Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2011

MARKING SCHEME

MATHEMATICS
(PROJECT MATHS)

FOUNDATION LEVEL

Contents

Introduction

The Foundation Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared one common question on Paper 1, i.e. question 8, Section C, with the examination for all other candidates. The marking scheme used for the common elements was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2011

Mathematics
(Project Maths – Phase 2)

Paper 1

Foundation Level

Friday 10 June   Afternoon 2:00 – 4:30

300 marks

Model Solutions – Paper 1

Note that the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	125 marks	5 questions
Section B	Contexts and Applications	125 marks	2 questions
Section C	Functions and Graphs (old syllabus)	50 marks	1 question

Answer questions as follows:

In Section A, answer **all five** questions

In Section B, answer **both** Question 6 **and** Question 7

In Section C, answer Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all five** questions from this section.

Question 1 (25 marks)

(a) The table below shows a list of real numbers. For each one, tick (✔) the correct box to say whether it is *rational* or *irrational*.

Number	rational	irrational
7	✔	
$\frac{2}{3}$	✔	
$\sqrt{2}$		✔
$\sqrt{4}$	✔	
π		✔
3·14	✔	

(b) Show the number $\sqrt{2}$ on the number line below.

(c) Ireland's national debt, in euro, at the end of 2010 was approximately $9{\cdot}3\times10^{10}$.

(i) Write this as an ordinary whole number.

Ans: = 93 000 000 000

(ii) Write the number out in words.

Ninety three billion
Ninety three thousand million
93 billion

Question 2 **(25 marks)**

(a) Write 8% as a decimal.

Answer: _____0·08_____

**(b)** A company borrows €15 000 for five years at an interest rate of 8% per annum (APR). If the company makes no repayments, how much will it owe at the end of the five years.

$$F = 15000(1·08)^5$$
$$= 22039·92115$$
$$\approx €22039·92$$

[8]

Question 3 **(25 marks)**

The diagram shows a right cone with measurements as follows:
    Radius of base: 5 cm
    Vertical height: 12 cm
    Slant height: 13 cm.

(a) Find the volume of the cone, in cm^3, correct to two decimal places.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (5^2)(12)$$

$$V \approx 314 \cdot 16\, cm^3$$

(b) Find the total surface area of the cone, in cm^2, correct to two decimal places.

$T.S.A = \pi rl + \pi r^2$

$T.S.A = \pi (5)(13) + \pi (5)^2$

$T.S.A = 65\pi + 25\pi$ **OR**

$T.S.A = 90\pi$

$T.S.A \approx 282 \cdot 74\, cm^2$

$T.S.A = \pi rl + \pi r^2$

$T.S.A = \pi (5)(13) + \pi (5)^2$

$T.S.A = 204 \cdot 2035 + 78 \cdot 5398$

$T.S.A \approx 282 \cdot 74\, cm^2$

Question 4   **(25 marks)**

(a) Evaluate $\dfrac{4x-8}{3x+2}$ when $x=-2$.

$$\frac{4(-2)-8}{3(-2)+2} = \frac{-16}{-4} = 4$$

(b) Let $f(x)=4x-8$ and let $g(x)=3x+2$.
Find the value of x for which $f(x)=g(x)$.

$$4x-8=3x+2$$

$$4x-3x=2+8$$

$$x=10$$

Question 5 **(25 marks)**

(a) Solve the equation $x^2 - 12x + 11 = 0$.

$(x-1)(x-11) = 0$

OR

$\Rightarrow x = \{1, 11\}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, b = -12, c = 11$

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(1)(11)}}{2(1)}$

$x = \dfrac{12 \pm \sqrt{100}}{2}$

$x = \dfrac{12 \pm 10}{2}$

$x = \{1, 11\}$

(b) Solve the equation $2y^2 - 7y - 5 = 0$, giving your answers correct to two decimal places.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2, b = -7, c = -5$

$x = \dfrac{7 \pm \sqrt{7^2 - 4(2)(-5)}}{2(2)}$

$x = \dfrac{7 \pm \sqrt{89}}{4}$

$x = \dfrac{7 \pm 9 \cdot 43398\ldots}{4}$

$x \approx \{-0 \cdot 61, 4 \cdot 11\}$

[11]

Answer **both** Question 6 **and** Question 7.

Question 6 (75 marks)

(a) A piece of machinery in a factory is valued every three years. The table below shows the value every three years from the end of 1996 to the end of 2008.

Year	1996	1999	2002	2005	2008
Value at end of year /€	256 000	128 000	64 000	32 000	16 000

(i) Describe fully the pattern in the value of the equipment over time.

The value halves every three years

(ii) Suppose that this pattern continued. Complete the table below for the next two valuations.

Year	2011	2014
Value at end of year /€	8000	4000

(iii) The machinery will be scrapped when its value is €1000 or less. When will this happen?

Answer: At the end of year 2020

[At end of 2017, the value will be €2000]

[At end of 2020, the value will be €1000]

(b) Peter had to do the following calculation:

$$\frac{132{\cdot}068 - 63{\cdot}142}{4{\cdot}82}$$

First, he made an estimate of what his answer should be. Then, he used his calculator to do the calculation. He got the answer 118·968.

Because of his estimate, he realised this answer could not be correct.

(i) Show how Peter might have made a suitable estimate.

$$\frac{130 - 60}{5} = \frac{70}{5} = 14$$

(ii) Find the correct answer to the calculation.

$$\frac{132{\cdot}068 - 63{\cdot}142}{4{\cdot}82} = \frac{68.926}{4{\cdot}82} = 14{\cdot}3$$

(iii) Peter got his answer by keying the following into his calculator:

132·068 − 63·142 ÷ 4·82 =

Explain why this did not give the correct answer to the calculation.

He should have used brackets around 132·068 - 63·142
i.e. the top line of the fraction

OR

He should have pressed the "=" key after he had keyed in 132·068 - 63·142,
i.e. the top line of the fraction

[13]

(c) Ciara spends €1600 each year on heating her house. By adding extra insulation in the attic, she can reduce this by 15%.

(i) How much will Ciara save on her heating bills each year if she gets the insulation?

$$1600 \times \frac{15}{100} = €240$$

OR

$$1600 \times 0 \cdot 15 = €240$$

(ii) It will cost €920 to have the extra insulation put in. Ciara will get a grant of €200 towards this cost. How many years will it take for the savings on her heating bills to recover the rest of the cost?

$(€920 - €200) = €720$, i.e. Ciara's cost

$$\frac{720}{240} = 3 \text{ years}$$

It will take 3 years at the given fuel cost

[14]

Question 7   **(50 marks)**

A company makes and sells clocks. The more they charge for
the clocks, the fewer they will sell. In particular, they have
noticed that:

- If they charge €15 per clock, they will sell 300 clocks per
  month.
- For every extra euro charged, the number sold per month
  falls by twenty.

(a) Complete the table below showing the number of clocks sold at the given prices.

Price per clock /€	15	16	17	18	19	20	25
Number sold per month	300	280	260	240	220	200	100

(b) Draw a graph to represent the number of clocks sold per month for prices from €15 to €25.

[15]
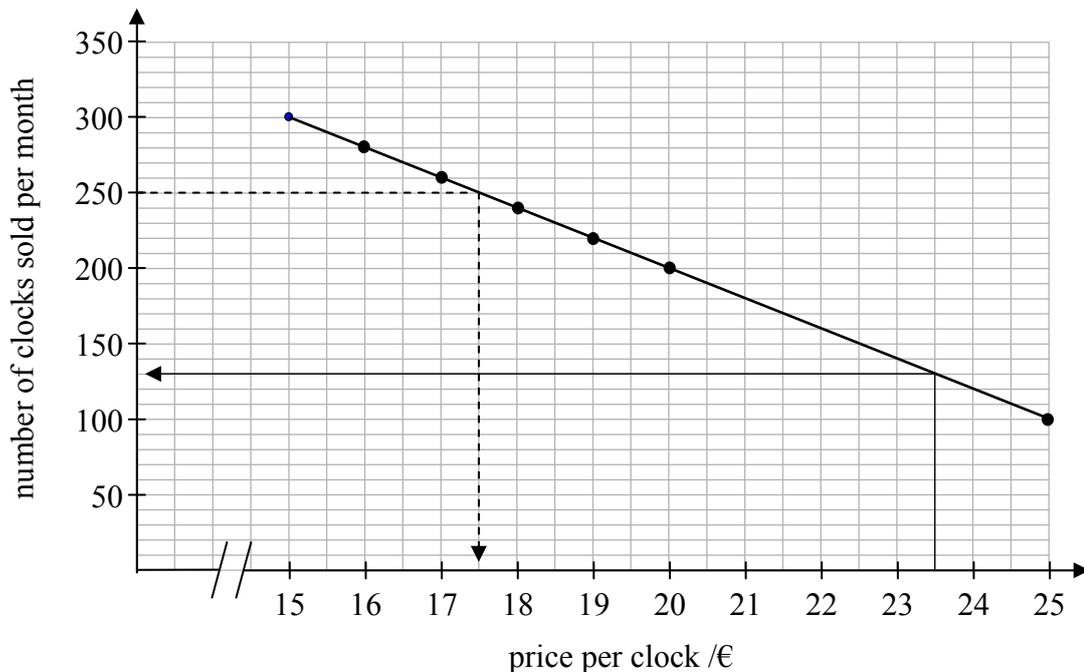
(c) Use the graph to estimate the number of clocks sold per month if the company charges €23·50 per clock.

Answer: <u>130 clocks</u>

(d) Use the graph to work out how much the company should charge per clock in order to sell 250 clocks per month.

Answer: <u>€17·50</u>

(e) Andrew, Betty and Charlie are each trying to write a formula that gives the number of clocks sold per month, n, when the price of each clock is €x. Here is what each of them writes:

Andrew: $n = 300 - 20x$

Betty: $n = 300 - 20(x + 15)$

Charlie: $n = 300 - 20(x - 15)$

Which one of these three formulae is correct, if any? Explain your answer.

According to the table in part (a),

if the price per clock is €15, the number of clocks sold is 300 etc.

Andrew: 300 - 20 (15) = 0,   not correct

Betty: 300 - 20 (15 + 15) = -300,   not correct

Charlie: 300 - 20 (15 - 15) = 300   correct
300 - 20 (16 - 15) = 280
300 - 20 (17 - 15) = 260 etc.

Therefore, the only correct formula is Charlie's

(f) Dónal also writes a formula. His formula is $n = 600 - 20x$. He notices that, no matter what the value of x is, his formula always gives the same value for n as Charlie's formula. Explain why this is so.

Because, Charlie's formula, when simplified, is the same as Donal's

Donal's formula	Charlie's formula
N = 600 – 20x	N = 300 – 20 (x – 15)
	= 300 – 20x + 300
	= 600 – 20x

[16]

Answer **Question 8** from this section.

A model solution for question 8 is incorporated into the marking scheme for section C.
See page 25.

Answer **Question 8** from this section.

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C
No of categories	2	3	4
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5
10 mark scale		0, 8 10	0, 6, 9, 10
15 mark scale		0, 10, 15	0, 9, 13,15
20 mark scale		0, 15, 20	0, 12, 17, 20
25 mark scale		0, 20, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1
    (a)   10C
    (b)   5B
    (c)(i) 5B
    (d)(ii) 5B

Question 2
    (a)   15B
    (b)   10C

Question 3
    (a)   20C*
    (b)   5C*

Question 4
    (a)   20C
    (b)   5C

Question 5
    (a)   20C
    (b)   5C*

Section B

Question 6
    (a)   (i)   10B
    (a)   (ii)   15C
    (a)   (iii)   10C
    (b)   (i)&(ii) 15C
    (b)   (iii)   5B
    (c)   (i)   15C
    (c)   (ii)   5C

Question 7
    (a)   20C
    (b)   15C
    (c)   5B
    (d)   5B
    (e)&(f) 5C

Detailed marking notes

Section A

Question 1

(a)   Scale 10C
             High partial credit: 3, 4 or 5 boxes correct.

             Low partial credit: 1 or 2 boxes correct.

(b)   Scale 5B
             Partial credit: Answer between 1 and 2, i.e. $1 \leq$ answer ≤ 2.
                                             $\sqrt{2} = 1 \cdot 4$ or $1 \cdot 414$ written.

             Accept answers: $1 \cdot 4 \leq$ answer $\leq 1 \cdot 5$

(c) (i)   Scale 5B
             Partial credit: Error with decimal point.
                                             Some meaningful work with 93 and / or 10^{10}.

(c) (ii)   Scale 5B
             Partial credit: Any correct step.
                                             Some meaningful work with 93 and / or 10^{10}.

Question 2

(a)   Scale 15B
             Partial credit: Error with decimal point.
                                             Any meaningful work with 8%.

(b)   Scale 10C
             High partial credit: Correct method but error(s) made in calculating.

             Low partial credit: Correct interest found for one year or finds simple interest.
                                                  Correct formula for compound interest written and stops.

Question 3

(a)   Scale 20C*

        High partial credit: Correct method but error(s) made in calculating.

        Low partial credit: Any work of merit – formula written, identifies $r = 5$ or $h = 12$.

(b)   Scale 5C*

        High partial credit: Correct method but error(s) made in calculating.
                                         One term fully worked out.

        Low partial credit: Any work of merit– formula written; $r = 5$ or $h = 12$ in this part.

Question 4

(a)   Scale 20C

        High partial credit: Correct substitution shown AND correct evaluation of either the numerator or the denominator.
                                         Incorrect substitution AND finishes correctly, e.g. $x = 2$.

        Low partial credit: Any work of merit.

(b)   Scale 5C
Accept correct solution from trial and error, if verified.
Accept correct solution from graph.

        High partial credit: $f(x)$ and $g(x)$ equated and some clear work towards isolating x.

        Low partial credit: $f(x)$ and $g(x)$ equated only.
                                         Effort at solution by trial and error.
                                         Effort at graph.

Question 5

(a)   Scale 20C
Note: Accept correct solutions from trial and error, if **both** are verified.

        High partial credit: Correct factors but roots not found.
                                         Incorrect factors but finished correctly.
                                         One correct solution from trial and error, must be verified.

        Low partial credit: Incorrect factors and did not finish.
                                         Effort at solution by trial and error.

(b)   Scale 5C*

        High partial credit: Correct method but error(s) made in calculating.

        Low partial credit: Any correct work, e.g.: effort at factoring;
                                                             correct formula written;
                                                       a, b or c identified.

Section B

Question 6

(a)   (i)   <u>Scale 10B</u>
            Partial credit: Incomplete description, e.g. "it's going down".

      (ii)   <u>Scale 15C</u>
            High partial credit: Two correct entries.

            Low partial credit: One correct entry.
                                  Any work of merit.

      (iii)   <u>Scale 10C</u>
            Full credit: 2020 without work.

            High partial credit: Incorrect answer, with reasonable work:
                                  e.g. 2011 = 8000, 2012 = 4000, 2013 = 2000, 2014 = 1000

            Low partial credit: 2014, 2017, 2023 etc., without work.

            No credit: Other years without work

(b)   (i) and (ii)
            <u>Scale 15C</u>
            High partial credit: Either **part (i) OR part (ii)** correct.

            Low partial credit: Work of merit in either part (i) or part (ii).

      (iii)   <u>Scale 5B</u>
            Partial credit: "Should have used brackets" and stops.
                                    An explanation that refers to order of operations but is
                                    unclear, e.g."BOMDAS"

(c)   (i)   <u>Scale 15C</u>
            High partial credit: Correct method but error(s) made in calculating.

            Low partial credit: Any correct step, e.g. writes 0·15 or 15/100.

(c)   (ii)   <u>Scale 5C</u>
            High partial credit: Correct method, not correctly completed.
                                  Omits the 200, otherwise correct.

            Low partial credit: Subtracts 200 and stops.

Question 7

(a)   <u>Scale 20C</u>

        High partial credit: All but one entry correct and consistent with the rule
                                     $300 - k(x - 15)$.

        Low partial credit: At least one correct or consistent step.

(b)   <u>Scale 15C</u>

        High partial credit: Four correct points plotted.

        Low partial credit: 1, 2 or 3 correct points.
                                         Any line with a negative slope.

        If the line is drawn by using only 2 plotted points, infer the remaining points from the
        line and if it only passes through 2 correct points, then award *high partial* credit.
        If it passes through all correct points, award *full credit*.

        Points only plotted: if all points are correct, award *full credit*.

(c)   <u>Scale 5B</u>

        Note: *Full credit:* Within tolerance of ± 9.

        Partial credit: Outside of tolerance of ± 9 but within tolerance of ± 20.
                                         Some relevant work shown.

(d)   <u>Scale 5B</u>

        Note: *Full credit:* $17 < $ answer $ < 18$.

        Partial credit: Outside of the range $\{17 < $ answer $ < 18\}$ but within the range
                                         $\{16{\cdot}50 \le $ answer $ \le 17\}$ OR $\{18 \le $ answer $ \le 18{\cdot}50\}$.
                                         Shows some relevant work.

(e) and (f) <u>Scale 5C</u>

        High partial credit: **Either** (e) **OR** (f) correct.

        Low partial credit: Shows some relevant work in either part, e.g.
                                         substitutes a value of *x* into any of the given formulae or
                                       attempts to simplify Charlie's formula in part (f);
                                       says "Charlie" with no explanation or work in part (e).

Marking Scheme – Paper 1, Section C

General Guidelines for Examiners – Paper 1, Section C

1. Penalties of three types are applied to candidates' work as follows:
    *   Blunders - mathematical errors/omissions   (-3)
    *   Slips - numerical errors   (-1)
    *   Misreadings   (provided task is not oversimplified)   (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3) note that
    *   any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
    *   if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    *   a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 8

Graph	**30 (20, 10) marks**	**Att (8, 4)**
Values	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Evaluation	**20 marks**	**Att 8**
Graph	**10 marks**	**Att 4**

Draw the graph of the function
$$f : x \to 2x^2 - x - 5, \text{ for } -3 \le x \le 3, \ x \in \mathbb{R}.$$

Table method   **20 marks**   **Att 8**

x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
$-x$	3	2	1	0	-1	-2	-3
-5	-5	-5	-5	-5	-5	-5	-5
$f(x)$	16	5	-2	-5	-4	1	10

*   Accept correct $f(x)$ values without work

Blunders (-3)
B1 x values added on when calculating $f(x)$ values
B2   Consistent errors across full line otherwise slips apply
B3 $f(x)$ not evaluated for an x value in domain or some x value omitted

Slips (-1)
S1   Each incorrect or omitted value in the body of the table
S2   Each incorrect or omitted $y / f(x)$ value from candidates work

Misreadings (-1)
M1 -5 treated as 5 across the line

Attempts (8 marks)
A1   Any four values in the table
A2   Function treated as linear e.g. $x^2 = 2x$ or x;
                                   $2x^2 = 4x$ or x.

$$f(x) = 2x^2 - x - 5$$
$$f(-3) = 2(-3)^2 - (-3) - 5 = 16$$
$$f(-2) = 2(-2)^2 - (-2) - 5 = 5$$
$$f(-1) = 2(-1)^2 - (-1) - 5 = -2$$
$$f(0) = 2(0)^2 - (0) - 5 = -5$$
$$f(1) = 2(1)^2 - (1) - 5 = -4$$
$$f(2) = 2(2)^2 - (2) - 5 = 1$$
$$f(3) = 2(3)^2 - (3) - 5 = 10$$

Blunders (-3)
B1   Consistent errors in the evaluation of $2x^2$
B2 -5 omitted from the evaluation
B3   Each incorrect $f(x)$ value when no work is shown to a maximum of 3 provided that at least one
     $f(x)$ value is correct
     All $f(x)$ values incorrect without work $\rightarrow$ 0 marks. Otherwise slips applied when work is
   shown

Slips (-1)
S1   Each incorrect or omitted value from the evaluation after substitution
S2   Each incorrect or omitted $f(x)$ value, calculated from candidates work

Misreadings (-1)
M1 -5 consistently treated as 5 in the evaluation.

Graph   **10 marks**   **Att 4**

On the graph: $f(x) = 7$, $f(2.5)$
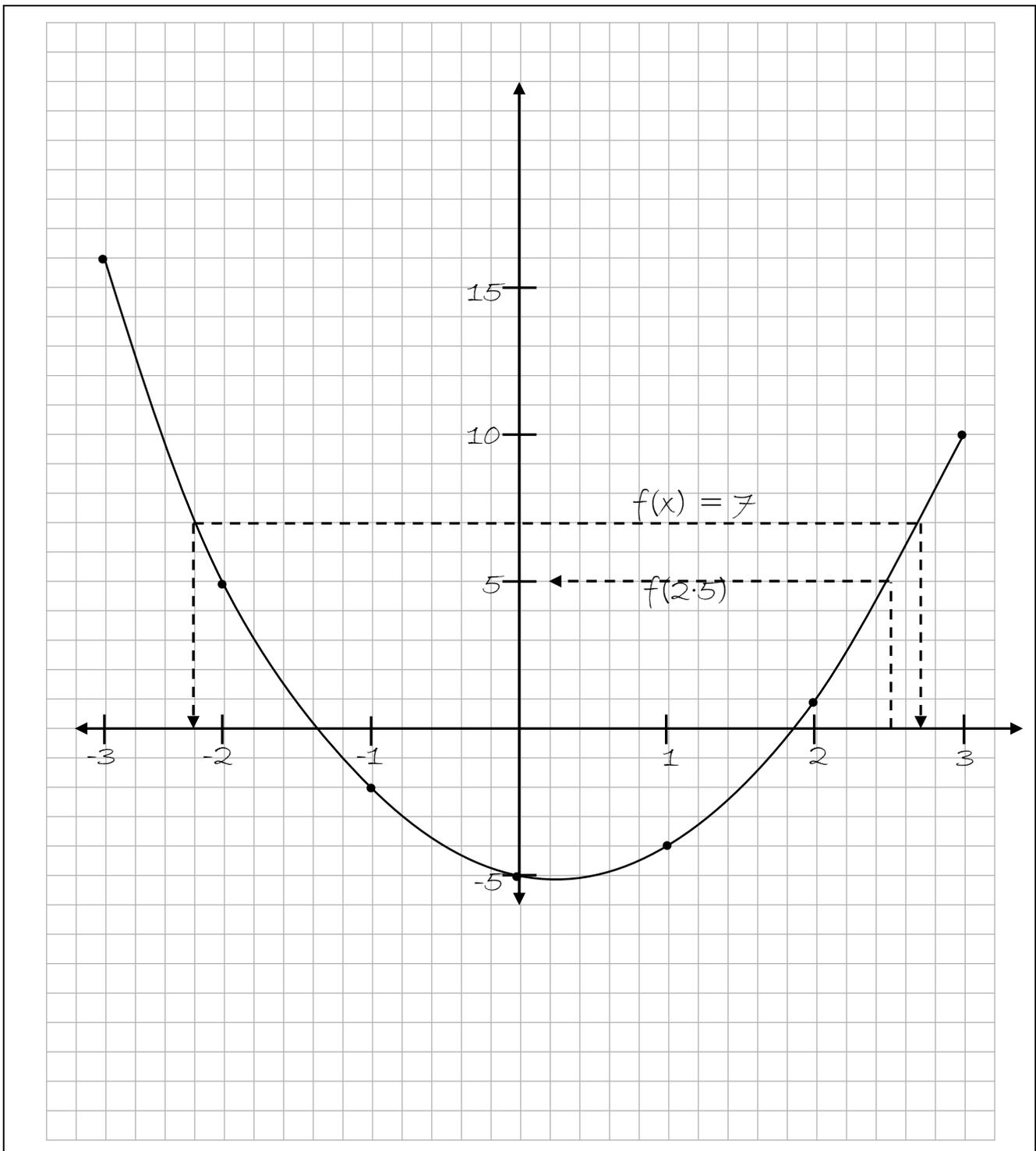
* Accept values from candidates work
* Fully correct graph drawn with no work shown: award 30 marks

Blunders (-3)
B1   Blunders in scales on axis or axes (once only)

Slips (-1)
S1   Each point from table plotted incorrectly
S2   Each pair of successive points not joined to a maximum of 3
S3   Not a smooth curve
S4   The graph of the function is not in the conventional position or orientation.

Attempts (4 marks)
A1   At least two of the candidates points plotted
A2   Any U-shaped graph
A3   Axes drawn

Values	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Use your graph to estimate
(i)   the value of $f(2{\cdot}5)$

(ii)   the minimum value of $f(x)$

(iii)   the values of x for which $f(x) = 7$

(iv)   the range of values of x for which $f(x)$ is increasing.

(i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**
(iii)	**5 marks**	**Att 2**
(iv)	**5 marks**	**Att 2**

(i) $f(2{\cdot}5) = 5$
(ii) $-5{\cdot}1$
(iii) $x = -2{\cdot}2,\ x = 2{\cdot}7$
(iv) $0{\cdot}2 < x \le 3$

*   Accept candidates values from graph
*   Allow tolerance of $\pm 0{\cdot}3$ units on x-axis, $\pm 0{\cdot}5$ on y-axis

Blunders (-3)
B1   Value omitted or extra value applies in parts (i) and (ii)

Slips (-1)
S1   Answers indicated correctly on axis but not specified
S2   Increasing part of graph indicated but no x value written down

Misreadings (-1)
M1   Gives the value of x corresponding to the minimum of $f(x)$ in part (i)

Attempts (2 marks)
A1   Effort at reading value(s) from graph
A2   Correctly solving equation algebraically; part (ii)

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 2

Foundation Level

Monday 13 June   Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note that the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

| Section A | Concepts and Skills | 150 marks | 6 questions |
| Section B | Contexts and Applications | 150 marks | 2 questions |

Answer **all eight** questions, as follows:

In Section A, answer:

      Questions 1 to 5 and

      either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1   (25 marks)

(a)   Give an example of an experiment with two outcomes that are *equally likely*, stating clearly what the two outcomes are.

E.g.:   tossing a coin

Outcomes: "Heads" or "Tails"

(b)   Give an example of an experiment with two outcomes that are **not** equally likely, stating clearly what the two outcomes are.

E.g.:   tossing a damaged coin

Outcomes: "Heads" or "Tails"

E.g.:   whether it will rain tomorrow

Outcomes: Yes or No

Question 2 **(25 marks)**

A girl and a boy are each asked to think of a whole number from 1 to 10. The outcome of this experiment is recorded as a pair of numbers. For example, if the girl picks 3 and the boy picks 1, this is recorded as (3, 1).

(a) Write out **three** possible outcomes of this experiment.

(3, 1) (3, 2) (5, 9) etc.

(b) How many different possible outcomes are there?

Answer: 100

[i.e. 10 × 10]

(c) Write out all of the outcomes in which the two children pick the same number.

(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) (7, 7)

(8, 8) (9, 9) (10, 10)

(d) Suppose that all numbers are equally likely, and that one child's choice has no effect on the other's choice. What is the probability that the two children will pick the same number?

$$\frac{10}{100} \Rightarrow \frac{1}{10}$$

Question 3 **(25 marks)**

Some scientists were studying a certain kind of ant. They selected a sample of 39 of these ants and measured the length of each ant's body, in millimetres. The results are shown in this stem-and-leaf plot:

```
4 | 3
4 | 6  6  8  9
5 | 0  0  1  1  1  2  4
5 | 5  7
6 | 0
6 | 8  9
7 | 0  1  2  4
7 | 5  5  7  8  8  8  9
8 | 0  0  1  1  2  3  4  4
8 | 5  5  8
```

Key: 4 | 3 means 4·3 mm.

(a) What is the length of the longest ant?

Answer: ___8·8 mm___

(b) What is the median length of the ants in the sample?

Answer: ___7·2 mm___

(c) Describe the shape of the distribution.

It has two "peaks"

(d) Suggest a reason why the distribution might have this shape.

Possibly two different types of ant e.g. males / females or adults and juveniles

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Question 4 (25 marks)

The points *A*, *B*, and *C* have co-ordinates as follows:

A (–4, 1)

B (–1, –5)

C (4, 5)

(a) Plot *A*, *B*, and *C* on the diagram, and show the triangle *ABC*.
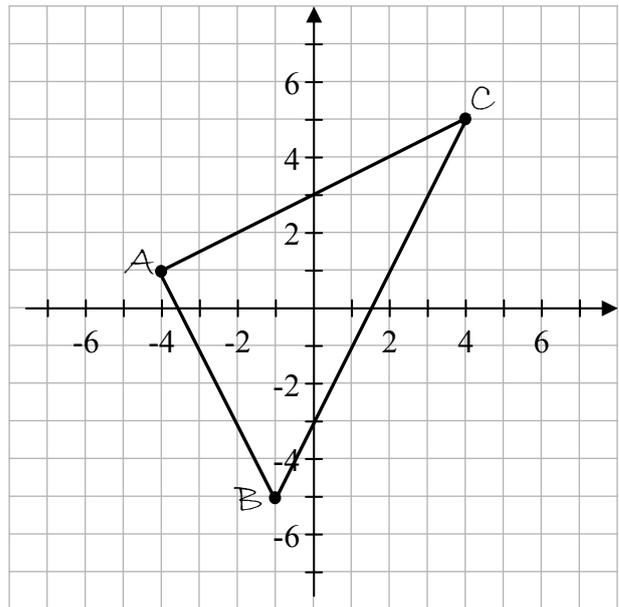
(b) Find the slope of *AB* and the slope of *AC*.

$$\text{Slope } AB = \frac{Rise}{Run} = \frac{-6}{3} = -2$$ **OR** $$\text{Slope } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{-1+4} = \frac{-6}{3} = -2$$

$$\text{Slope } AC = \frac{Rise}{Run} = \frac{4}{8} = \frac{1}{2}$$ **OR** $$\text{Slope } AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{4+4} = \frac{4}{8} = \frac{1}{2}$$

slope of *AB* = ___–2___   slope of *AC* = ___$\frac{1}{2}$___

(c) Show how to use your answers to part **(b)** to decide whether this triangle is right-angled at *A*.

$$\text{Slope } AB \times \text{Slope } AC = (-2)\left(\frac{1}{2}\right) = -1$$

Yes, the triangle is right-angled because AB and AC are perpendicular

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Question 5 (25 marks)

The line *l* has equation $5x + 12y - 60 = 0$. It cuts the *x*-axis at *A* and the *y*-axis at *B*.

(a) Find the co-ordinates of *A* and the co-ordinates of *B*.

x-axis: $y=0$ ⇒ $5x=60$ ⇒ $x=12$ ⇒ $A(12, 0)$

y-axis: $x=0$ ⇒ $12y=60$ ⇒ $y=5$ ⇒ $B(0, 5)$

(b) The point *P* has co-ordinates (5, 3). Show the point *P* and the line *l* on a co-ordinate diagram.

B

5

$P(5, 3)$

5

10   12

A

(c) Prove that *P* does not lie on *l*.

$l:\ 5x+12y-60=0$

Substitute $(5,3)$

⇒ $5(5)+12(3)-60=0$

⇒ $1=0$, which is false

So, P does not lie on l

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Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) In the diagram, *M* is the midpoint of [*AB*] and is also the midpoint of [*CD*].

Show that | *AC* | must be equal to | *BD* |.
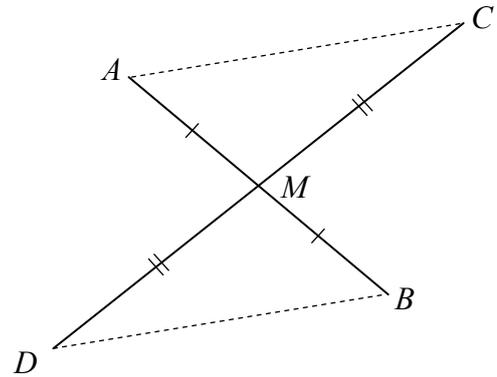
|*AM*| = |*MB*|

|∠*AMC*| = |∠*DMB*| (vertically opposite)

|*CM*| = |*MD*|

⇒   Δ*AMC* ≡ Δ*DMB* (S.A.S)

∴ |*AC*| = |*DB*|

(b) Construct an angle of 60°, without using a protractor or setsquare.

Show all construction lines clearly.

[36]
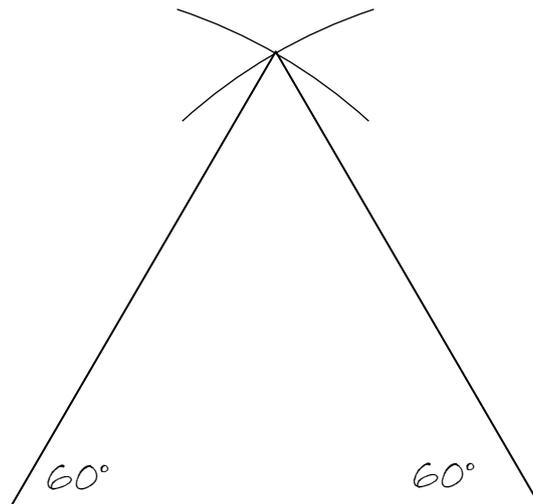
OR

Question 6B

**(a)** The diagram below shows a triangle with one side extended.
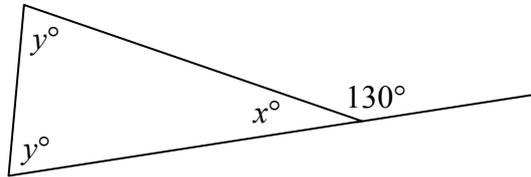
(i) Find the value of x.

$$x = (180° - 130°) = 50°$$

(ii) The other two angles in the triangle are equal. Each is $y°$. Find the value of y.

$$2y° = 130°$$
$$y° = 65°$$

(b) The diagram below shows another triangle with one side extended.
Find the value of s and the value of t.
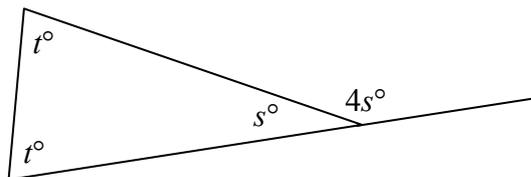
$$4s° + s° = 180°$$
$$5s° = 180°$$
$$\Rightarrow \quad s = 36°$$

$$180° - s° = 2t°$$
$$180° - 36° = 2t$$
$$144° = 2t$$
$$72° = t$$

OR

$$4s° = 2t°$$
$$4(36) = 2t$$
$$144° = 2t$$
$$72° = t$$

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Answer **Question 7** and **Question 8**.

Question 7   **(75 marks)**

Whenever a baby is born, one of the things measured and recorded is the baby's weight. The birth-weights of a sample of babies are summarised in the table below.

Weight in kg	2·2 – 2·6	2·6 – 3·0	3·0 – 3·4	3·4 – 3·8	3·8 – 4·2	4·2 – 4·6
Number of babies	12	40	64	56	24	4

(Source: simulated data, based on multiple sources)

(a)   How many babies were in the sample?

Answer: _200 babies_

(b)   Draw a histogram of the data.
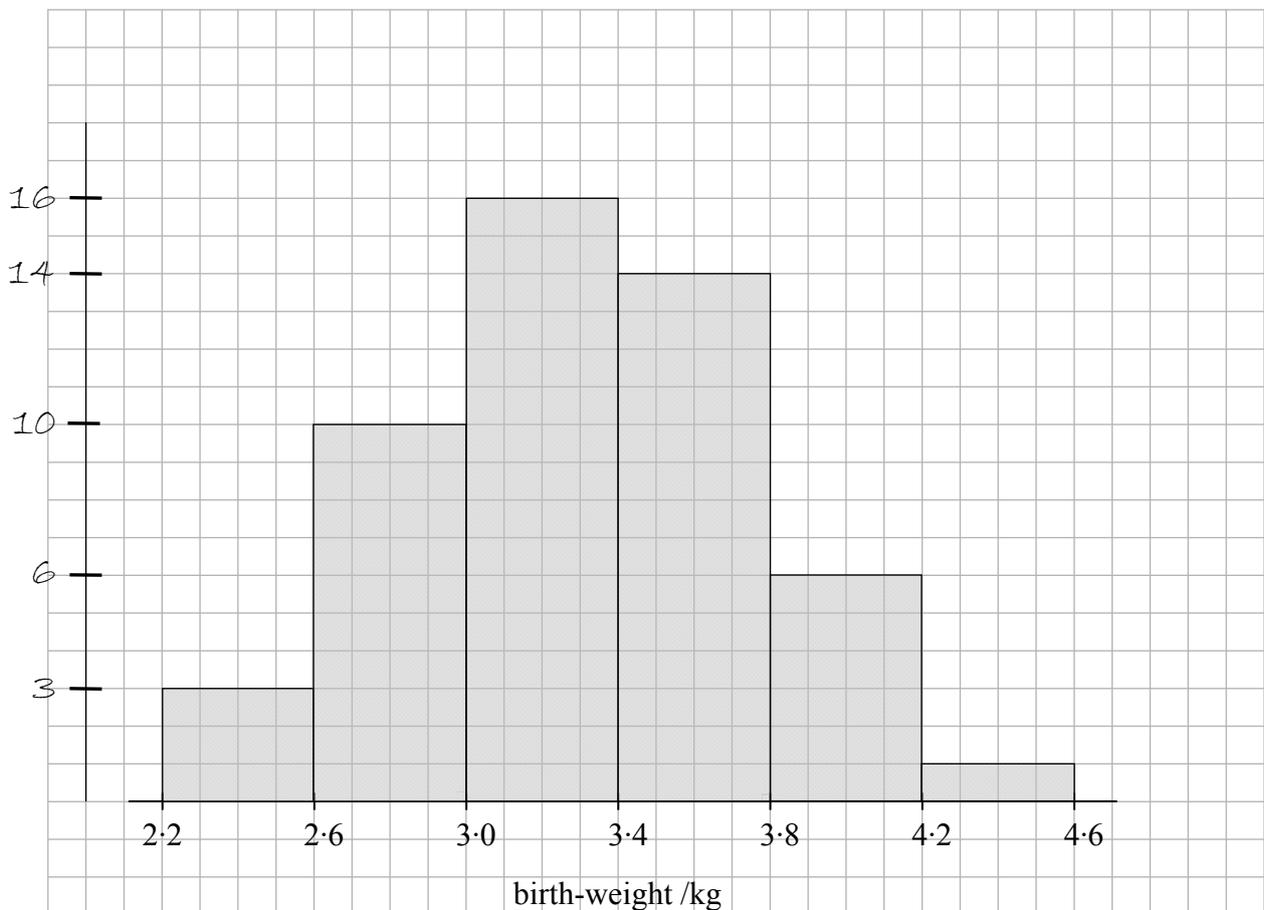
(c)   Complete the following sentence, by using the table and/or the histogram to make an estimate:

"On average, these babies weighed about _3·3_ kg at birth."

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(d) One of the babies weighed 3·675 kg when she was born. How would you describe this baby's weight in comparison to the other babies?

Slightly above average

(e) A weight of less than 2·5 kg is called a "low birth-weight".
Estimate the number of low-birth-weight babies in this sample.

9, (taking data to be evenly distributed in the interval).

OR

7 or 8, (taking data to be more smoothly distributed overall).

Approximately 60 000 babies were born in Ireland in 2005. According to a survey, 20% of the mothers smoked cigarettes during the pregnancy. Suppose that our sample was chosen from among these babies whose mothers smoked.

(f) What is the size of the population from which the sample was drawn?

20% of 60 000 = 12 000

(g) Using the information from the sample, estimate the number of low-birth-weight babies in that population.

e.g. $\frac{9}{200} \times 12000 = 540$

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(h) Explain why the sample cannot tell us *exactly* how many such babies were in the population.

Because, in the sample, some random variation can be expected

(i) The mean birth-weight for all babies born in Ireland that year was 3·51 kg. Do you think that the information from our sample shows that smoking during pregnancy affects the baby's birth weight? Explain your answer.

YES, as the sample mean is lower than the mean

The mean birth rate is 3·51 kg

The mean birth rate for the sample is 3·3 kg

OR

No, Not sure, Can't tell ...

The babies seem lighter but it might be just random variation in the sample

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Question 8 (75 marks)

(a) A jeweller is making a pendant. The design consists of two silver triangles on a rectangular background of copper. The design is shown in the diagram.

The bigger triangle is an enlargement of the smaller triangle. The scale factor of the enlargement is 2.

3 cm

O
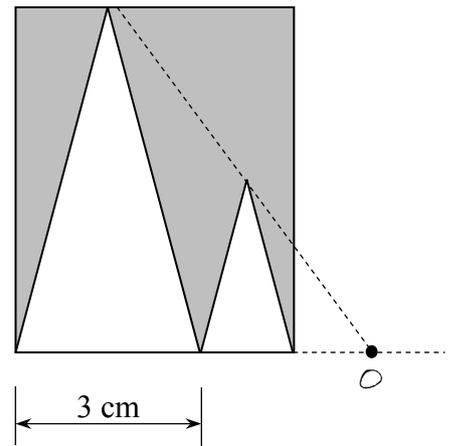
(i) On the diagram, find the centre of enlargement.

(ii) The width of the bigger triangle is 3 cm, as shown. Find the width of the smaller triangle.

Given the Scale factor $= 2 \Rightarrow$ width $= \frac{1}{2}(3) = 1 \cdot 5$ cm

(iii) The height of the smaller triangle is 2·8 cm. Find the **area** of the bigger triangle.

Given the Scale factor $= 2 \Rightarrow$ height $= 2(2 \cdot 8) = 5 \cdot 6$ cm

Area $= \frac{1}{2}(3)(5 \cdot 6) = 8 \cdot 4$ cm^2

(iv) What fraction of the area of the copper rectangle is covered by the silver triangles?

Copper rectangle:

$h = 5 \cdot 6$

$w = 3 + 1 \cdot 5 = 4 \cdot 5$

Area $= 4 \cdot 5 \times 5 \cdot 6 = 25 \cdot 2$

Area Big Δ + Area Small Δ

$8 \cdot 4 + \frac{1}{2}(1 \cdot 5)(2 \cdot 8)$ OR $8 \cdot 4 + \dfrac{8 \cdot 4}{2^2}$

$8 \cdot 4 + 2 \cdot 1$ $8 \cdot 4 + 2 \cdot 1$

$10 \cdot 5$ $10 \cdot 5$

Fraction $= \dfrac{10 \cdot 5}{25 \cdot 2} = \dfrac{5}{12}$

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(b) The jeweller is making some earrings to go with the pendant. Each earring is an isosceles triangle. The triangle is half copper and half silver, as shown in the diagram. The measurements are as shown.

d

4 cm

2 cm
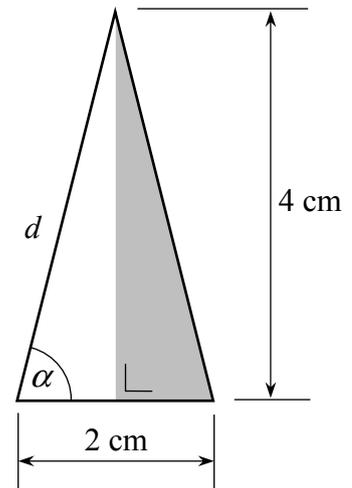
(i) Use Pythagoras' theorem to find d, the length of one of the sloping sides. Give your answer correct to one decimal place.

$$d^2 = 4^2 + 1^2$$
$$d^2 = 17$$
$$d = \sqrt{17} \approx 4 \cdot 1 \ cm$$

d 4

1

(ii) Find $|\angle \alpha|$, correct to the nearest degree.

$$\sin \alpha = \frac{4}{4 \cdot 1} \qquad \Rightarrow \quad \alpha \approx 77°$$

OR

$$\cos \alpha = \frac{1}{4 \cdot 1} \qquad \Rightarrow \quad \alpha \approx 76°$$

OR

$$\tan \alpha = \frac{4}{1}$$
$$\alpha = 75 \cdot 96...° \qquad \Rightarrow \alpha \approx 76°$$

4·1 4

α 1

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(c) The jeweller needs a drawing of the earring design. She wants the drawing to be bigger than the actual earring.

Construct, as accurately as you can, a drawing of the earring at a scale of 2:1. That is, each centimetre in reality should be 2 centimetres in your drawing.

8 cm

4 cm
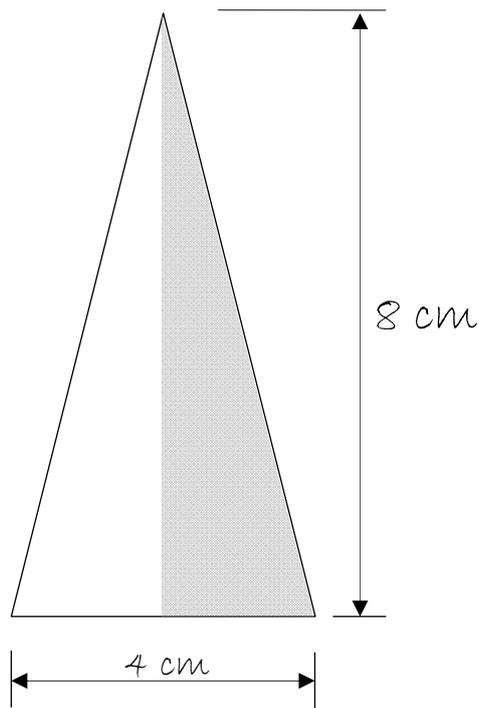
Marking Scheme – Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C
No of categories	2	3	4
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5
10 mark scale		0, 8 10	0, 6, 9, 10
15 mark scale		0, 10, 15	0, 9, 13,15
20 mark scale		0, 15, 20	0, 12, 17, 20
25 mark scale		0, 20, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	20C
(b)	5C

Question 2

(a)	20C
(b)(c)&(d)	5C

Question 3

(a)	15C
(b)	5B
(c)&(d)	5C

Question 4

(a)	15C
(b)&(c)	10C

Question 5

(a)	10B
(b)	10C
(c)	5C

Question 6A

(a)	10C
(b)	15C

Question 6B

(a)(i)	10B
(a)(ii)	10C
(b)	5B

Section B

Question 7

(a)	20B
(b)	10C
(c)	10B
(d)	10B
(e)	5C
(f)	10B
(g)&(h)	5C
(i)	5B

Question 8

(a)(i)	5B
(a)(ii)	25B*
(a)(iii)	20C*
(a)(iv)	5C
(b)(i)&(ii)	5C*
(c)	15C

Detailed marking notes

Section A

Question 1
(a)   <u>Scale 20C</u>

        High partial credit: A correct experiment but the two possible outcomes are not stated.
An experiment with more than two equally likely outcomes, with the outcomes **stated.**

        Low partial credit: An experiment with more than two outcomes.
An experiment with more than two equally likely outcomes, with the outcomes **not stated.**

(b)   <u>Scale 5C</u>

        High partial credit: A correct experiment but the two possible outcomes are not stated.
A correct experiment, outcomes stated, but the two outcomes are equally likely.

        Low partial credit: An incorrect experiment with the outcomes stated.

        Note here that it is possible that an answer that appears to be incorrect may be made correct by the use of an appropriate sample space.

Question 2
(a)   <u>Scale 20C</u>

        High partial credit: Two correct outcomes.

        Low partial credit: One correct outcome.

(b), (c) and (d)

        <u>Scale 5C</u>

        High partial credit: Parts (b) **AND** (c) correct.
Part (d) correct.

        Low partial credit: Part (b) **OR** (c) correct.
Work of merit in any of parts (b), (c) or (d).

Question 3

(a)   <u>Scale 15C</u>

            High partial credit: Answer = 88 or 4·3.

            Low partial credit: Answer = 43 or 8 or 9 or any incorrect two digit value from the data in the table.

(b)   <u>Scale 5B</u>

            Partial credit: Median given as 7·0, 7·1 or 7·4.
                                         Mode (i.e. 5·1 and / or 7·8)
                                         Mean (i.e. 67·69...)

(c) and (d) <u>Scale 5C</u>

            High partial credit: Either (c) **OR** (d) correct.
.

            Low partial credit: Any work of merit in either (c) or (d):
                                         e.g. any reasonable reference to the shape of the distribution in (c) or any meaningful attempt at a correct reason for the shape of the distribution in (d).

Question 4

(a)   <u>Scale 15C</u>

            High partial credit: Two or three points plotted correctly without showing the triangle.
                                         x-axis and y-axis obviously interchanged, (work otherwise correct) or co-ordinates treated as (y, x).

            Low partial credit: One point correctly plotted.

(b) and (c) <u>Scale 10C</u>

            High partial credit: Either (b) **OR** (c) correct.

            Low partial credit: Work of merit in (b) or (c), e.g. correct formula identified, with / without substitution.
                                         As above with partial calculation or no calculation.

Question 5

(a)   Scale 10B
            Partial credit: Attempt at substitution into *x* and / or *y*.
                                        $x = 0$ OR $y = 0$ written.
                                        $5x + 12y = 60$ written.

(b)   Scale 10C
            High partial credit: Correctly scaled axes and <u>either</u> point P or line *l* drawn.

            Low partial credit: Axes only drawn.

(c)   Scale 5C
            High partial credit: Error(s) in substitution / calculation **and** finishes correctly.
                                        Calculations correct but omits conclusion.

            Low partial credit: Correct substitution for *x* and / or *y* and no further work.
                                        Error(s) in substitution / calculation and does **not** finish.

Question 6A

(a)   Scale 10C
            High partial credit: Two correct steps but conclusion omitted.

            Low partial credit: Any correct step.

(b)   Scale 15C
            High partial credit: Angle constructed outside of tolerance of ±5° but within
                                        tolerance of ±10°.

            Low partial credit: Angle accurate to within ±5° **but not constructed.**
                                        Angle constructed but outside of tolerance of ±10°.

Question 6B

(a)   (i)   Scale 10B
            Partial credit: Any use of 180° or 130°.
                                        Any acute angle other than 50°.
                                        Any correct step.

      (ii) Scale 10C
            High partial credit: $2y \neq 130°$ used and finishes.

            Low partial credit: $2y$ written or some other meaningful work.

(b)   Scale 5B
            Partial credit: Writes 5s or 2t or meaningful use of 180°.
                                        Correctly finds one value.

Section B

Question 7

(a)   <u>Scale 20B</u>
           Partial credit: Any work indicating addition of any of the six frequency values in the table, if not correct or if not an obvious misreading.

(b)   <u>Scale 10C</u>
           High partial credit: Diagram largely correct but with some errors.

           Low partial credit: Any work of merit.

(c)   <u>Scale 10B</u>
           Full credit: Any point estimate in $3 \cdot 0 \leq$ answer $\leq 3 \cdot 5$.
                                     Any reasonable interval estimate containing $3 \cdot 3$.

           Partial credit: Shows some relevant work.

(d)   <u>Scale 10B</u>
           Partial credit: Shows some relevant work.

(e)   <u>Scale 5C</u>
           High partial credit: Answer given as 0, 1 or 12.

           Low partial credit: Shows some relevant work.

(f)   <u>Scale 10B</u>
           Partial credit: Correct method but error(s) made in calculating.
                                       Any correct step, e.g. writes $0 \cdot 2$ or $0 \cdot 20$ or 20/100.

(g) and (h) <u>Scale 5C</u>
           High partial credit: Either (g) **OR** (h) correct.

           Low partial credit: Work of merit in (g) or (h).

(i)   <u>Scale 5B</u>
           Partial credit: Shows some relevant work.

Question 8

(a) (i) <u>Scale 5B</u>
        Partial credit: Any meaningful work towards finding the centre of an
                                        enlargement.

   (ii) <u>Scale 25B*</u>
        Partial credit: Makes some use of 3 and 2
                                        $1 \leq$ answer $< 1 \cdot 5$ (measurement from given diagram)

  (iii) <u>Scale 20C*</u>
        High partial credit: One incorrect dimension used with correct formula and finishes.
                                          Correct formula filled in but calculation not done.

        Low partial credit: Correct formula and no other meaningful work.
                                          Any attempt at multiplying correct dimension(s).

  (iv) <u>Scale 5C</u>
        High partial credit: Correct method with some errors.

        Low partial credit: Correct area of rectangle or of either triangle, written or used.
                                          Any correct relevant step.

(b) (i) and (ii)
        <u>Scale 5C*</u>
        High partial credit: Either (i) **OR** (ii) correct.

        Low partial credit: Work of merit in (i) or (ii):
                                          e.g. states Pythagoras theorem in (i);
                                          Any use of 4 or 1;
                                          Any correct trigonometric ratio for α in (ii).

(c)   <u>Scale 15C</u>
        High partial credit: One side correct.

        Low partial credit: Any triangle drawn.

Marcanna Breise as ucht Freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar.   Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$ 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0