Coimisiún na Scrúduithe Stáit

An Ardteistiméireacht 2011

Aistriúchán
Ar Scéim Mharcála

MATAMAITIC FHEIDHMEACH

Ardleibhéal

Treoirlínte Ginearálta

1 Cuirtear trí chineál pionóis i bhfeidhm ar obair iarrthóirí mar a leanas:

Sciorrthaí	– sciorrthaí uimhriúla	S(-1)
Botúin	– earráidí matamaiticiúla	B(-3)
Míléamh	– mura bhfuil sé tromchúiseach	M(-1)

Botún tromchúiseach nó ábhar ar lár nó míléamh as a leanann róshimpliú:
– tabhair an marc i leith iarrachta, agus an marc sin amháin.

Tugtar marcanna i leith iarrachta mar a leanas: 5 (iarr 2).

2 Sa scéim mharcála, taispeántar réiteach ceart amháin ar gach ceist.
In a lán cásanna, tá modhanna eile ann atá chomh bailí céanna.

1. **(a)** Ligtear cáithnín saor ó fhos ag A. Titeann sé go ceartingearach agus é ag gabháil thar an dá phointe B and C.

Sroicheann sé B tar éis t soicind agus tógann sé $\frac{2t}{7}$ soicind titim ó B go dtí C, fad slí 2.45 m.

Faigh luach t.

AB

$$s = ut + \tfrac{1}{2}ft^2$$

$$h = 0 + \tfrac{1}{2}gt^2$$

5

AC

$$s = ut + \tfrac{1}{2}ft^2$$

$$h + 2.45 = 0 + \tfrac{1}{2}g\left(\frac{9t}{7}\right)^2$$

5

$$\tfrac{1}{2}gt^2 + \tfrac{1}{4}g = 0 + \tfrac{1}{2}g\left(\frac{81t^2}{49}\right)$$

5

$$2t^2 + 1 = \frac{162t^2}{49}$$

$$64t^2 = 49$$

$$\Rightarrow \quad t = \frac{7}{8} \text{ s}$$

5

20

1. **(b)** Luasghéaraíonn carr go haonfhoirmeach ó fhos dó go dtí luas v in t_1 soicind.
Leanann sé ar aghaidh ar an luas tairiseach sin ar feadh t soicind agus luasmhoillíonn sé ansin go haonfhoirmeach chun fois in t_2 soicind.

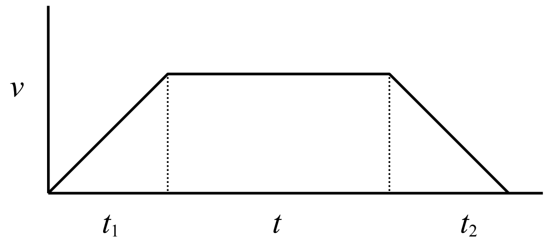
Is é $\frac{3v}{4}$ an meánluas ar an aistear.

(i) Tarraing graf luais is ama le haghaidh ghluaisne an chairr.

(ii) Faigh $t_1 + t_2$ i dtéarmaí t.

(iii) Dá gcuirfí an teorainn luais $\frac{2v}{3}$ i bhfeidhm, faigh, i dtéarmaí t, an t-am ba lú a ghlacfadh an t-aistear, dá mbeadh an luasghéarú agus an luasmhoilliú mar a bhí i gcuid (ii).

(i)



5, 5

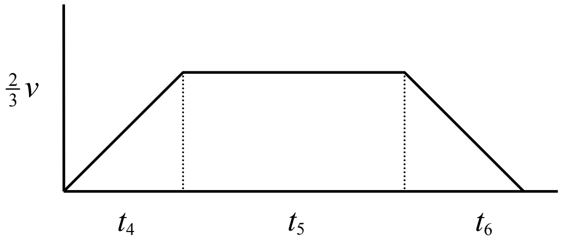
(ii) meánluas $= \dfrac{\frac{1}{2}t_1v + tv + \frac{1}{2}t_2v}{t_1 + t + t_2}$

$$\frac{3v}{4} = \frac{\frac{1}{2}t_1v + tv + \frac{1}{2}t_2v}{t_1 + t + t_2}$$ 5

$$\frac{3}{4} = \frac{\frac{1}{2}t_1 + t + \frac{1}{2}t_2}{t_1 + t + t_2}$$

$$3t_1 + 3t + 3t_2 = 2t_1 + 4t + 2t_2$$

$$\Rightarrow t_1 + t_2 = t$$ 5



(iii) $\frac{1}{2}t_1v + tv + \frac{1}{2}t_2v = \frac{1}{2}t_4\left(\frac{2v}{3}\right) + t_5\left(\frac{2v}{3}\right) + \frac{1}{2}t_6\left(\frac{2v}{3}\right)$ 5

$$3t_1v + 6tv + 3t_2v = 2t_4v + 4t_5v + 2t_6v$$

$$3t_1 + 6t + 3t_2 = 2t_4 + 4t_5 + 2t_6$$

$$9t = 2(t_4 + t_6) + 4t_5$$

$$t_4 + t_6 = \tfrac{2}{3}t$$

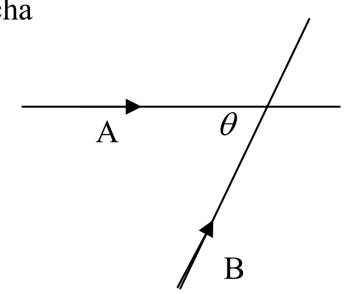
$$9t = 2\left(\tfrac{2}{3}t\right) + 4t_5$$

$$\Rightarrow t_5 = \tfrac{23}{12}t$$

$$\Rightarrow t_4 + t_5 + t_6 = \tfrac{2}{3}t + \tfrac{23}{12}t = \tfrac{31}{12}t$$ 5

30

2. **(a)** Gabhann dhá charr, A agus B, ar feadh dhá bhóthar dhíreacha a thrasnaíonn a chéile ar uillinn θ, áit a bhfuil $\tan\theta = \frac{4}{3}$.



Tá carr A ag gabháil i dtreo an phointe trasnaithe ar luas aonfhoirmeach 5 m s^{-1}.
Tá carr B ag gabháil i dtreo an phointe trasnaithe ar luas aonfhoirmeach 10 m s^{-1}.

Ag meandar ar leith, tá an dá charr 100 m ón bpointe trasnaithe agus iad ag druidim leis an bpointe trasnaithe.

Faigh **(i)** treoluas A i leith B
(ii) an fad slí is lú idir na cairr.

(i)

$$\vec{V}_A = 5\vec{i} + 0\vec{j}$$

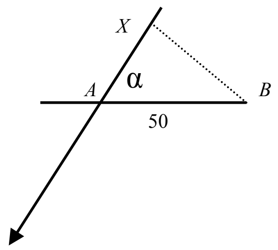
$$\vec{V}_B = 6\vec{i} + 8\vec{j}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \qquad 5$$

$$= -\vec{i} - 8\vec{j} \qquad 5$$

$$\text{méid} = \sqrt{65}\ \text{m s}^{-1} \qquad 5$$

$$\text{treo} = \text{Siar } \tan^{-1} 8 \text{ Ó Dheas} \qquad 5$$



(ii)

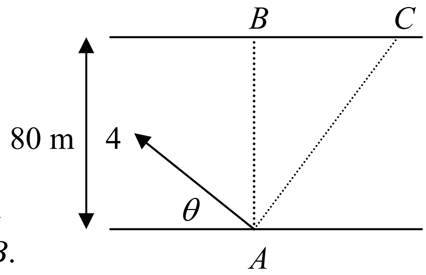
$$|BX| = 50\sin\alpha \qquad 5$$

$$= 50\left(\frac{8}{\sqrt{65}}\right)$$

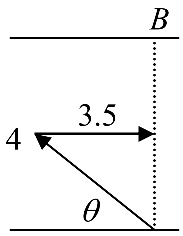
$$= 49.6\ \text{m} \qquad 5$$

30

2 **(b)** Is féidir le bean bád a rámhaíocht ar luas 4 m s^{-1} in uisce marbh. Rámhaíonn sí trasna abhann atá 80 m ar leithead.
Sníonn an abhainn ar luas tairiseach 3.5 m s^{-1} comhthreomhar leis na bruacha díreacha.
Is mian léi talamh a bhaint amach idir *B* agus *C*.
Tá an pointe *B* díreach ar aghaidh an phointe tosaithe *A* amach agus tá an pointe *C* $20\sqrt{3}$ m síos an abhainn ó *B*.

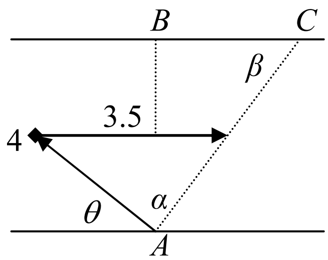


Más é θ an treo a dtéann sí, faigh an raon luachanna ar θ má bhaineann sí talamh amach idir *B* agus *C*.



$$\cos\theta = \frac{3.5}{4}$$
$$\theta = 28.955°$$

5
5



$$\tan\beta = \frac{80}{20\sqrt{3}}$$
$$\beta = 66.59°$$

5

$$\frac{\sin\alpha}{3.5} = \frac{\sin\beta}{4}$$
$$\sin\alpha = 0.8029$$
$$\alpha = 53.41°$$

$$\theta = 180 - 66.59 - 53.41$$
$$= 60°$$

$$28.955 \le \theta \le 60$$

5

20

3. **(a)** Déantar cáithnín a theilgean ó phointe P ar thalamh chothrománach.
Is é 35 m s^{-1} luas an teilgin ar uillinn $\tan^{-1} 2$ leis an gcothrománach.
Buaileann an cáithnín sprioc arb é $x\vec{i} + 50\vec{j}$ a shuíomh-veicteoir i leith P.

Faigh **(i)** luach x
(ii) uillinn teilgin eile i dtreo is go mbuailfidh an cáithnín an sprioc.

(i)

$$35\cos\alpha . t = x$$

$$t = \frac{x}{7\sqrt{5}} \qquad 5$$

$$35\sin\alpha . t - 4.9t^2 = 50 \qquad 5$$

$$35\left(\frac{2}{\sqrt{5}}\right).\left(\frac{x}{7\sqrt{5}}\right) - 4.9\left(\frac{x}{7\sqrt{5}}\right)^2 = 50$$

$$x^2 - 100x + 2500 = 0$$

$$x = 50 \qquad 5$$

(ii)

$$35\cos\alpha . t = 50$$

$$t = \frac{10}{7\cos\alpha}$$

$$35\sin\alpha . t - 4.9t^2 = 50$$

$$35\sin\alpha . \left(\frac{10}{7\cos\alpha}\right) - 4.9\left(\frac{10}{7\cos\alpha}\right)^2 = 50 \qquad 5$$

$$50\tan\alpha - 10\left(1 + \tan^2\alpha\right) = 50$$

$$\tan^2\alpha - 5\tan\alpha + 6 = 0$$

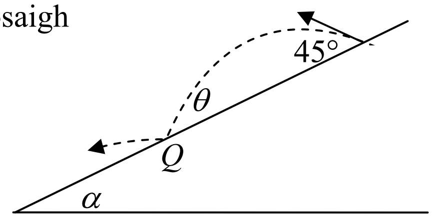
$$(\tan\alpha - 2)(\tan\alpha - 3) = 0$$

$$\tan\alpha = 3$$

$$\alpha = 71.6° \qquad 5$$

25

3 **(b)** Tá plána claonta ar uillinn α leis an gcothromán.
Déantar cáithnín a theilgean síos an plána ar luas tosaigh 10 m s^{-1} ar uillinn 45º leis an bplána claonta.
Tá plána an teilgin ceartingearach agus cuimsíonn sé an líne is mó fána.
Buaileann an cáithnín an plána ag Q ar uillinn tuirlingthe θ, áit a bhfuil $\tan\theta = \frac{1}{4}$.



(i) Faigh luach α .
(ii) Más é $5\sqrt{33}$ an méid atá i dtreoluas na hathphreibe ag Q, faigh luach e, comhéifeacht an chúitimh.

(i)

$$r_j = 0$$

$$0 = 10\sin 45.t - \tfrac{1}{2} g\cos\alpha.t^2$$

$$\Rightarrow \quad t = \frac{10\sqrt{2}}{g\cos\alpha}$$

5

$$v_i = 10\cos 45 + g\sin\alpha\left(\frac{10\sqrt{2}}{g\cos\alpha}\right)$$

5

$$= 5\sqrt{2} + 10\sqrt{2}\tan\alpha$$

$$v_j = 10\sin 45 - g\cos\alpha\left(\frac{10\sqrt{2}}{g\cos\alpha}\right)$$

5

$$= -5\sqrt{2}$$

$$\tan\theta = \frac{-v_j}{v_i}$$

$$\frac{1}{4} = \frac{5\sqrt{2}}{5\sqrt{2} + 10\sqrt{2}\tan\alpha}$$

$$\tan\alpha = 1.5 \quad \Rightarrow \quad \alpha = 56.3°$$

5

(ii)

$$v_i = 20\sqrt{2}$$

$$v_j = 5e\sqrt{2}$$

$$5\sqrt{33} = \sqrt{\left(20\sqrt{2}\right)^2 + \left(5e\sqrt{2}\right)^2}$$

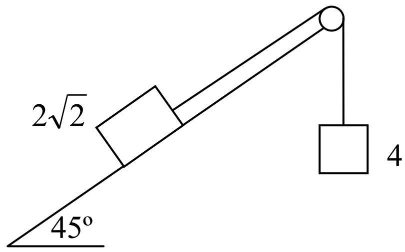
$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

5

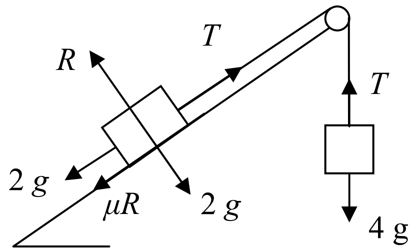
25

4. (a) Tá bloc, ar mais dó $2\sqrt{2}$ kg, ar fos ar phlána garbh atá claonta ar 45º leis an gcothromán. Tá sé ceangailte le téad éadrom dhoshínte a ghabhann thar ulóg mhín, éadrom, fhosaithe, de cháithnín, ar mais dó 4 kg, atá ar crochadh saor faoi dhomhantarraingt.

Is é $\frac{1}{4}$ comhéifeacht na frithchuimilte idir an bloc agus an plána.



Faigh luasghéarú na maise 4 kg.



5

$$4g - T = 4f$$ 5

$$T - 2g - \mu R = 2\sqrt{2}f$$ 5

$$T - 2g - \tfrac{1}{4}(2g) = 2\sqrt{2}f$$

$$4g - 2g - \frac{1}{2}g = \left(4 + 2\sqrt{2}\right)f$$

$$\frac{3g}{2} = \left(4 + 2\sqrt{2}\right)f$$

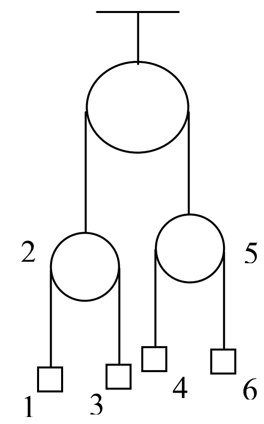
$$f = \frac{3g}{2\left(4 + 2\sqrt{2}\right)}$$

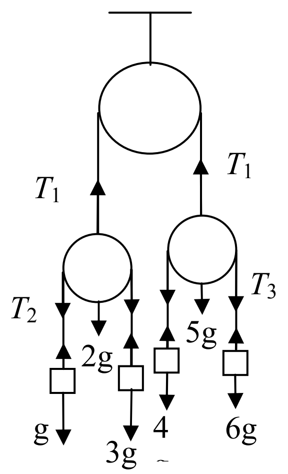
$$\Rightarrow \quad f = 2.15 \text{ m s}^{-2}$$ 5

20

4 **(b)** Ta ulóg mhín, ar mais di 2 kg, ceangailte le téad éadrom dhoshínte atá ag gabháil thar ulóg mhín, éadrom, fhosaithe d’ulóg mhín, ar mais di 5 kg.
Tá dhá cháithnín, ar maiseanna dóibh 1 kg agus 3 kg, ceangailte le chéile le téad éadrom dhoshínte atá ag gabháil thar an ulóg 2 kg.
Tá dhá cháithnín, ar maiseanna dóibh 4 kg agus 6 kg, ceangailte le chéile le téad éadrom dhoshínte atá ag gabháil thar an ulóg 5 kg.

Faigh an teannas i ngach téad díobh nuair a ligtear an córas saor ó fhos.





5

$$6g - T_3 = 6(c + a)$$
$$T_3 - 4g = 4(c - a) \quad \Rightarrow \quad 24g - 5T_3 = 24a$$
5

$$3g - T_2 = 3(b - a)$$
5
$$T_2 - g = (b + a) \quad \Rightarrow \quad 6g - 4T_2 = -6a$$
5

$$T_1 - 2T_2 - 2g = 2a$$
$$2T_3 + 5g - T_1 = 5a \quad \Rightarrow \quad 2T_3 - 2T_2 + 3g = 7a$$
5

$$2\left\{\frac{24g - 24a}{5}\right\} - 2\left\{\frac{6g + 6a}{4}\right\} + 3g = 7a$$

$$\Rightarrow \quad a = 4.8 \ \text{m s}^{-2}$$

$$\left.\begin{aligned} T_1 &= 73 \ \text{N} \\ T_2 &= 21.9 \ \text{N} \\ T_3 &= 24 \ \text{N} \end{aligned}\right\}$$
5

30

5. **(a)** Sféar mín P, ar mais dó $2m$ kg, atá ag gabháil ar luas $u\ \text{m s}^{-1}$, imbhuaileann sé go díreach sféar mín Q, ar mais dó $3m$ kg, atá ag gabháil i bhfritreo ar luas $u\ \text{m s}^{-1}$. Is é e comhéifeacht an chúitimh idir na sféir agus tá $0 < e < 1$.

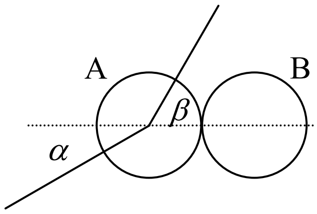
(i) Taispeáin go ndéanfaidh P athphreab i gcás na luachanna uile ar e.

(ii) Cad é an raon luachanna ar e mar a ndéanfaidh Q athphreab?

(i) PCM	$2m(u) + 3m(-u) = 2mv_1 + 3mv_2$	5
NEL	$v_1 - v_2 = -e(u+u)$	5
	$v_1 = \dfrac{-u(1+6e)}{5}$, $v_2 = \dfrac{u(-1+4e)}{5}$	5
	$v_1 = \dfrac{-u(1+6e)}{5} < 0 \quad \forall\ e, \quad 0<e<1$	5
(ii)	$v_2 = \dfrac{u(-1+4e)}{5} > 0$ $4e > 1$ $e > \dfrac{1}{4}$	
	$\Rightarrow \dfrac{1}{4} < e < 1$	5
		25

5 **(b)** Sféar mín A, ar mais dó m, atá ag gabháil ar luas u, imbhuaileann sé sféar mín B, atá comhionann leis agus atá ar fos.

Roimh an imbhualadh agus ina dhiaidh, déanann treo ghluaisne A na huillinneacha α agus β, faoi seach, le líne na lárphointí ag meandar an tuinsimh.



Is é e chomhéifeacht an chúitimh idir na sféir.

(i) Má tá $\tan\alpha = k\tan\beta$, faigh k, i dtéarmaí e.

(ii) Más é $\frac{7}{8}mu\cos\alpha$ méid na ríge a dháiltear ar gach sféar díobh de thoradh an imbhuailte, faigh luach e.

(i) PCM $m(u\cos\alpha) + m(0) = mv_1 + mv_2$ — 5

NEL $v_1 - v_2 = -e(u\cos\alpha - 0)$ — 5

$$v_1 = \frac{u\cos\alpha(1-e)}{2}$$

$$v_2 = \frac{u\cos\alpha(1+e)}{2}$$ — 5

$$\tan\beta = \frac{u\sin\alpha}{v_1} = \frac{2u\sin\alpha}{u\cos\alpha(1-e)} = \frac{2\tan\alpha}{1-e}$$

$$\tan\beta = \frac{2k\tan\beta}{1-e}$$

$$1-e = 2k$$

$$\Rightarrow k = \frac{1-e}{2}$$ — 5

(ii)

$$I = mv_2 - m(0)$$

$$\frac{7}{8}mu\cos\alpha = \frac{1}{2}mu\cos\alpha(1+e)$$

$$e = \frac{3}{4}$$ — 5

25

6. **(a)** Tugtar an fad slí, x, atá cáithnín ó phointe fosaithe, O, mar

$$x = a\sin(\omega t + \varepsilon)$$

áit ar tairisigh dheimhneacha iad a, ω agus ε .

(i) Taispeáin gur gluaisne shimplí armónach í gluaisne an cháithnín.

Cáithnín atá ag gabháil faoi ghluaisne shimplí armónach, tosaíonn sé ó phointe atá 1 m ó lár na gluaisne ar luas 9.6 m s^{-1} agus faoi luasghéarú 16 m s^{-2}.

(ii) Ríomh a, ω agus ε.

(i)

$$x = a\sin(\omega t + \varepsilon)$$

$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$ 5

$$\ddot{x} = -a\omega^2\sin(\omega t + \varepsilon)$$
$$= -\omega^2 x$$ 5

(ii)

$$\ddot{x} = \quad \omega^2 x$$
$$16 = \omega^2(1)$$
$$\Rightarrow \omega = 4 \text{ rad s}^{-1}$$ 5

$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$
$$9.6 = \text{a}(4)\cos\varepsilon$$
$$\Rightarrow a\cos\varepsilon = 2.4$$

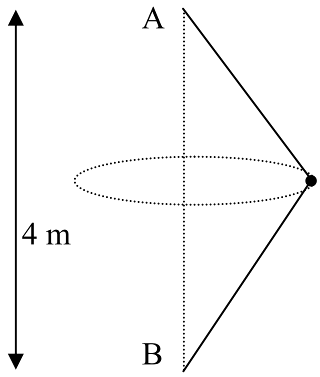
$$x = a\sin(\omega t + \varepsilon)$$
$$1 = a\sin\varepsilon$$

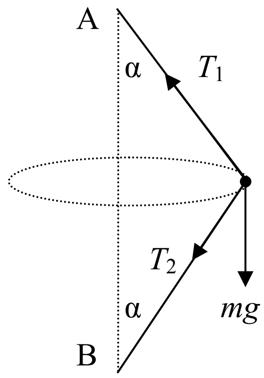
$$\frac{a\sin\varepsilon}{a\cos\varepsilon} = \frac{1}{2.4}$$
$$\tan\varepsilon = \frac{5}{12} \Rightarrow \varepsilon = 0.395 \text{ rad}$$ 5

$$a = \frac{1}{\sin 0.395} = 2.6 \text{ m}$$ 5 | 25

6 **(b)** Dhá phionna fhosaithe iad A agus B.
Ta A 4 m go ceartingearach lastuas de B.
Mais *m* kg, atá ceangailte de A agus B le dhá théad éadroma dhoshínte atá ar comhfhad, ℓ, déanann sí ciorcal cothrománach faoi threoluas uilleach aonfhoirmeach ω.

Más é 11: 9 cóimheas na dteannas sa dá théad, faigh luach ω.





$$T_1 \sin\alpha + T_2 \sin\alpha = m(\ell \sin\alpha)\omega^2$$
$$T_1 + T_2 = m\ell\omega^2$$ 5

$$\tfrac{11}{9}T_2 + T_2 = m\ell\omega^2$$
$$T_2 = \tfrac{9}{20} m\ell\omega^2$$ 5

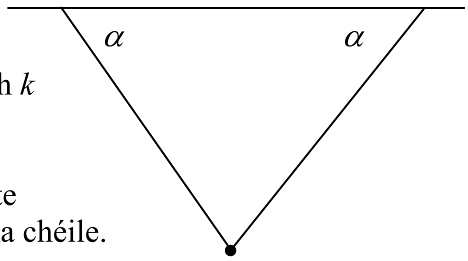
$$T_1 \cos\alpha - T_2 \cos\alpha = mg$$
$$T_1 - T_2 = \frac{mg}{\cos\alpha} = \tfrac{1}{2} mg\ell$$ 5

$$\tfrac{11}{9}T_2 - T_2 = \tfrac{1}{2} mg\ell$$
$$T_2 = \tfrac{9}{4} mg\ell$$ 5

$$\tfrac{9}{20} m\ell\omega^2 = \tfrac{9}{4} mg\ell$$
$$\omega^2 = 49$$
$$\omega = 7 \text{ rad s}^{-1}$$ 5

25

7. **(a)** Tá cáithnín, ar mais dó 24 kg, ceangailte de dhá théad éadroma leaisteacha a bhfuil fad nádúrtha 33 cm iontu agus an tairiseach leaisteach *k* ag gach aon cheann díobh.

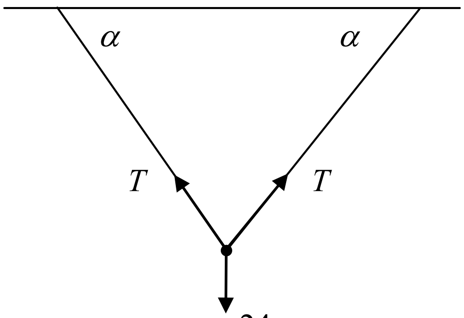


Tá foircinn eile na dtéad ceangailte de dhá phointe ar an leibhéal cothrománach céanna atá 64 cm óna chéile.

Déanann gach téad uillinn α leis an gcothromán, áit a bhfuil $\tan\alpha = \frac{3}{4}$.

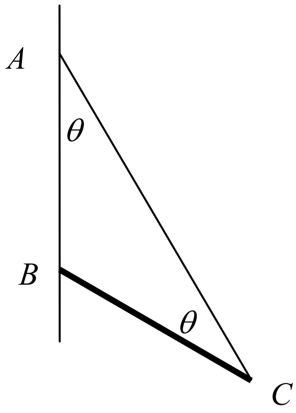
(i) Taispeáin gurb é 7 cm an síneadh i ngach téad díobh.

(ii) Faigh luach *k*.



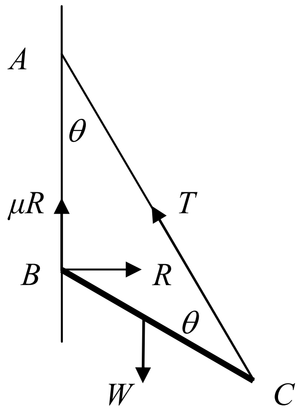
(i)	$\cos\alpha = \dfrac{32}{33+x}$	5	
	$\dfrac{4}{5} = \dfrac{32}{33+x}$		
	$x = 7\text{ cm}$	5	
(ii)	$2T\sin\alpha = 24g$	5	
	$2T\left(\dfrac{3}{5}\right) = 24g$		
	$T = 20g$	5	
	$T = kx$		
	$20g = k(0.07)$		
	$\Rightarrow k = 2800\ \text{N m}^{-1}$	5	25

7 **(b)** Bata aonfhoirmeach BC, ar fad dó $2p$ agus ar meáchan dó W, tá sé ar fos agus i gcothromaíocht sa chaoi go bhfuil B i dteagmháil le balla garbh ceartingearach.
Tá foirceann amháin de théad éadrom dhoshínte greamaithe de phointe A ar an mballa go ceartingearach lastuas de B, agus tá an foirceann eile ceangailte de C.



Is é μ comhéifeacht na frithchuimilte idir an bata agus an balla.

Má tá $|\angle CAB| = |\angle BCA| = \theta$, cruthaigh go bhfuil $\mu \geq \tan\theta$.



$$T\sin\theta(2p) = W\sin 2\theta(p)$$
$$T\sin\theta(2p) = W 2\sin\theta\cos\theta(p)$$
$$T = W\cos\theta$$ 5

$$R = T\sin\theta$$ 5

$$\mu R + T\cos\theta = W$$ 5
$$\mu T\sin\theta + T\cos\theta = \frac{T}{\cos\theta}$$
$$\mu\sin\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta}$$ 5
$$\mu\sin\theta = \sin\theta\tan\theta$$
$$\mu = \tan\theta$$

$$\Rightarrow \mu \geq \tan\theta$$ 5

25

8. **(a)** Cruthaigh gurb é $\frac{1}{3}m\ell^2$ móimint na táimhe ag lann aonfhoirmeach chearnógach, ar mais di m agus ar fad sleasa di 2ℓ, thart timpeall aise trína lárphointe atá comhthreomhar le ceann amháin de na sleasa.

Bíodh M = mais in aghaidh an aonaid achair

mais na heiliminte $= \mathrm{M}\{2\ell\, \mathrm{d}x\}$

móimint táimhe na heiliminte $= \mathrm{M}\{2\ell\, \mathrm{d}x\}x^2$ — 5

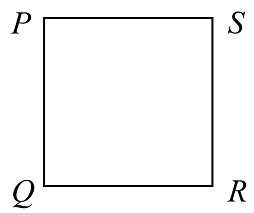
móimint táimhe na lainne $= 2\ell\,\mathrm{M}\int_{-\ell}^{\ell} x^2\,\mathrm{d}x$ — 5

$= 2\ell\,\mathrm{M}\left[\frac{x^3}{3}\right]_{-\ell}^{\ell}$ — 5

$= 4\,\mathrm{M}\frac{\ell^4}{3}$

$= \frac{1}{3}m\ell^2$ — 5 — 20

8 **(b)** Is féidir le lann chearnógach *PQRS*, ar fad sleasa di 60 cm agus ar mais di *m*, saorchasadh thart timpeall aise cothrománaí tríd an bpointe *P* atá ingearach le plána na lainne.

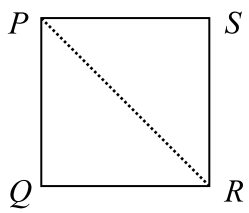


Ligtear an lann saor ó fhos nuair atá *PS* cothrománach.

(i) Faigh treoluas uilleach na lainne nuair a bheidh *PR* ceartingearach.

Déantar mais *m* a cheangal des an lann ag *R*. Cuirtear gluaisne sa chomhluascadán.

(ii) Faigh peiriad ascaluithe beaga an chomhluascadáin agus uaidh sin, nó ar mhodh eile, faigh an fad atá sa luascadán simplí coibhéiseach.



(i)

$$\text{Gnóchan san FC} = \text{Caillteanas san FP}$$

$$\tfrac{1}{2} I\omega^2 = mgh$$

$$\tfrac{1}{2}\left\{\tfrac{4}{3} m(0.3)^2 + \tfrac{4}{3} m(0.3)^2\right\}\omega^2 = mg\left(0.3\sqrt{2} - 0.3\right)$$ 5, 5

$$\omega^2 = \frac{g\left(\sqrt{2}-1\right)}{0.4} = 10.1482$$

$$\omega = 3.19 \text{ rad s}^{-1}$$ 5

(ii)

$$\left.\begin{aligned} I &= \tfrac{8}{3} m(0.3)^2 + m\left(0.6\sqrt{2}\right)^2 \\ &= 0.96m \\ Mgh &= mg\left(0.3\sqrt{2}\right) + mg\left(0.6\sqrt{2}\right) \\ &= 0.9\sqrt{2}\, mg \end{aligned}\right\}$$ 5

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$= 2\pi\sqrt{\frac{0.96m}{0.9\sqrt{2}\, mg}}$$

$$= 1.74 \text{ s}$$ 5

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.96}{0.9\sqrt{2}\, g}}$$

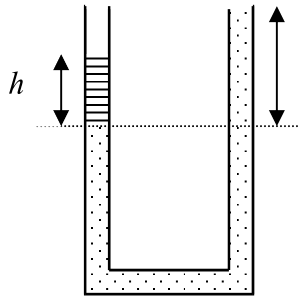
$$\Rightarrow L = \frac{0.96}{0.9\sqrt{2}} = 0.75 \text{ m}$$ 5

30

9. **(a)** U-fheadán, ar achar trasghearrtha dó 0.15 cm^2, coinníonn sé ola ar dlús coibhneasta di 0.8.

Tá dromchla na hola 12 cm ó bharr an dá bhrainse araon den U-fheadán.

Cad é an toirt uisce is féidir a dhoirteadh isteach i mbrainse amháin sula gcuireann an ola sa bhrainse eile thar maoil?



$1000gh = 800g(0.24)$	5
$h = 0.192\,\text{m}$	5
$\text{Toirt} = h\,A$ $= 0.192\left(0.15\times10^{-4}\right)$	5
$= 2.88\times10^{-6}\ \text{m}^3$	5
	20

9 **(b)** Nuair a dhéantar dlúthshorcóir aonfhoirmeach a lonnú i leacht A, snámhann sé go ceartdíreach agus $\frac{1}{3}$ dá ais faoin leacht.
Nuair a lonnaítear an sorcóir aonfhoirmeach leacht B, snámhann sé go ceartdíreach agus $\frac{3}{5}$ dá ais faoin leacht.
Cén codán d'ais an tsorcóra a bheidh faoin leacht nuair a snámhann an sorcóir go ceartdíreach i meascán aonfhoirmeach de thoirteanna cothroma den dá leacht?

A

$$B_A = W$$

$$\frac{\frac{1}{3}Ws_A}{s} = W$$

$$\Rightarrow s_A = 3s \qquad 5$$

B

$$B_B = W$$

$$\frac{\frac{3}{5}Ws_B}{s} = W$$

$$\Rightarrow s_B = \tfrac{5}{3}s \qquad 5$$

A + B

$$B_M = W$$

$$\frac{yWs_M}{s} = W$$

$$\Rightarrow s_M = \tfrac{1}{y}s \qquad 5$$

$$s_A V + s_B V = s_M(2V) \qquad 5$$

$$s_A + s_B = 2s_M$$

$$3s + \tfrac{5}{3}s = \tfrac{2}{y}s \qquad 5$$

$$\tfrac{14}{3}s = \tfrac{2}{y}s$$

$$\Rightarrow y = \frac{3}{7} \qquad 5$$

30

10. **(a)** Má tá

$$x^2 \frac{dy}{dx} - xy = 7y$$

agus $y = 1$ nuair $x = 1$, faigh luach y nuair $x = 2$.

$$x^2 \frac{dy}{dx} = xy + 7y$$

$$\frac{dy}{dx} = \frac{y(x+7)}{x^2}$$

$$\int \frac{1}{y}\,dy = \int \frac{x+7}{x^2}\,dx \qquad 5$$

$$\int \frac{1}{y}\,dy = \int \left(\frac{1}{x} + \frac{7}{x^2}\right) dx$$

$$\ln y = \ln x - \frac{7}{x} + C \qquad 5$$

$$y = 1,\ x = 1$$

$$\Rightarrow\ C = 7 \qquad 5$$

$$\ln y = \ln x - \frac{7}{x} + 7$$

$$\ln y = \ln 2 - \frac{7}{2} + 7$$

$$= 4.1931$$

$$\Rightarrow\ y = e^{4.1931} = 66.23 \qquad 5 \qquad 20$$

10 (b) Is é an luasmhoilliú atá faoi cháithnín atá ag gabháil ina líne dhíreach ná

$$\frac{v^2}{400} + 16 \quad \text{m s}^{-2}$$

áit arb é *v* a luas ag am ar bith *t*.

Más é 40 m s^{-1} a luas tosaigh, faigh

(i) an fad slí a ghabhann sé sula dtagann sé chun fois
(ii) meánluas an cháithnín le linn na gluaisne.

(i)

$$v\frac{dv}{dx} = -\left(\frac{v^2}{400} + 16\right)$$

$$= -\left(\frac{v^2 + 80^2}{400}\right)$$

$$\int_{40}^{0} \frac{v}{v^2 + 80^2}\, dv = -\frac{1}{400}\int_{0}^{x} dx \qquad 5$$

$$\left[\tfrac{1}{2}\ln\left(v^2 + 80^2\right)\right]_{40}^{0} = \left[-\frac{x}{400}\right]_{0}^{x} \qquad 5$$

$$\tfrac{1}{2}\ln\left(\frac{40^2 + 80^2}{80^2}\right) = \frac{x}{400}$$

$$x = 200\ln\left(\frac{5}{4}\right)$$

$$x = 44.63 \text{ m} \qquad 5$$

(ii)

$$\frac{dv}{dt} = -\left(\frac{v^2 + 80^2}{400}\right)$$

$$\int_{40}^{0} \frac{1}{v^2 + 80^2}\, dv = -\frac{1}{400}\int_{0}^{t} dt$$

$$\left[\tfrac{1}{80}\tan^{-1}\left(\frac{v}{80}\right)\right]_{40}^{0} = \left[-\frac{t}{400}\right]_{0}^{t} \qquad 5$$

$$t = 5\tan^{-1}\left(\frac{1}{2}\right)$$

$$= 2.32 \qquad 5$$

$$\text{meánluas} = \frac{44.63}{2.32} = 19.24 \text{ m s}^{-1} \qquad 5$$

30

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba choir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] ×15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an table thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0