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LEAVING CERTIFICATE EXAMINATION, 1999

57005

MATHEMATICS - ORDINARY LEVEL - PAPER 1 (300 marks)

THURSDAY, 10 JUNE - MORNING, 9.30 - 12.00

Attempt **SIX QUESTIONS** (50 marks each).

Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

1. **(a)** IR£40 is divided between two pupils in the ratio 7:3. How much does each pupil get?

(b) A car journey of 559 kilometres took 6 hours and 30 minutes.

(i) Calculate the average speed, in km/hr, for the journey.

(ii) If the average petrol consumption for the journey was 8.3 kilometres per litre, calculate the number of litres of petrol used, correct to the nearest litre.

(c) A holiday complex consists of three different types of chalet.

Chalet Type	Number of chalets	Number of people per chalet	Weekly rent per chalet
Type A	12	5	IR£300
Type B	20	6	IR£350
Type C	14	8	IR£450

During one week in July all chalets are fully occupied.

(i) Calculate the number of people staying in the chalets at the holiday complex that week.

(ii) Calculate the total amount of rent paid for that week.

In the last week of September, a 35% discount is offered on the weekly rent of a Type C chalet. Calculate the weekly rent on a type C chalet for the last week in September.

2. **(a)** Solve for x

$$2(x + 8) = 7x.$$

(b) Write as a power of 2

(i) 8

(ii) $8^{\frac{4}{3}}$.

Solve for x the equation

$$8^{\frac{4}{3}} = \frac{2^{5x-4}}{\sqrt{2}}.$$

(c) Solve for x

$$\frac{3}{2x-1} = 1 + \frac{2x}{x+2}, \quad x \neq \frac{1}{2} \text{ and } x \neq -2.$$

3. **(a)** Express p in terms of q and r when

$$\frac{p-3r}{q} = 5, \quad q \neq 0.$$

(b) Solve for x and y

$$x + 2y = 6$$
$$x^2 + y^2 = 17.$$

(c) Show that $6x^2 + 5x - 4$ is a factor of $6x^3 + 17x^2 + 6x - 8$.

Hence, or otherwise, find the roots of $6x^3 + 17x^2 + 6x - 8 = 0$.

4. **(a)** Let $z = 5 + 4i$, where $i^2 = -1$.

Plot

(i) z

(ii) $z - 4i$

on an Argand diagram.

(b) Let $u = 3 - 6i$.

(i) Calculate $|u|$.

(ii) Show that $iu + \dfrac{u}{i} = 0$.

(iii) Express $\dfrac{u}{u+3i}$ in the form $p + qi$, $p, q \in \mathbf{R}$.

(c) Let $w = i - 2$.

Express w^2 in the form $a + bi$, $a, b \in \mathbf{R}$.

Hence, solve

$$kw^2 = 2w + 1 + ti$$

for real k and real t.

5. **(a)** The nth term of a sequence is given by

$$T_n = \frac{n}{n+1}.$$

(i) Find T_2, the second term.

(ii) Show that $T_2 + T_3 > 1$.

(b) The first two terms of a geometric series are $2 + \frac{2}{3} + \ldots$

(i) Find r, the common ratio.

(ii) Write down the third and fourth terms of the series.

(iii) Show that S_6, the sum to 6 terms, is $3 - \frac{1}{3^5}$.

(c) The nth term of a series is given by

$$T_n = 4n + 1.$$

(i) Write down, in terms of n, an expression for T_{n-1}, the $(n\text{-}1)$st term.

(ii) Show that the series is arithmetic.

(iii) Find S_{20}, the sum of the first 20 terms of the series.

6. **(a)** Let $f(x) = 2(3x-1), \quad x \in \mathbf{R}$.

Find the value of x for which $f(x) = 0$.

(b) Differentiate from first principles

$$x^2 + 5x$$

with respect to x.

(c) Let $f(x) = x^3 - 6x^2 + 12$ for $x \in \mathbf{R}$.
Find the derivative of $f(x)$.

At the two points (x_1, y_1) and (x_2, y_2), the tangents to the curve $y = f(x)$ are parallel to the x axis, where $x_2 > x_1$.

Show that

(i) $x_2 - x_1 = 4$

(ii) $y_2 = y_1 - 32$.

7. **(a)** Differentiate

$$2x^3 - 7$$

with respect to x.

(b) **(i)** Find $\frac{dy}{dx}$ when $y = (3 - 7x)^5$.

(ii) Find $\frac{dy}{dx}$ when $y = \frac{x^2}{1-x}$, $x \neq 1$.

Show that $\frac{dy}{dx} = 0$ at $x = 0$.

(c) The speed, v, in metres per second, of a body after t seconds is given by

$$v = 3t\,(4 - t).$$

(i) Find the acceleration at each of the two instants when the speed is 9 metres per second.

(ii) Find the speed at the instant when the acceleration is zero.

8. Let $f(x) = 2x^3 - 5x^2 - 4x + 3$ for $x \in \mathbf{R}$.

(i) Complete the table

x	−1.5	−1	0	1	2	3	3.5
$f(x)$	−9						13.5

(ii) Find the derivative of $f(x)$.

Calculate the co-ordinates of the local minimum and show that the co-ordinates of the local maximum are $\left(\frac{-1}{3}, \frac{100}{27}\right)$.

(iii) Draw the graph of

$$f(x) = 2x^3 - 5x^2 - 4x + 3$$

for $-1.5 \leq x \leq 3.5$.

(iv) Write the equation $2x^3 - 5x^2 - 6x + 6 = 0$ in the form

$$2x^3 - 5x^2 - 4x + 3 = ax + b, \quad a, b \in \mathbf{Z}.$$

Hence, use your graph to estimate the solutions of the equation

$$2x^3 - 5x^2 - 6x + 6 = 0.$$