# LEAVING CERTIFICATE EXAMINATION 2002 

# Marking Scheme <br> MATHEMATICS - FOUNDATION LEVEL 

## PAPER 2

## General Instructions to Examiners:

Note: The slips, misreadings, blunders and attempts listed in the Marking scheme are not exhaustive.

1. Penalties:
(a) Numerical slips e.g. $4 \times 8=36$
(b) Misreading, if not serious
(c) Mathematical blunders, omissions
(d) Serious blunders, misreadings or omissions may result in the loss of all marks for a particular section or may result in the attempt mark being awarded.
(e) The same error in the same section of a question is penalised once only.
2. Marking scripts:
(a) Mark scripts in red or a colour not used by the candidate.
(b) Mark deductions as ( -1 ) or ( -3 ) on the script where they occur.
(c) Show section marks in the right hand margin.
(d) Indicate attempt marks on the right margin as Att. 4, for example.
(e) Show total marks awarded for each question on the left hand margin near the start of the question and ring the mark.
(f) Worthless or irrelevant work should be marked 0 .
(g) Scrutinise all pages and indicate by marking pages.
(h) Mark all questions, including cancelled non-repeated questions and allow the highest scoring answers.

## 3. Attempt Marks:

(a) If deductions result in a mark which is lower than the attempt mark, award the attempt mark.
(b) The attempt mark for a section is the final mark for that section and so a mark between 0 and the mark may not be awarded.
(c) The attempt mark must be awarded for any relevant work.
(d) Particular cases or verifications qualify for the attempt mark in general.

## QUESTION 1



- Square ... 10 marks, circle ... 5 marks, shaded area ... 5 marks
- Units not required
- Accept $\pi=\frac{22}{7}$ or $\pi$ value from calculator
- $\quad$ Accept $12.56-16=3.44$


## Blunders (-3)

B1 Incorrect area of square formula.
B2 Incorrect area of circle formula - must include $\pi$.
B3 Failure to subtract.
B4 Mistakes circle for sphere using
i. Area $=\frac{4}{3} \pi r^{3} \Rightarrow 33.49$ when $r=2$ or 267.95 when $r=4$.
ii. Area $=4 \pi r^{2} \Rightarrow 50.24$ when $r=2$ or 200.96 when $r=4$.

B5 Takes square as a cube, i.e. uses Area $=a^{3}$ or $6 a^{2}$
Slips (-1)
S1 Takes $r=4 \Rightarrow$ area of circle $=50.24 \mathrm{~cm}^{2}$.
S2 No round off, incorrect round off.
S3 Error in calculation, square or circle.
Attempts (8)
A1 Area $=4+4$ and stops, for square.
A2 Additional material on diagram, for square / circle.
A3 Any incorrect formula with substitution, for square / circle.
A4 $a^{2}$ or $a b$ only...att. $4, \pi r^{2}$ only, ...att.2.
A5 Any circle formula from formula sheet only, for square / circle.
A6 $r=2$, circle only.
A7 Some attempt at Pythagoras e.g. $x^{2}=4^{2}+4^{2}$ for circle. However $4^{2}$ only merits 9 marks for square.
(b) The diagram below shows a stained section of carpet.


Offsets of lengths $4,6,5,10,12,8$ and 9 centimetres are measured at intervals of 5 centimetres along $[c d]$.
(i) Calculate the area of the stained section using Simpson's Rule.
(ii) Find, in $\mathrm{cm}^{2}$, the area of the smallest rectangular rug that could be used to cover the stained section.

## Solution

(i) Area $=\frac{h}{3}[F+L+$ TOFE $]$

$$
\begin{aligned}
& =\frac{5}{3}[4+9+2(5+12)+4(6+10+8)] \\
& =\frac{5}{3}[4+9+2(17)+4(24)]=\frac{5}{3}[4+9+2(17)+4(24)] \\
& =\frac{5}{3}[4+9+34+96]=\frac{5}{3}[143]=238.3 \mathrm{~cm}^{2}
\end{aligned}
$$

(ii) Smallest rectangular rug $=(5 \times 6) \times 12=360 \mathrm{~cm}^{2}$.

- Units not required


## Blunders (-3)

B1 Incorrect relevant formula e.g. 'four odd and twice even'.
B2 Omits 2 or 4 in equation: omits $2 \Rightarrow 210 \mathrm{~cm}^{2}$

$$
\text { omits } 4 \Rightarrow 118.3 \mathrm{~cm}^{2}
$$

B3 Uses forward or backward rectangulation
Forward: $\quad 5[4+6+5+10+12+8]=5 \times 45=225 \mathrm{~cm}^{2}$
Backward: $5[6+5+10+12+8+9]=5 \times 50=250 \mathrm{~cm}^{2}$.
B4 Uses trapezoidal method i.e.

$$
\begin{aligned}
& 5\left[\frac{1}{2}(4+6)+\frac{1}{2}(6+5)+\frac{1}{2}(5+10)+\frac{1}{2}(10+12)+\frac{1}{2}(12+8)+\frac{1}{2}(8+9)\right] \\
& =5[5+5.5+7.5+11+10+8.5]=5 \times 47.5=237.5 \mathrm{~cm}^{2} .
\end{aligned}
$$

B5 Omits $h$ or uses incorrect $h$.
B6 $\frac{h}{3}=143 \Rightarrow h=429$

B7 Fails to finish $\frac{5}{3}[143]$ and stops.
B8 Area of rug $=5 \times 4=20 \quad$ or $=5 \times 6=30$

$$
\begin{array}{ll}
\text { or }=5 \times 5=25 & \text { or }=5 \times 10=50 \\
\text { or }=5 \times 8=40 & \text { or }=5 \times 9=45
\end{array}
$$

Slips (-1)
S1 Each incorrect altitude.
S2 Interchanges odd and even

$$
\begin{aligned}
\text { Area } & =\frac{5}{3}[4+9+2(6+10+8)+4(5+12)] \\
& =\frac{5}{3}[13+48+68]=\frac{5}{3}[129]=215 \mathrm{~cm}^{2}, \text { merits } 24 \text { marks. }
\end{aligned}
$$

S3 Area of rug = $30 \times 4=120 \quad$ or $=30 \times 6=180$

$$
\begin{gathered}
\text { or }=30 \times 5=150 \quad \text { or }=30 \times 10=300 \\
\text { or }=30 \times 8=240 \quad \text { or }=30 \times 9=270 \\
\text { or }=5 \times 12=60
\end{gathered}
$$

Attempts (10, 2)
A1 Simpson's Rule, formula only
A2 Worked diagram.
A3 Area of rug $=5 \times($ any number other than above $)$
or $=30 \times$ ( any number other than above) .
A4 Area of rug $=30 \mathrm{~cm}^{2}$.

Part (a)
20 Marks
30 Marks
20 Marks
.
ng dimensions: A trapezium has the follo
the length of $[a b]$ is 8 cm the length of $[d c]$ is 12 cm , and the height is 7 cm .

Find the area of the trapezium.


Solution

$$
\text { Area }=\frac{h(a+b)}{2}=\frac{7(8+12)}{2}=\frac{7(20)}{2}=70 \mathrm{~cm}^{2}
$$

- Units not required


## Blunders (-3)

B1 Incorrect area formula e.g. area $=\frac{a h}{2}$

$$
\Rightarrow \quad \frac{12 \times 7}{2}=42 \quad \text { or } \quad \frac{8 \times 7}{2}=28
$$

B2 Correct formula, incorrect substitution e.g. $\frac{12(8+7)}{2}=90, \frac{8(12+7)}{2}=76$
B3 Failure to divide by $2 \Rightarrow$ Area $=140 \mathrm{~cm}^{2}$
B4 $8+12=20$ only, apply $2 \times(-3)$ i. no multiplication by $h$. ii. no division by 2 .

B5 $8+12$ and stops, apply $2 \times(-3)+(-1)$
B6 $8 \times 12=96$, apply $3 \times(-3)$ i. multiplication for addition.
ii. no multiplication by 7 .
iii. no division by 2 .

B7 $8 \times 7=56$ or $12 \times 7=84$
apply $2 \times(-3)$,
i.e. omission and no division by 2 .

B8 $8+7=15$ or $12+7=19$
apply $3 \times(-3)$
B9 $8+12+7=27$
apply $2 \times(-3)$
B10 $8 \times 12 \times 7=672 \quad$ apply $2 \times(-3)$

## Slips (-1)

S1 Numerical $8 \times 7=54$

## Attempts (8)

A1 Additional material on a diagram
A2 Correct or relevant formula only.
(i) Find in $\mathrm{cm}^{3}$, the volume of a metal sphere of diameter 18 cm .

Take $\pi=3.14$

## Solution:

(i) Volume of sphere $=\frac{4}{3} \pi r^{3}=\frac{4}{3} \times \pi \times 9^{3}=3052.08 \mathrm{~cm}^{3}$.

- No penalty for decimal round off in (i) or (ii).
- Accept $\pi=\frac{22}{7}$ or $\pi$ value from calculator.
- Accept $972 \pi$ in (i) for full marks.


## Blunders (-3)

B1 Ignores $\pi$ in calculations $\Rightarrow$ Volume $=972$.
B2 $r^{3}=3 r$.
B3 Incorrect volume formula involving $\pi$, e.g. $\pi r^{2} h$ or $\frac{1}{3} \pi r^{2} h$.
B4 Uses surface area formula $4 \pi r^{2}$.
B5 Uses other area formula e.g. $\pi r^{2}, \pi r l, 2 \pi r h$ apply 2(-3)
B6 $\frac{4}{3} \times \pi \times 9^{3}$ and stops or evaluates incorrectly

## Misreadings (-1)

M1 Uses $r=18 \Rightarrow$ volume $=24416.64 \mathrm{~cm}^{3}$.

Slips (-1)
S1 Numerical errors.

Attempts (6).
A1 $18 \times 18=324$.
A2 $9 \times 9=81$
A3 Diagram only
A4 Diagram of cone, cylinder or hemisphere with measurements included.
A5 Correct or relevant formula only, circle or volume.
A6 Incorrect formula with some substitution
A7 $9 \times 3.14$ or $18 \times 3.14$.

Att. 6
(ii) Two such spheres are melted down and recast as a single cylinder of diameter 12 cm .

Calculate the height of the cylinder.

## Solution:

(ii)


Blunders (-3)
B1 Ignores $\pi$ in calculations.
B2 $r^{2}=2 r$
B3 Incorrect volume formula, must involve $\pi$, e.g. $\frac{4}{3} \pi r^{3}$ or $\frac{1}{3} \pi r^{2} h$.
B4 Uses formula for surface area, i.e. area $=2 \pi r h \Rightarrow h=\frac{\text { area }}{2 \times \pi \times r}$
This approach leads to $h=162$
B5 Final answer contains $\pi$.
B6 Uses other area formula e.g. $4 \pi r^{2}$, $\pi r l$, apply $2 \times(-3)$
B7 Incorrect transposition when using Volume $=\pi r^{2} h$

## Misreadings (-1)

M1 Uses $r=12$, this gives $h=13.5 \mathrm{~cm}$
M2 Fails to multiply volume of sphere by 2 .
M3 Halves volume of sphere, this gives $h=13.5 \mathrm{~cm}$

## Slips (-1)

S1 Numerical errors.
S2 Uses multiplication key for division
$h=6104.16 \div 3.14 \times 6 \times 6=69984$
$h=4 \times 6104.16 \div 3.14 \times 12 \times 12=1119744$

## Attempts (6)

A1 Diagram only
A2 Correct formula only
A3 Any formula containing $\pi$, with some substitution.


- Accept answers on a diagram
- Accept answer written as $65^{\circ}-180^{\circ}=115^{\circ}$.


## Blunders (-3)

B1 Sum of the internal angles $=90^{\circ}$ or $100^{\circ}$ or $200^{\circ}$ or $360^{\circ}$

$$
\begin{array}{ll}
\text { Sum }=90^{\circ} \Rightarrow x=12.5^{\circ} & \text { Sum }=100^{\circ} \Rightarrow x=17.5^{\circ} \\
\text { Sum }=200^{\circ} \Rightarrow x=67.5^{\circ} & \text { Sum }=360^{\circ} \Rightarrow x=147.5^{\circ}
\end{array}
$$

B2 Straight angle $=90^{\circ}$ or $100^{\circ}$ or $200^{\circ}$ or $360^{\circ}$
B3 Incorrect isosceles $x=65^{\circ}$, no penalty then for $y=130^{\circ}$, but ( -3 ) for $y=50^{\circ}$, or $x=50^{\circ}$, no penalty then for $y=115^{\circ}$.
B4 Incorrect alternate angles e.g. $y=65^{\circ}$
B5 Incorrect corresponding angles e.g. $y=x$
B6 $x=180-65=115^{\circ}$, i.e. omits 2

Slips (-1)
S1 Numerical 180-65 = 105

## Misreadings (-1)

M1 Angle given $=85^{\circ}$ or similar excluding $60^{\circ}, 90^{\circ}$.
M2 $\quad y$ indicated as adjacent internal angle $\Rightarrow y=57.5^{\circ}, 65^{\circ}$ or $50^{\circ}$ in (i).
Attempts (2, 2)
A1 Additional material on a diagram, equal angles indicated.
A2 Visual inspection, e.g. $x=60^{\circ}$ or $y=120^{\circ}$
A3 Clear use of protractor or graphical approach ( $\pm 5^{\circ}$ )
(b) $\quad a b c d$ is a parallelogram.
$|a d|=7 \mathrm{~cm},|d c|=6 \mathrm{~cm}$ and $|\angle a b c|=70^{\circ}$
Find:
(i) the length of [bc]
(ii) the measure of $\angle b c d$
(iii) the measure of $\angle a d c$
(iv) the measure of $\angle e a b$


## Solution

$\begin{array}{ll}\text { (i) } & |b c|=7 \\ \text { (iii) } & |\angle a d c|=70^{\circ}\end{array}$
(ii) $|\angle b c d|=180-70=110^{\circ}$
(iv) $\quad|\angle e a b|=70^{\circ}$

- Accept answers written on diagram.
- Accept sides or angles indicated.
- Accept $109^{\circ} 60^{\prime}$ for $110^{\circ}$.


## Blunders ( -3 )

B1 $|b c|=6$
B2 $\quad|\angle b c d|=70^{\circ}$
B3 $\quad|\angle a d c|=110^{\circ}$
B4 $\quad|\angle e a b|=110^{\circ}$, but allow for full marks if $|\angle a d c|=110^{\circ}$
B5 Interior opposite angles sum to $90^{\circ} / 100^{\circ} / 200^{\circ} / 360^{\circ}$
$\Rightarrow|\angle b c d|=20^{\circ} / 30^{\circ} / 130^{\circ} / 290^{\circ}$.
B6 Straight angle $=90^{\circ} / 100^{\circ} / 200^{\circ} / 360^{\circ}$ e.g. $|\angle e a b|=250^{\circ}$ for the $360^{\circ}$ case.

Slips (-1)
S1 Numerical e.g. $180^{\circ}-65^{\circ}=105^{\circ}$
Attempts (2, 2, 2, 2)
A1 Additional material on a diagram, apply each time to (iii) and (iv) only.
e.g. (i) diagonals drawn (ii) $\Delta a e b$ drawn (iii) $\square$ abfe completed,
single diagram having diagonal and $\square$ abfe only merits $2 \times(2)$.
A2 Sides or angles measured e.g. $|b c|=2.5 \mathrm{~cm}$ or 1 inch.
A3 $|b c|=70^{\circ}$ or $110^{\circ}$
A4 $|\angle b c d|$ or $|\angle a d c|$ or $|\angle e a b|=6$ or 7 or 13 .
A5 $7 \times 6$ or $7+6$, apply each time.
(c) $[p r]$ is the diameter of a circle with centre $c . t$ is a point on the circle.
$|p t|=9 \mathrm{~cm},|r t|=12 \mathrm{~cm}$ and $|\angle p r t|=37^{\circ}$.

Find:
(i) the measure of $\angle p t r$
(ii) the measure of $\angle t p r$
(iii) the length of $[\mathrm{cr}]$
(iv) the area of the triangle $t p r$.


Solution
(i) $|\angle p t r|=90^{\circ}$
(ii) $|\angle t p r|=53^{\circ}$
(iii) $|p r|^{2}=9^{2}+12^{2}=225 \Rightarrow|p r|=15 \mathrm{~cm}$
$\Rightarrow|c r|=7.5 \mathrm{~cm}$
(iv) Area $=\frac{1}{2} \times 12 \times 9=54 \mathrm{~cm}^{2}$

- Accept answers written on diagram.
- Accept $|\angle t p r|=\tan ^{-1}\left(\frac{12}{9}\right)=53.13^{\circ}$


## Blunders (-3)

B1 $\quad|\angle p t r|=180^{\circ}$ or $360^{\circ}$ or $37^{\circ}$ or $53^{\circ}$
B2 $|\angle t p r|=180^{\circ}$ i.e. sum of the angles in triangle.
B3 $|\angle t p r|=37^{\circ}$ i.e. isosceles
B4 $|\angle t p r|=180^{\circ}-2\left(37^{\circ}\right)=106^{\circ}$, not applied if B1 applied above.
B5 $\quad|\angle t p r|=180^{\circ}-37^{\circ}=143^{\circ}$
B6 $|\angle t p r|=360^{\circ}-37^{\circ}=323^{\circ}$

Slips (-1)
S1 Numerical errors e.g. $180-37=133$
S2 In (iii) no square root sign i.e. $|p r|=225 \Rightarrow|c r|=112.5$.
S3 In (iii) or (iv), failure to divide by 2.
S4 $\quad|p r|^{2}=225$ and stops, apply $2 \times(-1)$
S5 $\sqrt{12^{2}-9^{2}}=\sqrt{63}$
S6 Error in Pythagoras
S7 In (iv), incorrect perpendicular height

$$
\text { e.g. } \frac{1}{2}(9 \times 15)=67.5 \text { or } \frac{1}{2}(12 \times 15)=90
$$

## Misreadings (-1)

M1 Angle given $=73^{\circ}$

Attempts (2, 2, 2, 2)
A1 Additional material on a diagram
A2 Clear use of protactor in (i) and (ii), or use of ruler in (iii).
A3 In (iii), $12 \leq|p r| \leq 21$
A4 In (iii), $|p r|$ or $|c r|=180^{\circ}$
A5 $\quad$ In (iv), Area $=9+12+15=36$
Worthless
W1 $\quad 37 \pm 9,37 \pm 12$
W2 $\quad 37 \times$ or $\div 9,37 \times$ or $\div 12$.

Part (a)
Part (b)
Part (c)
Part (a)

## )

$\qquad$
5

20 Marks
Att. 8
15 Marks
15 Marks
Att. 6
Att. 6
(a) Plot the points $a(-4,3)$ and $b(6,5)$ on graph paper. Find the midpoint of $[a b]$.


- Accept correct midpoint from graph.
- Accept plot on answer book paper.


## Blunders ( -3 )

B1 Error in axes, unsatisfactory calibration.
B2 One correct plot and stops
B3 Incorrect midpoint formula, e.g. $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$ or equivalent.
B4 Calculates 'endpoint' using $(-4,3)$ or $(6,5)$ as midpoints $\Rightarrow(-14,1)$ or $(16,7)$.
B5 One correct ordinate only, e.g. $\frac{-4+6}{2}=1$ or $\frac{3+5}{2}=4$
B6 Correct slope or distance $\Rightarrow \quad m=\frac{2}{10}$ or $|a b|=\sqrt{104}$.
B7 Incorrect slope or distance formula, apply 2(-3).
B8 Incorrect midpoint from graph.
Slips (-1)
S1 Each incorrect addition, subtraction or division.
S2 Each incorrect substitution.
S3 $\left(\frac{2}{2}, \frac{8}{2}\right)$ and stops.
S4 Interchanges $x$ and $y$ axes, interchanges + and - directions.
Misreadings (-1)
M1 Plots $(4,3)$ for $(-4,3)$ or similar.

## Attempts $(4,4)$

A1 Axes only.
A2 Copies correct formula from formula sheet only
A3 Joins $a$ to $b$.
A4 Translation (10,2).
(b) $\quad p(-4,-5)$ and $q(3,-7)$ are points.
(i) Find the length of $[p q]$
(ii) Find the slope of $p q$.
(iii) Find the equation of the line $p q$.

## Solution

$$
\begin{align*}
|p q|=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} & =\sqrt{(3+4)^{2}+(-7+5)^{2}}  \tag{i}\\
& =\sqrt{(7)^{2}+(-2)^{2}}=\sqrt{53}=7.28
\end{align*}
$$

(ii) Slope: $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-7+5}{3+4}=-\frac{2}{7}$
(iii) Equation: $y-y_{1}=m\left(x-x_{1}\right), \quad y+5=-\frac{2}{7}(x+4)$,
$7 y+35=-2 x-8, \quad 2 x+7 y=-43$

- Accept answer to (i) in surd form.
- In (i) accept correct length on graph.
- In (ii) accept correct slope on graph.
- In (iii) accept $y+5=-\frac{2}{7}(x+4)$ for full marks.


## Blunders (-3)

B1 Error in formula.
B2 Uses incorrect formula ie. midpoint or slope in (i). midpoint or distance in (ii), midpoint, slope or distance in (iii).

## Slips (-1)

S1 Each incorrect addition, subtraction, multiplication, division or squaring.
S2 In (i), omits square root.
S3 In (ii), $\frac{-7+5}{3+4}$ and stops.
S4 In (iii), incorrect slope used.
S5 Each incorrect substitution, maximum deduction (-3).
S6 Ordered pairs reversed.

## Misreadings (-1)

M1 Uses $a$ and / or $b$ from part (a), apply once only.
Attempts (2, 2, 2)
A1 Correct formula copied from formula sheet only.
A2 Incorrect formula with any substitution.
A3 Points plotted correctly or incorrectly (once only).
A4 7 only in (i), no work shown.
(c) The line $K$ has equation $3 y=-4 x+9$.
(i) Write down the slope of $K$.
(ii) Show that the point $(0,3)$ lies on the line $K$.
(iii) Find the equation of the line $L$, which passes through the point $(1,-2)$ and is perpendicular to $K$.

## Solution

$K: \quad 3 y=-4 x+9$
(i) $y=-\frac{4}{3} x+3 \Rightarrow$ slope $=-\frac{4}{3}$
(ii) $\quad(0,3) \in K: \quad 3 y=-4 x+9$

$$
3(3)=-4(0)+9 \Rightarrow 9=9 .
$$

(iii) $m_{\perp}=\frac{3}{4}$, point $(1,-2)$

Equation: $y+2=\frac{3}{4}(x-1)$

$$
4 y+8=3 x-3 \Rightarrow 3 x-4 y=11
$$

- In (iii), accept $y+2=\frac{3}{4}(x-1)$ for full marks.


## Blunders (-3)

B1 In (i), slope $= \pm 3, \pm 4, \pm 9, \pm \frac{9}{3}, \pm \frac{3}{9}, \pm \frac{4}{9}, \pm \frac{9}{4}$.
B2 Error in equation of the line formula.
B3 $y=-\frac{4}{3} x+c$ in (iii), however this merits full marks in (i).
Slips (-1)
S1 In (i), slope $= \pm \frac{3}{4}$ or $\frac{4}{3}$.
S2 In (i), uses $(0,3)$ and another incorrect point to calculate slope.
S3 Each incorrect multiplication in (ii).
S4 Reverses coordinates in (ii) and / or (iii).
S5 Incorrect slope in (iii), or $y+2=m(x-1)$.
S6 Each incorrect substitution, subject to a maximum of ( -3 ), in (ii) and (iii).

## Attempts (2, 2, 2)

A1 In (i), any correct transposition.
A2 Any attempt to draw line $K$.
A3 In (i) attempts to find a second point, $\neq(0,3)$, on $K$.
A4 In (i), copies correct slope formula from formula sheet.
A5 In (ii), attempt to plot $(0,3)$.
A6 Copies correct line equation formula in (iii).
A7 In (iii) draws perpendicular lines.
A8 In (iii), displays knowledge of $m_{1} m_{2}=-1$.
A9 In (iii), attempts to plot ( $1,-2$ ).

Part (a)
Part (b)
Part (c)
Part (a)
15 Marks
20 Marks
15 Marks
Att. 6
Att. 8
Att. 6
$15(5,5,5)$ Marks
Att. 6(2, 2, 2)
(a) Given that $\tan \mathrm{A}=\frac{12}{5}$ write down, the value of :
(i) $\quad \sin \mathrm{A}$
(ii) $\quad \cos \mathrm{A}$
(iii) $\tan \mathrm{B}$.


## Solution

(i) $\quad \sin A=\frac{12}{13}$
(ii) $\cos A=\frac{5}{13}$
(iii) $\tan B=\frac{5}{12}$

- Accept fractions or decimals,
e.g. $\sin \mathrm{A}=0.9231, \cos \mathrm{~A}=0.3846, \tan \mathrm{~B}=0.4167$, allow two decimal places.
- Accept $\sin \left(\frac{12}{13}\right), \cos \left(\frac{5}{13}\right), \tan \left(\frac{5}{12}\right)$ for $\sin \mathrm{A}=\frac{12}{13}$ etc.
- If $\sin A=\frac{5}{13}$, no penalty for $\cos A=\frac{12}{13}$ and $\tan B=\frac{12}{5}$


## Blunders (-3)

B1 Incorrect trigonometric ratio.
B2 In (i), substitutes into Sine Rule and stops.
Slips (-1)
S1 Inverted ratio, i.e. $\sin A=\frac{13}{12}, \cos A=\frac{13}{5}, \tan B=\frac{12}{5}$
S2 Truncation $67.38^{\circ}=67^{\circ}$ or $68^{\circ}$ apply once only, then calculates
$\sin 67^{\circ}=0.9205, \sin 68^{\circ}=0.9272$,
$\cos 67^{\circ}=0.3903, \cos 68^{\circ}=0.3746$
$\tan 23^{\circ}=0.4249, \tan 22^{\circ}=0.4040$

Attempts (2, 2, 2)
A1 $\sin =\frac{o p p}{h y p}$ only, same for tan and cos definitions.
A2 Mnemonic (SOHCAHTOA) or equivalent in (i), (ii), (iii), apply att. mark to all three.
A3 Writes Sine Rule or Cosine Rule.
A4 Correct $\sin / \cos /$ tan for some stated angle.
A5 Statement of, or attempt at, Pythagoras.

A6 The following table is a compendium of sources of incorrect answers.

|  | Degrees | Radians | Grads |
| :--- | :---: | :---: | :---: |
| $\sin 5$ | 0.0872 | -0.9589 | 0.0785 |
| $\sin 12$ | 0.2079 | -0.5366 | 0.1874 |
| $\sin 13$ | 0.2250 | 0.4201 | 0.2028 |
|  |  |  |  |
| $\cos 5$ | 0.9962 | 0.2837 | 0.9969 |
| $\cos 12$ | 0.9781 | 0.8439 | 0.9823 |
| $\cos 13$ | 0.9744 | 0.9074 | 0.9792 |
|  |  |  |  |
| $\tan 5$ | 0.0875 | -3.3805 | 0.0787 |
| $\tan 12$ | 0.2126 | -0.6359 | 0.1908 |
| $\tan 13$ | 0.2309 | 0.4630 | 0.2071 |

(b) Find $\tan X$ and write your answer as a decimal.

Hence, find the measure of the angle $X$, Correct to the nearest degree.


## Solution

$$
\tan X=\frac{5}{6}=0.8333 \Rightarrow X=\tan ^{-1}\left(\frac{5}{6}\right)=39.81^{\circ}=40^{\circ}
$$

- Accept correct answer for full marks
- Full marks for $40^{\circ}$.
- $\tan X=\frac{5}{6}$ and stops, apply S4.

Blunders ( -3 )
B1 Incorrect trigonometric ratio e.g. $\tan X=\frac{6}{5} \Rightarrow X=50^{\circ}$
Slips (-1)
S1 Truncation, $\Rightarrow \frac{5}{6}=0.8$, however accept 0.83 and 0.833 etc.
S2 Fails to round off, accept $39^{\circ}$ or $40^{\circ}$.
S3 Rounds to $38^{\circ}$.
S4 $\tan X$ not expressed as decimal, stops at fraction.
S5 RAD or GRAD mode on calculator.
RAD: $\tan ^{-1}(0.8333)=0.6947=1$
GRAD: $\tan ^{-1}(0.8333)=44.227=44$
S6 Incorrect reading from tables.

## Attempt (4, 4)

A1 Graphical $\pm 5^{\circ}$
A2 Measurement from diagram $=45^{\circ} \quad\left( \pm 5^{\circ}\right)$ for (ii).
A3 $\tan =\frac{o p p}{a d j}$ and stops, allow $\tan =\frac{a d j}{o p p}$ for attempt mark.
A4 Some attempt at Pythagoras e.g. $x^{2}=6^{2}+5^{2}$

$$
\text { or } \tan X=6^{2}+5^{2}
$$

A5 Some attempt to get area $\frac{1}{2} \times 6 \times 5=15$.
A6 $6 \times 5=30$

## Worthless

W1 $6 \pm 5$
(c) A point $p$ is on level ground, 80 m from the foot of a pole. From $p$, the angle of elevation of the top of the pole is $35^{\circ}$.

Calculate the height $h$ of the pole, correct to the nearest metre.


## Solution

$\tan 35^{\circ}=\frac{h}{80} \quad \Rightarrow \quad h=80 \tan 35^{\circ}=80 \times 0.7002=56.02=56 \mathrm{~m}$

- $\tan 35^{\circ}=\frac{h}{80} \ldots 8$ marks. $\quad h=80 \tan 35^{\circ}$ or $0.4838=\frac{h}{80} \ldots 11$ marks.

$$
h=56.02 \ldots 14 \text { marks. } \quad h=56 \mathrm{~m} \quad \ldots \text { 15marks. }
$$

## Blunders ( -3 )

B1 Incorrect trigonometric ratio e.g. $\tan 35^{\circ}=\frac{80}{h} \Rightarrow h=114 \mathrm{~m}$.
B2 Incorrect trigonometric function e.g. $\sin 35^{\circ}=\frac{h}{80} \Rightarrow h=46 \mathrm{~m}$.
B3 Uses $\tan 55^{\circ}=\frac{h}{80} \Rightarrow h=114 \mathrm{~m}$.
B4 Incorrect transposition.
B5 Correct subst. into Sine Rule and stops, e.g. $\frac{h}{\sin 35^{\circ}}=\frac{80}{\sin 55^{\circ}}$, apply $2 \times(-3)+(-1)$
B6 Incorrect substitution into Sine Rule and stops, apply $3 \times(-3)$
Slips (-1)
S1 No rounding off
S2 Rad or Grad mode on calculator.
Rad: $80 \times(0.4738)=37.91$
Grad: $80 \times(0.6128)=49.02$
S3 Incorrect reading from tables.
Attempts (6)
A1 Graphical (correct $\pm 5 \mathrm{~m}$ )
A2 Measurement from diagram, $h=3.8 \mathrm{~cm}$ or 1.5 inches.
A3 Definition of tan or mnemonic.
A4 Pythagoras.
A5 Some attempt to calculate area.
A6 $55^{\circ}$ and stops.
A7 $35 \times 80$ or $\frac{35}{80}$ or $\frac{80}{35}$ only.

## Worthless

W1 $80 \pm 35$.

## QUESTION 6



Solution

$$
4 \times 2 \times 3=24
$$

- Accept correct answer for full marks

Blunders ( -3 )
B1 $4+2+3=9$
B2 3 ! $=6$
B3 $4!\times 2!\times 3!=288$
B4 $4!+2!+3!=24+2+6=32$, apply $2 \times(-3)$
B5 144
B6 3 or 1
B7 $2 \times 2 \times 2=8$ or $3 \times 3 \times 3=27$ or $4 \times 4 \times 4=64$

## Slips (-1)

S1 Probability of a certain type $=\frac{1}{24}$.
S2 $4 \times 2 \times 3$ and stops.
S3 Each three types omitted from listing process.

| Selections | Marks |
| :---: | :---: |
| $23-21$ | 9 |
| $20-17$ | 8 |
| $16-13$ | 7 |
| $12-9$ | 6 |
| $8-5$ | 5 |

Attempts (4)
A1 Any multiple of the wrong answers listed above.
(b) A bag contains 4 white balls, 3 red balls, 2 green balls and 1 yellow ball. A ball is picked at random from the bag. What is the probability that:
(i) the ball is red
(ii) the ball is not green.
(iii) the ball is red or white?

## Solution

(i) $\mathrm{P}(\mathrm{red})=\frac{3}{10}$
(ii) $\mathrm{P}($ not green $)=\frac{8}{10}$
(iii) $\mathrm{P}($ red or white $)=\frac{7}{10}$

- Accept fraction, ratio, decimal or percentage format.

| fraction | $3 / 10$ | $8 / 10$ | $7 / 10$ |
| :---: | :---: | :---: | :---: |
| decimal | 0.3 | 0.8 | 0.7 |
| ratio | $3: 10$ | $8: 10$ | $7: 10$ |
| percentage | $30 \%$ | $80 \%$ | $70 \%$ |

- Simplification of fractions not required
- Accept candidates $n(\mathrm{~S})$ and $n(\mathrm{E})$ from listing


## Blunders (-3)

B1 Incorrect $n(\mathrm{~S})$, apply once only, but $\mathrm{S}(-1)$ for $n(\mathrm{~S})=6,7,8,9$.
B2 Incorrect $n(\mathrm{E})$.
B3 Fails to divide by $n(\mathrm{~S})$, apply once only i.e. $\mathrm{P}(\mathrm{red})=3$
B4 Inverted fraction, apply once only.
Slips (-1)
S1 $\quad \frac{3}{7} \Rightarrow 0.43, \frac{8}{2} \Rightarrow 4.00, \frac{7}{3} \Rightarrow 2.33$
S2 Fails to add in (iii) i.e. $\frac{3}{10}, \frac{4}{10}$
S3 $\quad \mathrm{P}($ red or white $)=\frac{3}{10} \times \frac{4}{10}$ or $\frac{3}{10} \times \frac{4}{9}$
S4 $\quad \mathrm{P}($ red or white $)=\mathrm{P}($ not green or not yellow $)=\frac{8}{10}+\frac{9}{10}=\frac{17}{10}$.
Misreadings ( -1 )
M1 $\quad \mathrm{P}($ red $)=\mathrm{P}($ not red $)=\frac{7}{10}$
$\mathrm{M} 2 \quad \mathrm{P}($ not green $)=\mathrm{P}($ green $)=\frac{2}{10}$

## Attempt (4, 2, 2)

A1 Any other fraction, decimal, ratio or percentage apart from those specified.
Worthless
W1 'Good' or 'poor' chance.
(c) 200 eggs were classified according to size (large or medium), and colour (brown or white). The results are given in the following table

|  | Brown | White |
| :---: | :---: | :---: |
| Large | 40 | 80 |
| Medium | 32 | 48 |

An egg is chosen at random. What is the probability that it is
(i) a white egg
(ii) a brown egg
(iii) a large, brown egg
(iv) a medium white egg

## Solution

(i) $\mathrm{P}($ white egg $)=\frac{128}{200}=\frac{16}{25}$
(ii) $\mathrm{P}($ brown egg $)=\frac{72}{200}=\frac{9}{25}$
(iii) $\mathrm{P}($ large brown egg $)=\frac{40}{200}=\frac{1}{5}$
(iv) $\mathrm{P}($ medium white egg $)=\frac{48}{200}=\frac{6}{25}$

- Accept fraction, ratio, decimal or percentage format.

| Fraction | $16 / 25$ | $9 / 25$ | $1 / 5$ | $6 / 25$ |
| :---: | :---: | :---: | :---: | :---: |
| Decimal | 0.64 | 0.36 | 0.2 | 0.24 |
| Ratio | $16: 25$ | $9: 25$ | $1: 5$ | $6: 25$ |
| Percentage | $64 \%$ | $36 \%$ | $20 \%$ | $24 \%$ |

- Simplification of fractions not required


## Blunders (-3)

B1 Incorrect $n(\mathrm{~S})$, apply once only.
B2 Incorrect $n(\mathrm{E})$
B3 Fails to divide by $n(\mathrm{~S})$, once only, e.g. $\mathrm{P}($ white $)=128$
B4 Inverted fraction, once only.
B5 Chooses two eggs each time in (iii) and (iv)
e.g. $\mathrm{P}($ large egg and brown egg $)=\mathrm{P}($ large egg $) \times \mathrm{P}($ brown egg $)$.

Slips (-1)
S1 $\quad \frac{128}{72}(=1.78), \quad \frac{72}{128}(=0.56)$,

$$
\frac{40}{160}(=0.25), \quad \frac{48}{152}(=0.32) .
$$

Misreadings (-1)
M1 $\quad \mathrm{P}($ white $)=\frac{80}{200} \quad$ or $\quad \mathrm{P}($ white $)=\frac{48}{200}$
M2 $\quad \mathrm{P}($ brown $)=\frac{40}{200} \quad$ or $\quad \mathrm{P}($ brown $)=\frac{32}{200}$
M3 $\quad \mathrm{P}($ white $)=\mathrm{P}($ not white $)=\frac{72}{200}$
M4 $\quad \mathrm{P}($ brown $)=\mathrm{P}($ not brown $)=\frac{128}{200}$
M5 $\quad \mathrm{P}($ large or brown $)=\frac{120}{200}+\frac{72}{200}=\frac{192}{200}, \quad$ apply $2 \times(-1)$
M6 $\mathrm{P}($ large or brown $)=\frac{120}{200}+\frac{72}{200}-\frac{40}{200}=\frac{152}{200}$
M7 $\quad \mathrm{P}($ medium or white $)=\frac{80}{200}+\frac{128}{200}=\frac{208}{200}, \quad$ apply $2 \times(-1)$
M8 $\quad \mathrm{P}($ medium or white $)=\frac{80}{200}+\frac{128}{200}-\frac{48}{200}=\frac{160}{200}$

Attempts (2, 2, 2, 2)
A1 Probability of any fraction, decimal, ratio or percentage not covered above.

## QUESTION 7

| Part (a) | 15 Marks | Att. 6 |
| :--- | :--- | :--- |
| Part (b) | 20 Marks | Att. 8 |
| Part (c) | 15 Marks | Att. 6 |
|  |  |  |
|  |  | Att. 6 |

(a) Draw a histogram to illustrate the data given in the table below.

Put the time in minutes on the horizontal axis.

| Time in minutes | $0-5$ | $5-10$ | $10-20$ |
| :---: | :---: | :---: | :---: |
| Frequency | 6 | 14 | 12 |

## Solution



- Accept histogram drawn in answer book.


## Blunders (-3)

B1 Last height $=12$
B2 Bar chart, last height $=12$, bars same width, B3 may also apply.
B3 Error in axes, apart from above, e.g. unsatisfactory calibration.
B4 Correct cumulative frequency curve, or trend graph
B5 Frequency curve, no addition, apply $2 \times(-3)$.
B6 Each rectangle omitted, subject to attempt mark.

Slips (-1)
S1 Numerical errors.
S2 Reverses axes.

Attempts (6)
A1 Attempt at drawing pie chart.
(b) The following table gives the results of 100 students in an examination.

| Marks | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No of <br> students | 15 | 25 | 35 | 20 | 5 |

Note: 0-20 means 0 marks or more, but less than 20 marks.
Copy and complete the cumulative frequency table below:

| Marks | $<20$ | $<40$ | $<60$ | $<80$ | $<100$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of <br> students | 15 | 40 |  |  |  |

Draw the cumulative frequency curve, putting the number of students on the vertical axis. Use your curve to estimate the median mark.

## Solution

| Marks | $<20$ | $<40$ | $<60$ | $<80$ | $<100$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of <br> students. | 15 | 40 | 75 | 95 | 100 |

Median $=43$

- Copy and complete the table: 5 marks.
- Draw the cumulative frequency curve: 10 marks.
- Estimate median: 5 marks.
- Join to origin not required.
- Accept median value read from candidate's own graph.


## Blunders (-3)

B1 Numbers in table subtracted instead of added, i.e. 15, 40, 5, 15, 10.
B3 Error in scales.
B4 Uses incorrect table for curve.
B5 Points not joined.

## Slips (-1)

S1 Straight line segments used to join points
S2 Numerical errors in any point
S3 Each point not plotted.
S4 Each point plotted incorrectly.
S5 Reverses axes.
S6 Required lines drawn, but median value not specified.
S7 Starts from incorrect axis when determining the median.

S8 Median outside tolerance of $\pm 3$.
S9 Plots on midpoints i.e. 10, 30, 50 etc.
Attempts (2, 4, 2)
A1 Attempt to draw bar chart, pie chart, trend graph, histogram.
A2 New table with <, containing original numbers
A3 50 identified on vertical or horizontal axes for estimation of median.
A4 Axes only for curve.
A5 $\quad$ Median $=50$.

(c) The ages of five students are: $15,17,18,17,18$.
(i) Find the mean age.
(ii) Find the standard deviation, correct to two places of decimals.

## Solution

$$
\begin{aligned}
\bar{x} & =\frac{15+17+18+17+18}{5}=\frac{85}{5}=17 \\
\mathrm{SD} & =\sqrt{\frac{(15-17)^{2}+(17-17)^{2}+(18-17)^{2}+(17-17)^{2}+(18-17)^{2}}{5}} \\
& =\sqrt{\frac{(-2)^{2}+(0)^{2}+(1)^{2}+(0)^{2}+(1)^{2}}{5}}=\sqrt{\frac{6}{5}}=\sqrt{1.2}=1.10
\end{aligned}
$$

- Allow full marks for correct answer without work shown.
- Correct mean merits 10 marks, correct standard deviation merits 5 marks


## Blunders (-3)

B1 $15+17+18+17+18=85$ only
B2 Divides by 2 i.e. $\frac{85}{2}=42.5$

Slips (-1)
S1 Numerical errors.
S2 Each step omitted when finding SD, i.e. deviation, square, addition, mean, square root.

S3 Errors in signs $(-2)^{2}=-4$, once only

Attempts (4, 2)
A1 Any addition / proposed addition in (i) or (ii).
A2 Any work on table for standard deviation.
A3 Formula for SD and stops.

## QUESTION 8

| Part (a) | 10 Marks | Att. 4 |
| ---: | :--- | :--- |
| Part (b) | 25 Marks | Att. 10 |
| Part (c) | 15 Marks | Att. 6 |
| Part (a) | 10 Marks | Att. 4 |
| (a) | Use a ruler and compass to construct an angle of $60^{\circ}$. <br>  <br>  Show the construction lines clearly. |  |

## Solution



## Blunders ( -3 )

B1 No construction arcs shown.
B2 Distances outside tolerance ( $\pm 10 \%$ )
B3 Clear use of protractor, set square.
B4 Outside $\pm 5^{\circ}$ if protractor used.

Attempts (4)
A1 Any angle or triangle drawn or sketched.

Part (b)
(b)

The rectangle pqrt is an enlargement of the rectangle abct.
$|q r|=10.8 \mathrm{~cm}$ and $|b c|=4.5 \mathrm{~cm}$
(i) Write down the centre of the enlargement.
(ii) Calculate the scale factor of the enlargement.
(iii) The area of the rectangle pqrt is $92.16 \mathrm{~cm}^{2}$. Find the area of the rectangle abct.

(i) Centre of enlargement $=t$
(ii) Scale factor $=\frac{10.8}{4.5}=2.4$
(iii) Area of the rectangle $a b c t=\frac{92.16}{2.4^{2}}=\frac{92.16}{5.76}=16$

## Blunders ( -3 )

B1 $\quad$ Scale factor $=\frac{4.5}{10.8}=0.4167$
B2 $\quad$ Scale factor $=10.8 \times 4.5=48.6$
B3 Area $=\frac{92.16}{2.4}=38.4$ ie. Area $=\frac{92.16}{\text { candidate's scale factor }}$
B4 $\quad$ Area $=92.16 \times 2.4=221.184$
ie. $\quad 92.16 \times$ candidate's scale factor.
B5 Area $=92.16 \times 2.4^{2}=530.8416$
ie. $\quad 92.16 \times(\text { candidate's scale factor })^{2}$
B6 $\quad$ Area $=92.16 \times 10.8=995.3$ or $92.16 \times 10.8^{2}=10749.5424$

$$
92.16 \times 4.5=414.72 \text { or } 92.16 \times 4.5^{2}=1866.24, \text { apply } 2 \times(-3)
$$

B7 $\quad$ Area $=92.16 \div 10.8=8.5 \quad$ or $92.16 \div 10.8^{2}=0.79$

$$
92.16 \div 4.5=20.48 \text { or } 92.16 \div 4.5^{2}=4.55, \quad \text { apply } 2 \times(-3)
$$

B8 $\quad$ Area $=10.8 \times 4.5=48.6$,
apply $2 \times(-3)$
Slips (-1)
S1 $2.4^{2}=4.8$ or 2.8 or similar.
S2 Uses multiplication for division i.e. $92.16 \div(2.4 \times 2.4)=92.16$.
Attempts (2, 4, 4)
A1 Copies diagram, once only, att. 4.
A2 Estimation of scale factor e.g. $k=2, k=3, k=0.5$ etc., att. 4.
A3 $k=10.8 \pm 4.5$
(c) Construct a circle of radius 4 cm .


Mark $c$, the centre of the circle, and $d$, a point on the circle, as shown.

Show how to construct the tangent to the circle at the point $d$.


## Blunders (-3)

B1 Incorrect radius, outside tolerance of $\pm 0.5 \mathrm{~cm}$.
B2 Line intersects circle at two points.
B3 Line fails to intersect circle.
B4 Circle constructed without compass (radius $\approx 4 \mathrm{~cm}$ ).
Misreadings ( -1 )
M1 Semicircle drawn or circle with diameter 4 cm .
Slips (-1)
S1 Fails to mark $c$ and or $d$.
S2 Fails to draw radius.
S3 Tangent does not pass through $d$.
Attempts $(4,2)$
A1 Circle sketched (freehand).
A2 Tangent, radius or diameter drawn and stops.

