**AN ROINN OIDEACHAIS** M. 31

LEAVING CERTIFICATE EXAMINATION, 1997

# APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 27 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 10 m/s$^2$.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

**Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.**

**1.** A car starts from rest with uniform acceleration. In the tenth second of its motion it travels a distance of 38 m.

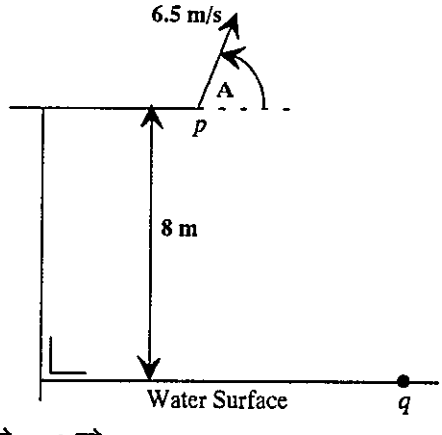
*(i)* Find the acceleration of the car.
*(ii)* Find the speed of the car at the end of the tenth second.
*(iii)* Find the time it takes the car to travel 91.125 m from rest.

**2.** The velocity of boat A is $4\vec{i} + 3\vec{j}$ km/hr.
The velocity of boat B is $x\vec{i} + y\vec{j}$ km/hr.
The velocity of A relative to B is $5\vec{j}$ km/hr.
Find the value of $x$ and the value of $y$.

At noon, A and B sail from port Q with velocities as described above. After how many hours will they be 20 km apart?

What is the direction, to the nearest degree, and distance of each boat from Q when they are 20 km apart?

**3.** A projectile is fired upwards from a point $p$ with an initial speed of 6.5 m/s inclined at an angle A to the horizontal where $\tan A = \frac{12}{5}$.



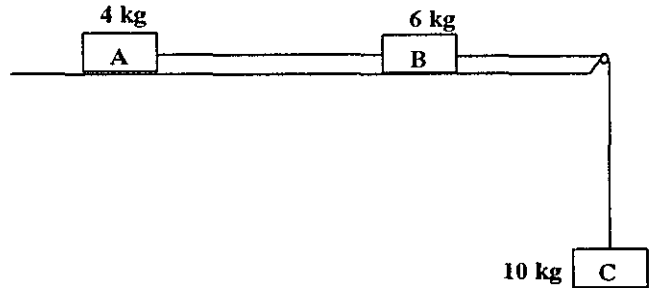

The point $p$ is 8 m above the surface of the water in a pool. The projectile strikes the water at the point $q$.

Find

*(i)* the time it takes the projectile to reach $q$
*(ii)* the velocity of the projectile at $q$ in terms of $\vec{i}$ and $\vec{j}$
*(iii)* the angle between the path of the projectile and the vertical as it enters the water at $q$. Give your answer correct to the nearest degree.

**OVER→**

**4.** Particles A and B, of mass 4 kg and 6 kg respectively, resting on a rough horizontal table, are connected by a light taut inextensible string. The coefficient of friction between A and the table is $\frac{1}{4}$ and between B and the table is $\frac{1}{2}$.




Particle B is connected by a second light inextensible string passing over a smooth pulley at the edge of the table to a third particle C, of mass 10 kg, hanging freely.

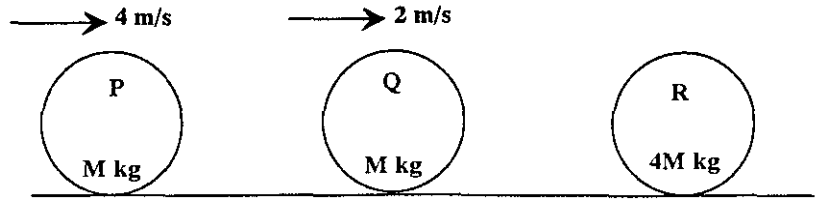
The system is released from rest.

*(i)* Show on separate diagrams the forces acting on each particle.

*(ii)* Write down the equation of motion for each particle.

*(iii)* Find the common acceleration of the particles and the tension in each string.

**5.** A smooth sphere P of mass M kg moving with a velocity of 4 m/s collides directly with a second smooth sphere Q, also of mass M kg, moving in the same direction with a velocity of 2 m/s on a smooth horizontal table.




After the collision, spheres P and Q keep moving in the same direction and the ratio of their velocities is

velocity of P : velocity of Q = 5 : 7.

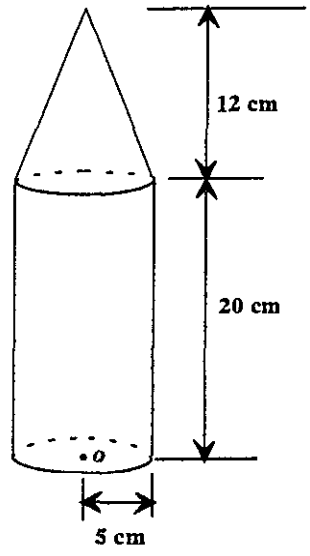
Find the velocity of P after the collision and the velocity of Q after the collision.

Find the coefficient of restitution for the collision.

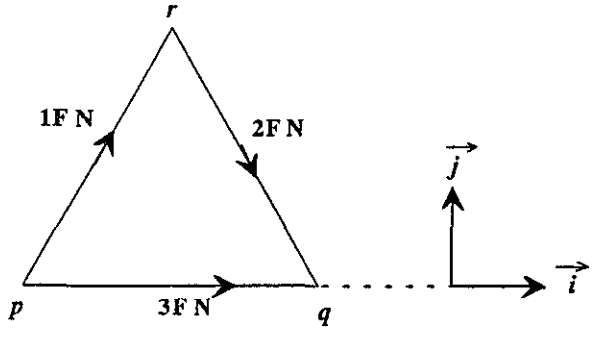
Sphere Q now goes on to collide directly with a stationary sphere R of mass 4M kg. This collision reverses the direction of Q and also causes Q to lose 75% of its kinetic energy.
Find the value of the coefficient of restitution for this collision.

**6.** *(a)* A uniform solid is in the shape of an upright cylinder surmounted by a right circular cone. The mass of the cylinder is five times the mass of the cone. The cylinder has radius of length 5 cm and height 20 cm. The height of the cone is 12 cm.

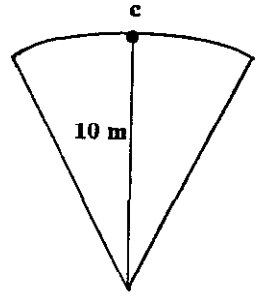
Find the distance of the centre of gravity of the solid from *o*, the centre point of the base of the cylinder. Give your answer correct to two places of decimals.
See Tables p. 40.




*(b)* $pqr$ is a triangle with $|pq| = |qr| = |rp|$. Forces 3F N, 2F N and 1F N act along the sides $[pq]$, $[rq]$ and $[pr]$ as shown. If the magnitude of the resultant is 63 N, find the value of F and give your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}$.



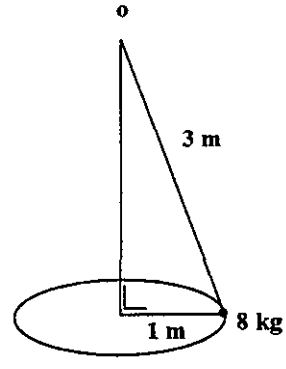

**7.** *(a)* A car is driven over a bridge whose highest point, *c*, is on a circular arc of radius 10 m.




*(i)* Show, on a diagram, the two forces acting vertically on the car as it is driven over the highest point, *c*, of the bridge.

*(ii)* What is the greatest speed at which the car may be driven over the highest point without leaving the ground?

*(b)* A particle of mass 8 kg is connected by means of a light inelastic string of length 3 m to a fixed point *o*. The particle describes a horizontal circle with constant speed $\sqrt{3}$ m/s and radius of length 1 m on a smooth horizontal table.



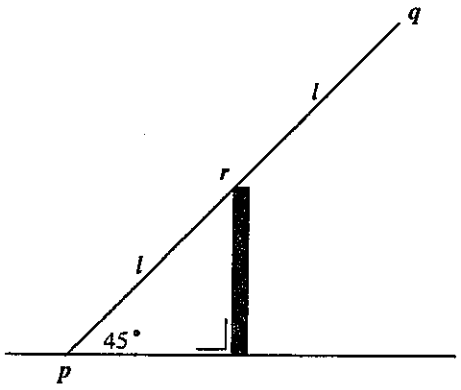

*(i)* Show that the value of the tension in the string is 72 N.

*(ii)* Calculate the normal reaction between the particle and the table, correct to two decimal places.

**OVER→**

**8.** A uniform rod, $[pq]$, of weight W and length $2l$, has end $p$ on rough horizontal ground and its midpoint $r$ against the top of a smooth vertical wall. The coefficient of friction at $p$ is $\mu$.

The rod is on the point of slipping when inclined at an angle of 45° to the horizontal.




*(i)* Show on a diagram all the forces acting on the rod.

*(ii)* Show that the normal reaction at $r$ is $\frac{W\sqrt{2}}{2}$.

*(iii)* Find the normal reaction at $p$ in terms of $W$.

**9.** A hollow right circular cone has an internal base radius length of 0.6 m and an internal height of 1 m. The cone is filled with water, closed and its horizontal base placed on a horizontal surface.

Calculate, in terms of $\pi$,

*(i)* the weight of the water in the cone

*(ii)* the downward thrust on the base due to the water

*(iii)* the force exerted by the curved surface of the cone on the water.

Density of water = 1000 kg/m$^3$.