100$.

(b) The circle C has equation $(x-2)^2 + (y+1)^2 = 8$.

(i) Find the co-ordinates of the two points at which C cuts the y-axis.

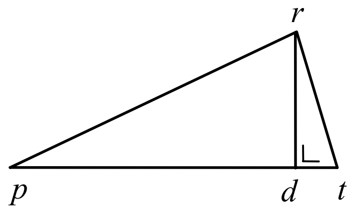
(ii) Find the equation of the tangent to C at the point $(4, 1)$.

(c) $a(-5,1)$, $b(3,7)$ and $c(9,-1)$ are three points.

(i) Show that the triangle abc is right-angled.

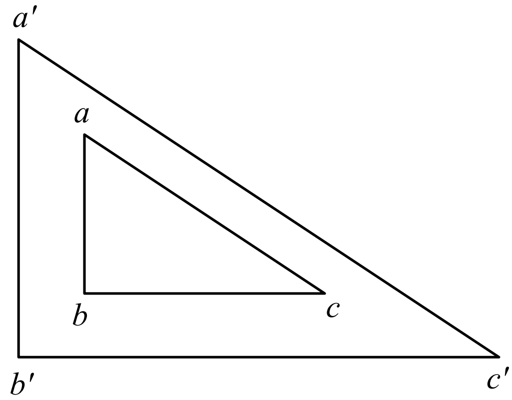
(ii) Hence, find the centre of the circle that passes through a, b and c and write down the equation of the circle.

4. **(a)** The area of the triangle rpt is 30 cm^2.
rd is perpendicular to pt.
Given that $|pt| = 12$ cm, calculate $|rd|$.



(b) Prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.

(c) The triangle $a'b'c'$ is the image of the triangle abc under an enlargement.



(i) Find, by measurement, the scale factor of the enlargement.

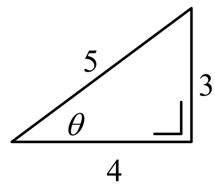
(ii) Copy the diagram and show how to find the centre of the enlargement.

(iii) Units are chosen so that $|bc| = 8$ units.
How many of these units is $|b'c'|$?

(iv) Find the area of triangle abc, given that the area of $a'b'c'$ is 84 square units.

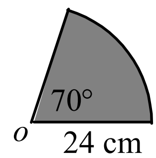
5. **(a)** Use the information given in the diagram to show that

$$\sin\theta + \cos\theta > \tan\theta.$$

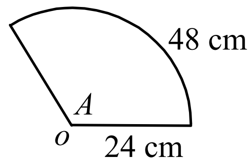


(b) A circle has radius 24 cm and centre o.

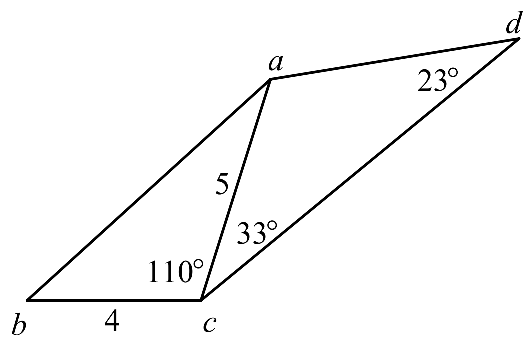
(i) Calculate the area of a sector which has $70°$ at o.
Take $\pi = \dfrac{22}{7}$.



(ii) An arc of length 48 cm subtends an angle A at o.
Calculate A, correct to the nearest degree.



(c) In the quadrilateral $abcd$, $|ac| = 5$ units, $|bc| = 4$ units, $|\angle bca| = 110°$, $|\angle acd| = 33°$ and $|\angle cda| = 23°$.



(i) Calculate $|ab|$, correct to two decimal places.

(ii) Calculate $|cd|$, correct to two decimal places.

6. **(a)** There are eight questions on an examination paper.

(i) In how many different ways can a candidate select six questions?

(ii) In how many different ways can a candidate select six questions if one particular question must always be selected?

(b) A meeting is attended by 23 men and 21 women.
Of the men, 14 are married and the others are single.
Of the women, 8 are married and the others are single.

(i) A person is picked at random. What is the probability that the person is a woman?

(ii) A person is picked at random. What is the probability that the person is married?

(iii) A man is picked at random. What is the probability that he is married?

(iv) A woman is picked at random. What is the probability that she is single?

(c) The digits 0, 1, 2, 3, 4, 5 are used to form four-digit codes. A code cannot begin with 0 and no digit is repeated in any code.

(i) Write down the largest possible four-digit code.

(ii) Write down the smallest possible four-digit code.

(iii) How many four-digit codes can be formed?

(iv) How many of the four-digit codes are greater than 4000?

7. **(a)** Calculate the mean of the following numbers:

1, 0, 1, 5, 2, 3, 9.

(b) The following cumulative frequency table refers to the ages of 70 guests at a wedding:

Age (in years)	< 20	< 40	< 60	< 90
Number of guests	6	23	44	70

(i) Copy and complete the following frequency table:

Age (in years)	0 – 20	20 – 40	40 – 60	60 – 90
Number of guests				

[Note: 20 – 40 means 20 years old or more but less than 40 etc.]

(ii) Using mid-interval values, calculate the mean age of the guests.

(iii) What is the greatest number of guests who could have been over 65 years of age?

(c) The grouped frequency table below refers to the marks obtained by 85 students in a test:

Marks	0 – 40	40 – 55	55 – 70	70 – 100
Number of students	16	18	27	24

[Note: 40 – 55 means 40 marks or more but less than 55 etc.]

(i) What percentage of students obtained 55 marks or higher?

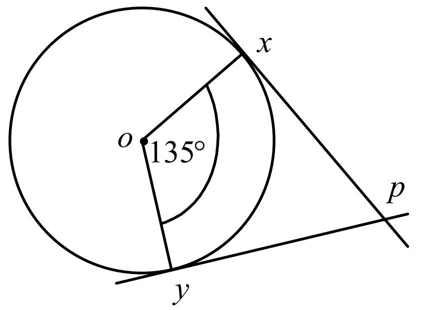
(ii) Name the interval in which the median lies.

(iii) Draw an accurate histogram to represent the data.

SECTION B

Attempt ONE question from this section.

8. **(a)** x and y are two points on a circle with centre o.
px and py are tangents to the circle, as shown.

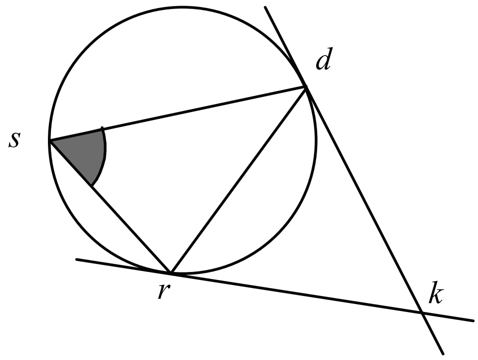


(i) Write down $|\angle pxo|$.

(ii) Given that $|\angle xoy| = 135°$, find $|\angle ypx|$.

(b) Prove that an angle between a tangent ak and a chord $[ab]$ of a circle has degree-measure equal to that of any angle in the alternate segment.

(c) The lines kd and kr are tangents to a circle at d and r respectively.
s is a point on the circle as shown.



(i) Name two angles in the diagram equal in measure to $\angle dsr$.

(ii) Find $|\angle rkd|$, given that $|\angle dsr| = 65°$.

(iii) Is $|dk| = |rk|$? Give a reason for your answer.

9. **(a)** Let $\vec{p} = -\vec{i} + 2\vec{j}$ and $\vec{w} = 3\vec{i} - 4\vec{j}$.

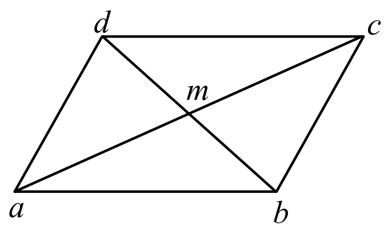
Express, in terms of $\vec{i}$ and $\vec{j}$,

(i) $2\vec{w}$

(ii) $2\vec{w} - \vec{p}$.

(b) $abcd$ is a parallelogram. The diagonals intersect at the point m.

Express each of the following as a single vector



(i) $\vec{ab} + \vec{bm}$

(ii) $\vec{ab} + \vec{ad}$

(iii) $\vec{ac} - \vec{ab}$

(iv) $\frac{1}{2}\vec{ac} + \frac{1}{2}\vec{db}$.

(c) Let $\vec{x} = 3\vec{i} + 4\vec{j}$ and $\vec{y} = 5\vec{i} + 12\vec{j}$.

(i) Show that $|\vec{x}| + |\vec{y}| > |\vec{x} + \vec{y}|$.

(ii) Write down $\vec{x}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$ and hence, show that $|\vec{x}|^2 + |\vec{x}^{\perp}|^2 = |\vec{x} - \vec{x}^{\perp}|^2$.

10. **(a)** **(i)** Calculate each of the following correct to three decimal places

$$\left(\frac{1}{2}\right)^6, \left(\frac{1}{2}\right)^7, \left(\frac{1}{2}\right)^8, \left(\frac{1}{2}\right)^9.$$

(ii) Hence, write down $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n$.

(b) The first term of a geometric series is 3. The second term of the series is 12.

(i) Write down the common ratio.
(ii) What is the fifth term of the series?
(iii) Calculate the sum of the first nine terms of the series.

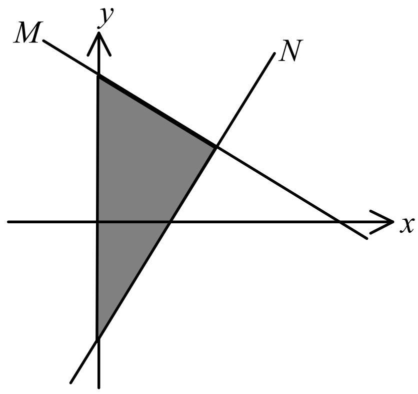
(c) **(i)** €100 is invested at 10% compound interest per annum.

Show that the value of the investment is less than €1000 after 24 years and more than €1000 after 25 years.

(ii) The sum to infinity of a geometric series is 2. The common ratio and the first term of the series are equal. Find the common ratio.

11. **(a)** The equation of the line M is $2x + y = 10$.
The equation of the line N is $4x - y = 8$.

Write down the three inequalities that define the shaded region in the diagram.



(b) A new ship is being designed. It can have two types of cabin accommodation for passengers — type A cabins and type B cabins.

Each type A cabin accommodates 6 passsengers and each type B cabin accommodates 3 passengers. The maximum number of passengers that the ship can accommodate is 330.

Each type A cabin occupies $50\,\text{m}^2$ of floor space. Each type B cabin occupies $10\ \text{m}^2$ of floor space. The total amount of floor space occupied by cabins cannot exceed $2300\ \text{m}^2$.

(i) Taking x to represent the number of type A cabins and y to represent the number of type B cabins, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The income on each voyage from renting the cabins to passengers is €600 for each type A cabin and €180 for each type B cabin. How many of each type of cabin should the ship have so as to maximise income, assuming that all cabins are rented?

(iii) What is the maximum possible income on each voyage from renting the cabins?