M27

AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 2000

MATHEMATICS - ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 8 JUNE - MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

Marks may be lost if all necessary work is not clearly shown.

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1. **(a)** Express 400 grammes as a fraction of 1 kilogramme. Give your answer in its simplest form.

(b)

1 euro = IR£ 0.787564
1 euro = DM 1.95583

(i) Calculate the value of IR£100 in euro, correct to two places of decimals.

(ii) Hence, calculate the value of IR£100 in Deutschmarks (DM), correct to two places of decimals.

(c) A person has annual tax free allowances of IR£7400.

The person pays income tax at the rate of 24% on the first IR£12 400 of taxable income and at the rate of 46% on the remainder.

(i) Calculate the amount of income tax paid on the first IR£12 400 of taxable income.

(ii) Calculate the person's gross income if the total annual income tax paid is IR£5138.

2. **(a)** Find the value of $5x - 3y$ when $x = \frac{5}{2}$ and $y = \frac{2}{3}$.

(b) Solve for x and y

$$x - 3y = 1$$
$$x^2 - y^2 = 0.$$

(c) Write as a power of 3

(i) 243
(ii) $\sqrt{27}$.

Hence, solve for x the equation

$$\sqrt{3}(3^x) = \left(\frac{243}{\sqrt{27}}\right)^2.$$

3. **(a)** Express p in terms of t and k when

$$tp - k = 7k, \qquad t \neq 0.$$

(b) **(i)** Show that $x = 2$ is a root of $3x^3 + 8x^2 - 33x + 10 = 0.$

(ii) Find the other roots of $3x^3 + 8x^2 - 33x + 10 = 0$.

(c) **(i)** $f(x) = ax^2 + bx - 8,$ where a and b are real numbers.

If $f(1) = -9$ and $f(-1) = 3$, find the value of a and the value of b.

(ii) Using your values of a and b from **(i)**, find the two values of x for which

$$ax^2 + bx = bx^2 + ax.$$

4. **(a)** Simplify

$$7(2+i) + i(11+9i)$$

and express your answer in the form $x + yi$ where $x, y \in \mathbf{R}$ and $i^2 = -1.$

(b) Let $w = 3 - i$.

(i) Plot w and $w + 6i$ on an Argand diagram.

(ii) Calculate $|w + 6i|$.

(iii) Express $\dfrac{1}{w+6i}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

(c) Let $z = 2+4i$.

(i) Express $z^2 + 28$ in the form $p + qi$ where $p, q \in \mathbf{R}$.

(ii) Solve for real k

$$k(z^2 + 28) = |z|(1+i).$$

Express your answer in the form $\dfrac{\sqrt{a}}{b}$ where $a, b \in \mathbf{N}$ and a is a prime number.

5. (a) The nth term of a sequence is given by $T_n = n^2 + 1$.

(i) Write down the first three terms of the sequence.

(ii) Show that $T_1 + T_2 + T_3 = T_4$.

(b) The first term of a geometric series is 1 and the common ratio is $\frac{11}{10}$.

(i) Write down the second, third and fourth terms of the series.

(ii) Calculate S_4, the sum of the first four terms. Give your answer as a decimal.

(c) The first three terms of an arithmetic series are 5 + 10 + 15 + ...

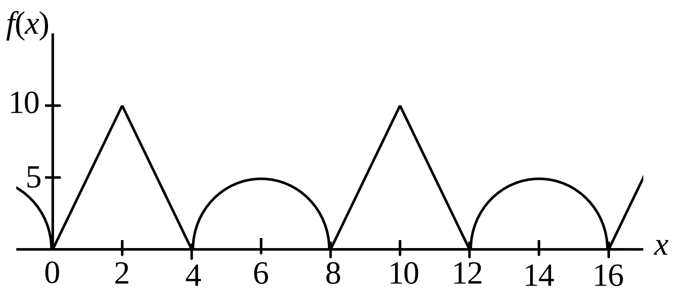
(i) Find, in terms of n, an expression for T_n, the nth term.

(ii) Find, in terms of n, an expression for S_n, the sum to n terms.

(iii) Using your expression for S_n, find the sum of the natural numbers that are both multiples of 5 and smaller than 1000.

6. (a) Differentiate $7x + 3$ from first principles with respect to x.

(b)



The graph shows portion of a periodic function $f : x \to f(x)$ which is defined for $x \in \mathbf{R}$.

(i) Write down the period and the range of $f(x)$.

(ii) Complete the following table:

x	2	8	14	20	26
$f(x)$					

(c) Let $g(x) = (2x+3)(x^2-1)$ for $x \in \mathbf{R}$.

(i) For what two values of x is the slope of the tangent to the curve of $g(x)$ equal to 10?

(ii) Find the equations of the two tangents to the curve of $g(x)$ which have slope 10.

7. **(a)** Differentiate with respect to x

(i) $4x^2 + 5$

(ii) $9x - x^3$.

(b) **(i)** Find $\dfrac{dy}{dx}$ when $y = \dfrac{2x-7}{x-1}$, $x \neq 1$.

(ii) Find $\dfrac{dy}{dx}$ when $y = \left(x^2 + 5x - 1\right)^3$.

(c) A car, starting at t = 0 seconds, travels a distance of s metres in t seconds where

$$s = 30t - \frac{9}{4}t^2.$$

(i) Find the speed of the car after 2 seconds.
(ii) After how many seconds is the speed of the car equal to zero?
(iii) Find the distance travelled by the car up to the time its speed is zero.

8. **(a)** Let $p(x) = 3x - 12$.

For what values of x is $p(x) < 0$ where x is a positive whole number?

(b) **(i)** Draw the graph of

$$g(x) = \frac{1}{x} \quad \text{for } -3 \leq x \leq 3, \; x \in \mathbf{R} \text{ and } x \neq 0.$$

(ii) Using the same axes and the same scales, draw the graph of

$$h(x) = x + 1 \quad \text{for } -3 \leq x \leq 3, \; x \in \mathbf{R}.$$

(iii) Use your graphs to estimate the values of x for which

$$\frac{1}{x} = x + 1.$$

(c) Let $f(x) = x^3 - 3x^2 + ax + 1$ for all $x \in \mathbf{R}$ and for $a \in \mathbf{R}$.

$f(x)$ has a turning point (a local maximum or a local minimum) at $x = -1$.

(i) Find the value of a.

(ii) Is this turning point a local maximum or a local minimum?
Give a reason for your answer.

(iii) Find the co-ordinates of the other turning point of $f(x)$.

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LEAVING CERTIFICATE EXAMINATION, 2000

MATHEMATICS – ORDINARY LEVEL – PAPER 2
(300 marks)

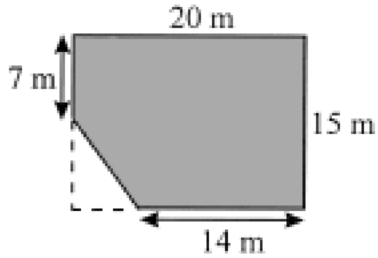
FRIDAY, 9 JUNE — MORNING, 9.30 to 12.00

Attempt 5 **Questions** from Section **A** and **ONE Question** from Section **B**.
Each question carries 50 marks.

Marks may be lost if necessary work is not clearly shown.

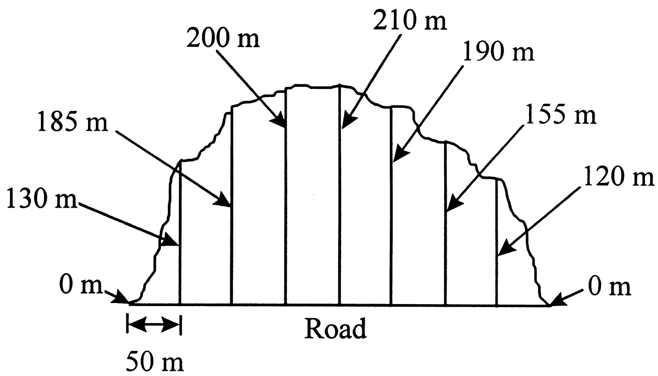
SECTION A

1. **(a)** Calculate the area of the shaded region in the diagram.



(b) The sketch shows a piece of land covered by forest which lies on one side of a straight road.

At equal intervals of 50 m along the road, perpendicular measurements of 130 m, 185 m, 200 m, 210 m, 190 m, 155 m and 120 m are made to the forest boundary.

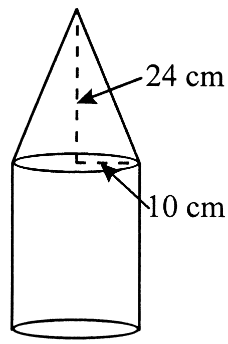


Use Simpson's Rule to estimate the area of land covered by the forest.

[See Tables, page 42.]

Give your answer in hectares. [Note: 1 hectare = 10 000 m^2.]

(c) A candle is in the shape of a cylinder surmounted by a cone, as in the diagram.



(i) The cone has height 24 cm and the length of the radius of its base is 10 cm.

Find the volume of the cone in terms of π.

(ii) The height of the cylinder is equal to the slant height of the cone.

Find the volume of the cylinder in terms of π.

(iii) A solid spherical ball of wax with radius of length r cm was used to make the candle.

Calculate r, correct to one decimal place.

2. **(a)** Find the coordinates of the midpoint of the line segment which joins the points $(2, -3)$ and $(-8, -6)$.

(b) $a(-2, -1)$, $b(1, 0)$ and $c(-5, 2)$ are three points.

(i) Show that $|ab| = \sqrt{10}$.

(ii) Find $|bc|$.

(iii) Hence, find the ratio $|ab| : |bc|$.

Give your answer in the form $m : n$ where m and n are whole numbers.

(c) **(i)** The line L has equation $3x - 4y + 20 = 0$.

K is the line through $p(0, 5)$ which is perpendicular to L.

Find the equation of K.

(ii) L cuts the x-axis at the point t.

K cuts the x-axis at the point r.

Calculate the area of the triangle ptr. Give your answer as a fraction.

3. **(a)** The circle C has equation $x^2 + y^2 = 16$.

(i) Write down the length of the radius of C.

(ii) Show, by calculation, that the point $(3, 1)$ is inside the circle.

(b) **(i)** Find the slope of the tangent to the circle $x^2 + y^2 = 29$ at the point $(2, 5)$.

(ii) Hence, find the equation of the tangent.

(c) **(i)** The end points of a diameter of a circle are $(-2, -3)$ and $(-4, 3)$.
Find the equation of the circle.

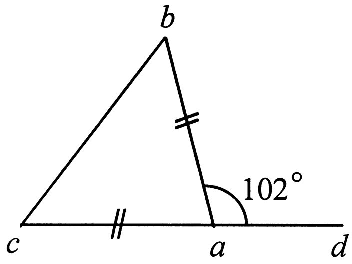
(ii) The circle cuts the y-axis at the points a and b. Find $|ab|$.

(iii) c and d are points on the circle such that $abcd$ is a rectangle.
Find the area of the rectangle $abcd$.

4. **(a)** In the diagram, $|ab| = |ac|$ and $|\angle bad| = 102°$.

(i) Find $|\angle cab|$.

(ii) Find $|\angle abc|$.



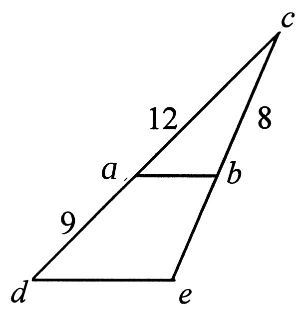
(b) Prove that in a right-angled triangle, the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

(c) The triangle *cde* is the image of the triangle *cab* under an enlargement with centre *c*.
$|ca| = 12$, $|ad| = 9$ and $|cb| = 8$.

(i) Find the scale factor of the enlargement.

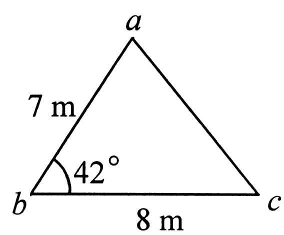
(ii) Find $|be|$.

(iii) The area of the triangle *cde* is 98 square units.
Find the area of the triangle *cab*.



5. **(a)** In the triangle *abc*, $|ab| = 7$ m, $|bc| = 8$ m and $|\angle abc| = 42°$.

Calculate the area of the triangle, correct to one place of decimals.



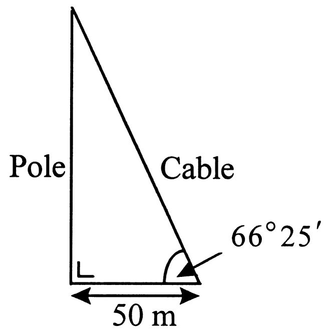
(b) The diagram shows a vertical pole which stands on level ground.

A cable joins the top of the pole to a point on the ground which is 50 m from the base of the pole.

The cable makes an angle of 66°25′ with the ground.

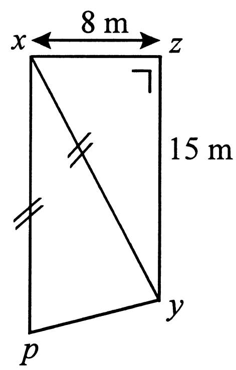
(i) Find the height of the pole, correct to the nearest metre.

(ii) Find the length of the cable, correct to the nearest metre.



(c) **(i)** In the diagram, the triangle *zxy* is right-angled.
$|zx| = 8$ m and $|zy| = 15$ m.
Find $|xy|$.

(ii) *xp* is parallel to *zy*.
$|xp| = |xy|$, as shown.
Calculate $|py|$, correct to the nearest metre.



6. **(a)** To go to work, a woman can walk or travel by bus or travel by car with a neighbour. To return home, she can walk or travel by bus.

(i) In how many different ways can the woman go to and return from work on any one day?

(ii) List all of these different ways.

(b) In a class, there are 15 boys and 13 girls. Four boys wear glasses and three girls wear glasses.

A pupil is picked at random from the class.

(i) What is the probability that the pupil is a boy?

(ii) What is the probability that the pupil wears glasses?

(iii) What is the probability that the pupil is a boy who wears glasses?

A girl is picked at random from the class.

(iv) What is the probability that she wears glasses?

(c) **(i)** How many different five-digit numbers can be formed from the digits 2, 3, 4, 5, 6? Each digit can be used once only in each number.

(ii) How many of the numbers are even?

(iii) How many of the numbers are less than 40 000?

(iv) How many of the numbers are both even and less than 40 000?

7. **(a)** Find the weighted mean of 11, 15, 19 and 21 if the weights are 2, 3, 1 and 2 respectively.

(b) The table shows the distribution of points obtained by 50 people who took a driving test.

Points obtained	0 – 20	20 – 40	40 – 80	80 – 100
Number of people	4	8	28	10

(i) Draw a histogram to illustrate the data.

(ii) To pass the driving test a person must obtain 65 points or more. What is the greatest possible number of people who passed the test?

(c) The table below refers to the number of emergency calls recorded at a fire station each week for 52 weeks.

Number of emergency calls	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70
Number of weeks	6	8	11	12	7	5	3

(i) Copy and complete the following cumulative frequency table:

Number of emergency calls	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Number of weeks	6						52

(ii) Draw the cumulative frequency curve.

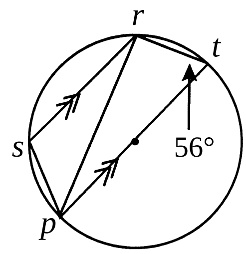
(iii) Use your graph to estimate the interquartile range.

(iv) Use your graph to estimate the number of weeks during which more than 56 emergency calls were recorded.

SECTION B

Attempt ONE question.

8. **(a)** In the diagram, [pt] is a diameter of the circle.
sr is parallel to pt and $|\angle ptr| = 56°$.

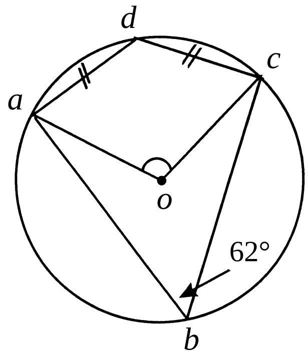


(i) Write down the value of $|\angle prt|$.

(ii) Find the value of $|\angle prs|$.

(b) Prove that the degree-measure of an angle subtended at the centre of a circle by a chord is equal to twice the degree-measure of any angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

(c) In the diagram, o is the centre of the circle.



a, b, c and d are points on the circle.

$|da| = |dc|$ and $|\angle abc| = 62°$.

(i) Find $|\angle aoc|$, where $\angle aoc$ is obtuse.

(ii) Find $|\angle adc|$.

(iii) Find $|\angle oad|$.

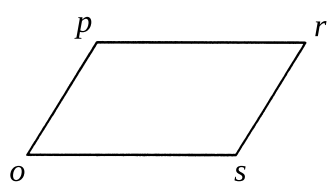
9. **(a)** Let $\vec{x} = \vec{i} + \vec{j}$ and $\vec{y} = 2\vec{i} + 5\vec{j}$.

Express, in terms of $\vec{i}$ and $\vec{j}$,

(i) $3\vec{x} + \vec{y}$

(ii) $\overrightarrow{xy}$.

(b) $osrp$ is a parallelogram where o is the origin.



(i) Copy the diagram and show on it $\vec{k}$ and $\vec{m}$ such that

$\vec{k} = \vec{s} + 2\vec{p}$ and $\vec{m} = 2\vec{s} + \vec{p}$.

(ii) Express $\vec{k} + \vec{m}$ in terms of $\vec{r}$.

(c) $\vec{a} = 5\vec{i} + 12\vec{j}$ and $\vec{b} = 3\vec{i} - 4\vec{j}$.

(i) Write down $\vec{a}^{\perp}$ and $\vec{b}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Evaluate $|\vec{a}^{\perp}|$ and $|\vec{b}^{\perp}|$.

(iii) Find the scalar k such that $|\vec{a}^{\perp} + \vec{b}^{\perp}| = k\left(|\vec{a}^{\perp}| - |\vec{b}^{\perp}|\right)$.
Give your answer in the form $\sqrt{n}$, where $n \in \mathbf{N}$.

10. **(a)** Expand $(1 + x)^3$ in ascending powers of x.

Show that $(1 + \sqrt{3})^3 = 10 + 6\sqrt{3}$.

(b) **(i)** Find the sum to infinity of the geometric series

$$\frac{4}{5} + \frac{4}{50} + \frac{4}{500} + \cdots$$

(ii) Hence, show that $1.\dot{8} = \frac{17}{9}$.

(c) A person invests IR£1000 at the beginning of each year for 3 consecutive years at 8% per annum compound interest. Tax at 24% is deducted at the end of each year from the interest earned.

Find

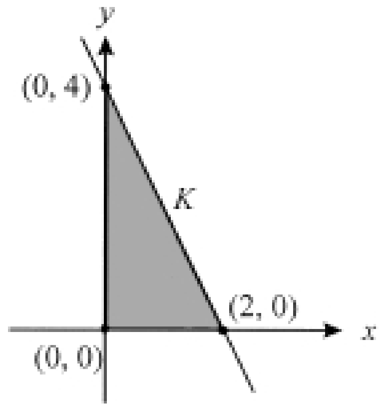
(i) the value of the first investment at the end of the third year, correct to the nearest penny

(ii) the total value of all the investments at the end of the third year, correct to the nearest penny.

11. **(a)** The line K passes through the points (2, 0) and (0, 4).

(i) Find the equation of the line K.

(ii) Write down three inequalities which define the shaded region in the diagram.



(b) Two types of machines, type A and type B, can be purchased for a new factory. Each machine of type A costs IR£1600. Each machine of type B costs IR£800. The purchase of the machines can cost, at most, IR£27 200.

Each machine of type A needs 90 m^2 of floor space in the factory.

Each machine of type B needs 54 m^2 of floor space.

The maximum amount of floor space available for the machines is 1620 m^2.

(i) If x represents the number of machines of type A and y represents the number of machines of type B, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The daily income from the use of each machine of type A is IR£75. The daily income from the use of each machine of type B machine is IR£42. How many of each type of machine should be purchased so as to maximise daily income?

(iii) What is the maximum daily income?