JUNIOR CERTIFICATE EXAMINATION

2011

MARKING SCHEMES

MATHEMATICS
HIGHER LEVEL

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PAPER 1

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GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
    *   Blunders - mathematical errors/omissions   (-3)
    *   Slips- numerical errors   (-1)
    *   Misreadings   (provided task is not oversimplified)   (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
    *   any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
    *   if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    *   a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5·50 may be written as €5,50.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 3,3
Part (c)	20 marks	Att 2,2,2,2

Part (a)	10 marks	Att 3

Peter and Anne share a lotto prize in the ratio $3\frac{1}{2}$ to $2\frac{1}{2}$.

Peter's share is €35 000.

✎ What is the total prize fund?

(a)	10 marks	Att 3

I

$3\frac{1}{2}$: $2\frac{1}{2}$

$3\frac{1}{2} + 2\frac{1}{2}$

$= 6$

€35,000 ÷ $3\frac{1}{2}$

€10,000 × 6   or   € 10,000 × 2½ = €25,000

                              €35,000 + €25,000

The total prize fund   =   €60,000

II

$3\frac{1}{2}$: $2\frac{1}{2}$

7   : 5

€35,000 ÷ 7 = €5,000

7 + 5 = 12

€5,000 × 12   or   €5,000 × 5 = €25,000

                              €35,000 + €25,000

The total prize fund   =   €60,000

* Other methods may be used
* €10,000 is 4 marks; €25,000 is 7 marks; €20,416·67 is 4 marks (B3 & B4 with work)

Blunders (-3)

B1 Correct answer no work shown. ✎
B2 Fails to finish - stops at €10,000 ×6, or €5,000 × 12 or €35,000 + €25,000 or equivalent
B3 Incorrect divisor
B4 Incorrect multiplier
B5 Decimal error

Slips (-1)
S1   Numerical errors to a maximum of −3

Misreadings (-1)
M1   Incorrect digit which does not oversimplify the question

Attempts (3 marks)
A1   3½ + 2½ and stops
A2   6 or 5,000 or 25,000 or 10,000 or 12 with no work shown
A3   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown, but note A2
W2   Work of no merit

Part (b)	20 (10,10) marks	Att (3,3)

(i) The diameters of Venus and Saturn are $1 \cdot 21 \times 10^4$ km and $1 \cdot 21 \times 10^5$ km .

✍ What is the difference between the diameters of the two planets? Give your answer in the form of $a \times 10^n$ where $n \in \mathbb{Z}$ and $1 \le a < 10$.

(ii) ✍ Write $\dfrac{\sqrt{3} \times 27}{3^2}$ in the form of 3^n where $n \in \mathbb{Q}$.

(b) (i)	10marks	Att3

I
1·21 × 10,000
12,100   **3m**

$\updownarrow$ Interchangeable

1·21 × 100,000
121,000   **4m**

121,000 – 12,100
108,900   **7m**

Difference: $1 \cdot 089 \times 10^5$ **10m**

II
$(1 \cdot 21 \times 10^5) - (1 \cdot 21 \times 10^4)$ **3m**
108,900   **7m**
$1 \cdot 089 \times 10^5$ **10m**

* One correct calculation 3 marks, two correct calculations 4 marks, subtraction 7 marks, conversion 10 marks

* 108,900 **only** is B1 and B5

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Reversed subtraction
B3 Index error
B4 Misplaced decimal
B5 Answer not given in required form

Slips (-1)
S1 Numerical errors to a maximum of −3

Misreadings (-1)
M1 Incorrect digit provided it doesn't oversimplify the question

Attempts (3 marks)
A1 10,000 or 100,000 or 12,100 or 121,000
A2 Digits 1089 with incorrectly placed decimal or index without work
A3 Any relevant step

Worthless (0)
W1 Incorrect answer, no work shown
W2 $1 \cdot 21 \times 10^1$ or $1 \cdot 21 \times 10^9$
W3 Work of no merit

(b) (ii)   **10 marks**   **Att3**

I

$$\frac{\sqrt{3} \times 27}{3^2}$$

$(3^{\frac{1}{2}} \times 3^3) \div 3^2$

$3^{7/2} \div 3^2$

$3^{\frac{3}{2}}$

II

$\frac{1 \cdot 732050808 \times 27}{9}$

$\frac{46 \cdot 7653718}{9}$

$5 \cdot 196152423$ **3m**

$5 \cdot 196152423 = 3^{3/2}$ **10m**

*   Accept $3^{1 \cdot 5}$ or $3^{1\frac{1}{2}}$ for full marks.

Blunders (-3)

B1   Correct answer no work shown. ✎

B2   Each index error

B3   Incorrect operation

B4   Fails to finish

Slips (-1)

S1   Numerical error to a maximum of −3

Misreadings (-1)

M1   Misreads a digit provided it doesn't oversimplify

Attempts (3 marks)

A1 $27 = 3^3$

A2 $\sqrt{3} = 3^{\frac{1}{2}}$

A3 $1 \cdot 732$

A4 $3^2 = 9$

A5 $\sqrt{}$ = power of ½

A6 $5 \cdot 196152423$

A7 $3\sqrt{3}$

A6   Some relevant step

Worthless (0)

W1   Incorrect answer with no work

W2   Work of no merit

Part (c)   **20 (5,5,5,5) marks**   **Att 2,2,2,2**

(i)   ✍   By rounding to the nearest whole number estimate the value of

$$\frac{\sqrt{(7\cdot 17)^2 + 14\cdot 59}}{8\cdot 29 - 1\cdot 64 \times 2\cdot 23}.$$

Then evaluate $\dfrac{\sqrt{(7\cdot 17)^2 + 14\cdot 59}}{8\cdot 29 - 1\cdot 64 \times 2\cdot 23}$, correct to one decimal place.

(ii) Úna and Conor were travelling to South Africa.
They bought 5760 rand in the bank.
The bank charged them €630, which included a 5%
service charge.

✍   What was the value of the euro in rand (the exchange rate) on that day?

(c) (i) Estimate   **5 marks**   **Att2**

$$\frac{\sqrt{(7\cdot 17)^2 + 14\cdot 59}}{8\cdot 29 - 1\cdot 64 \times 2\cdot 23}$$

$$\frac{\sqrt{7^2 + 15}}{8 - 2 \times 2}$$

$$\frac{\sqrt{49 + 15}}{8 - 4}$$

$$\frac{\sqrt{64}}{4}$$

$$\frac{8}{4}$$

$$= \quad 2$$

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Precedent error (*i.e.* incorrect order)
B3   Mishandles square root
B4   Incorrect squaring
B5   Incorrect use of indices
B6   Decimal error
B7   Mathematical error
B8   Calculates first, then rounds (*i.e.* 1·8, rounded to 2)

Slips (-1)
S1   Numerical errors to a maximum of −3
S2   Incorrect rounding to a max of −3 if it affects answer

Misreadings (-1)
M1   Misreads a digit, provided it doesn't oversimplify the question

(c) (i) Evaluate   **5 marks**   **Att 2**

$$\frac{\sqrt{(7\cdot17)^2 + 14\cdot59}}{8\cdot29 - 1\cdot64 \times 2\cdot23}$$

$$\frac{\sqrt{51\cdot4089 + 14\cdot59}}{8\cdot29 - 3\cdot6572}$$

$$\frac{\sqrt{65\cdot9989}}{4\cdot6328}$$

$$\frac{8\cdot123970704}{4\cdot6328}$$

$$1\cdot7535 \quad = \quad 1\cdot8$$

Blunders (-3)

B1   Apply once in (c) (i). Correct answer with no work shown. ✍
B2   Mishandles square root
B3   Incorrect squaring
B4   Precedent error
B5   Incorrect use of indices
B6   Decimal error

Slips (-1)
S1   Numerical error to a maximum of −3
S2   Incorrect or no rounding, apply once if it affects final answer

Misreadings (-1)
M1   Misreads a digit, provided it doesn't oversimplify the question

Attempts (2 marks)
A1   Any relevant step *e.g. squaring, multiplying, square root etc.*

Worthless (0)
W1   Incorrect answer no work shown
W2   Work of no merit

(c) (ii)   **10 (5,5) marks**   **Att 2,2**

I
105% = €630
1% = €630 ÷ 105
1% = €6
100% = €600   **5m**

€600 = 5760 rand
€1 = 5760 ÷ 600
€1 = 9·6 rand   **5m**

Value of euro in rand: €1 = 9·6 rand

II
5760 rand × 105%
= 6048 rand   **5m**

€630 = 6048 rand
€1 = 6048 ÷ 630
€1 = 9·6 rand   **5m**

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*   Accept final answer €1 = 9·6 rand with some work for 10 marks
*   Two parts in marking this question; dealing with the 5% and the conversion in any
         order. 5 marks each (but note first *)

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Incorrect operation
B3   Inverted division
B4   Decimal error

Slips (-1)
S1   Numerical error to a maximum of −3

Misreadings (-1)
M1   Misreads a digit provided it doesn't oversimplify the question

Attempts (2,2 marks)
A1   105% or 100% + 5%
A2   105% = €630 and stops
A3   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   Work of no merit

QUESTION 2

Part (a)	10 marks	Att 3
Part (b)	25 marks	Att 3,3,2
Part (c)	15 marks	Att 2,2,2
Part (a)	10 marks	Att 3

A computer salesperson is paid an annual salary of €30 000.
He is also paid a commission of 4% on sales.
Last year the salesperson earned €38 000.

✎   Calculate the value of the sales.

(a)	10 marks	Att 3

I
€38,000 − €30,000 = €8000
Commission of 4% = €8000
1% = €8,000 ÷ 4
1% = €2,000
100% = Sales = €200,000
II
€30,000 + 4% of Sales = €38,000
€30,000 + ·04 Sales = €38,000
·04 Sales = €38,000 − €30,000
·04 Sales = €8,000
Sales = €8,000 ÷ ·04
Sales = €200,000

Blunders (-3)

B1   Correct answer no work shown. ✎
B2   Decimal error
B3   Percentage error
B4   Incorrect transposition
B5   Mathematical error
B6   Expresses % as an incorrect fraction and continues
B7   In Method **I,** stops at €2,000
B8   Fails to finish

Slips (-1)
S1   Numerical errors to a maximum of −3

Misreadings (-1)
M1   Incorrect figure if it doesn't oversimplify the question *e.g.* uses €36,000

Attempts (3 marks)
A1   Indicates some knowledge of percentages *e.g.* 4% = 4/100 or ·04
A2   Subtraction involving €38,000 and €30,000 or €8,000 mentioned without work
A3   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   Adds €30,000 and €38,000
W3   Adds or subtracts 4 and €30,000 or 4 and €38,000
W4   Work of no merit

Part (b)   **25 (10,10,5) marks**   **Att 3,3,2**

Aoife is single and earned €40 000 last year. Aoife's tax credits are listed below.

Single Person Tax Credit	€1830
PAYE Tax Credit	€1830
Rent Allowance Tax Credit	€400
Trade Union Payment Tax Credit	€70

(i) ✍ Calculate Aoife's total tax credits.

The standard rate cut-off point for a single person was €36 400.
The standard rate of income tax was 20% and the higher rate was 41%.

(ii) ✍ Calculate the tax paid by Aoife on her income.

Aoife also had to pay a 2% income levy on her gross income.

(iii) ✍ Calculate Aoife's net income after all deductions had been made.

(b) (i)   **10 marks**   **Att 3**

€1,830 + €1,830 + €400 + €70 =   **7m**

Total tax credits = €4,130   **10m**

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Omits one tax credit
B3   Addition indicated but fails to complete

Slips (-1)
S1   Numerical error to a maximum of −3

Misreadings (-1)
M1   Incorrect number written *e.g.* €1,380 *etc.* provided it doesn't oversimplify the question

Attempts (3 marks)
A1   Adds two numbers together from list
A2   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   Work of no merit

(b) (ii)   **10 marks**   **Att 3**

€40,000 − €36,400 = €3,600	**3m**	
€36,400 × 20% = €7,280	**3m**	
		↕ Interchangeable
€3,600 × 41% = €1,476	**4m**	
€7,280 + €1,476 = €8,756	**7m**	
€8,756 − €4,130	**7m**	
The tax paid = €4,626	**10m**	

*   Accept candidate's tax credit figure from (b)(i)

*   If candidate gets 41% of €36,400 (€14,924) and 20% of €3,600 (€720) **and** continues correctly, this is one blunder (Total tax €15,644 minus tax credits €4,130 = €11,514 is worth 7 marks)

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Decimal error
B3   Percentage error
B4   20% of an incorrect figure but note *2
B5   41% of an incorrect figure but note *2
B6   Mishandles tax credits
B7   Mathematical error
B8   Fails to finish

Slips (-1)
S1   Numerical error to a maximum of −3

Misreadings (-1)
M1   Uses 21%
M2   Uses 40% or 42%

Attempts (3 marks)
A1   Finds 20% or 41% of any number and stops
A2   Writes 20% as 20/100, 1/5 or ·2 without any further work of merit
A3   Writes 41% as 41/100 or 0·41 without any further work of merit
A4   Some knowledge of tax paid *e.g.* writes tax paid = total tax – tax credits
A5   €7,280 **or** €1,476 with or without work
A6   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown, but note A2, A3 and A5
W2   Work of no merit

(b) (iii)   **5 marks**   **Att 2**

€40,000 × 2% = €800
€4,626 + €800 = € 5,426
€40,000 − € 5,426
Net income = €34,574

   * Accept candidate's "tax paid" figure from (b) (ii)

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Decimal error
B3   Percentage error
B4   Finds 2% of incorrect figure and continues
B5   Mathematical error
B6   Fails to finish
B7   Ignores "tax paid" figure when calculating net income

Slips (-1)
S1   Numerical errors to a maximum of −3

Attempts (2 marks)
A1   Finds 2% of any number and stops
A2   2% = ·02 or 2/100 or 1/50
A3   Demonstrates some knowledge of net income *e.g.* Net income = Gross – Tax
A4   Demonstrates some knowledge of income levy *e.g.* Levy = Gross × %
A5   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown, but note A2
W2   Work of no merit

Part (c)   **15 (5,5,5) marks**   **Att 2,2,2**

U is the universal set and P and Q are two subsets of U.
U = 30, #P = 16 and # Q = 6.

(i)   ✍   Find with the aid of a Venn diagram the minimum value of # $(P \cup Q)'$.

(ii)   ✍   Find with the aid of a Venn diagram the maximum value of # $(P \cup Q)'$.

U = u, #P = p, # Q = q and # $(P \cup Q)' = x$.

**(iii)**   ✍   Show with the aid of a Venn diagram, that if p>q and x is a maximum,
then u = p + x.

(c) (i)   **5 marks**   **Att 2**

16 + 6 = 22
30 −22 = 8

U (30)
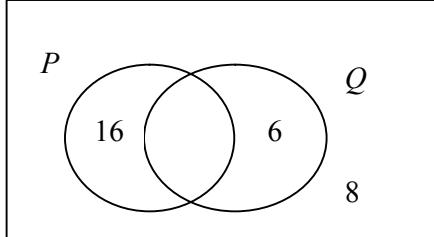
Minimum value of # $(P \cup Q)' = 8$

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*   Accept correct Venn diagram for full marks
*   Ignore notation

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Incorrect operation
B3   Venn diagram correct but no minimum included or stated

Slips (-1)
S1   Correct value of 8 for minimum with work and no Venn diagram or incorrect Venn
diagram

Attempts (2 marks)
A1   Venn diagram
A2   16 + 6 and stops
A3   30 – 6 or 30 – 16 and stops
A4   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   Work of no merit

(c) (ii)   **5 marks**   **Att 2**

16 – 6 = 10

30 – 16 = 14

U (30)
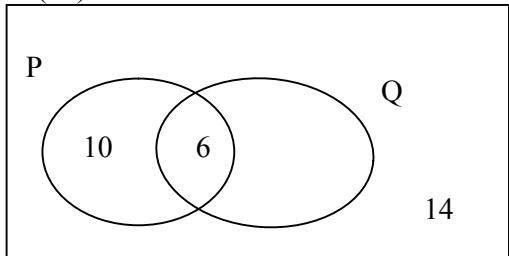
Maximum value of # $(P \cup Q)$ ´= 14

* Accept correct Venn diagram for full marks
* Ignore notation

Blunders (-3)

B1   Correct answer no work shown. ✍

B2   Incorrect operation

B3   Venn diagram correct but no maximum included or stated

Slips (-1)

S1   Correct value of 14 for maximum with work and no Venn diagram or incorrect Venn diagram

Attempts (2 marks)

A1   Venn diagram

A2   16 – 6

A3   Any relevant step

Worthless (0)

W1   Incorrect answer no work shown

W2   Work of no merit

**(c) (iii)**   **5 marks**   **Att 2**

0 in Q only for x to be a maximum

$u = p - q + q + x$

$u = p + x$

Venn diagram to give: $u = p + x$

I

II

III
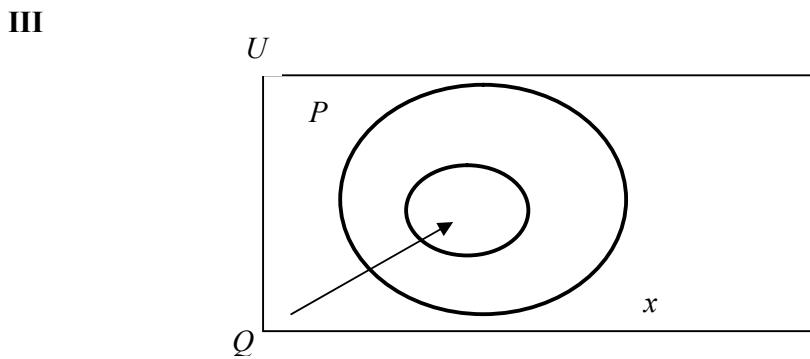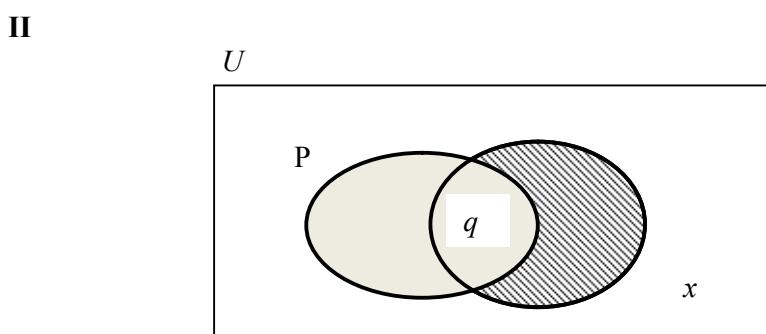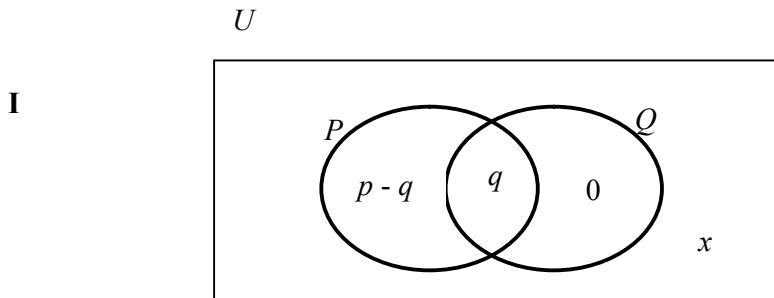
*   Accept correct Venn diagram for full marks **with conclusion** $u = p + x$

*   Ignore notation

Blunders (-3)

B1   Omits term from equation

B2   Fails to finish

B3   # Q/P $\neq 0$

B4   #P/Q $\neq p - q$

B5   # $(P \cap Q) \neq q$

Slips (-1)

S1 $u = p - q + q + x \rightarrow u = p + x$ only. No Venn diagram.

S2   No conclusion

Attempts (2 marks)
A1   Venn diagram
A2 $p - q$
A3   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2 $u = p + x$ only (Given)
W3   No work of merit

QUESTION 3

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 3,3
Part (c)	20 marks	Att 2,2,2,2

Part (a)	10 marks	Att 3

> ✍ Given that $t^2 - s = r$, express t in terms of r and s.

(a)	10 marks	Att 3

$t^2 - s = r$	**Given**	
$t^2 = r + s$	**7m**	
$t = \sqrt{r+s}$	**10m**	

* Two steps in this question, transposition and square root
* $t - s = r$ and continues correctly to get $t = r + s$ is worth 6 marks - Misread and B2.
* Finds s correctly in terms of t and r is 6 marks - Misread and B2. ($s = t^2 - r$)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Mishandles or fails to get square root
B3 Incorrect operation *e.g.* May attempt to square everything
B4 Transposition error

Misreadings (-1)

M1 Note *2 and *3
M2 $t^2 + s = r$ and continues correctly to get $t = \sqrt{r - s}$

Attempts (3 marks)

A1 Effort at square root
A2 $t^2 - s - r$ (with or without $= 0$)
A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown
W2 Work of no merit

Part (b)	20 (10,10) marks	Att (3,3)

> **(i)** ✍ Divide $3x^2 + 5x - 28$ by $x + 4$.
>
> **(ii)** ✍ Solve the equation $\dfrac{4x + 2}{5} - \dfrac{6 - x}{3} = -5$.

I

$3x^2 + 5x - 28 \div x + 4$

$$\frac{(3x - 7)(x + 4)}{x + 4}$$

$= 3x - 7$

II

$3x^2 + 5x - 28 \div x + 4$

$3x^2 + 12x - 7x - 28 \div x + 4$

$3x(x + 4) + 7(x + 4) \div x + 4$

$(3x - 7)(x + 4) \div x + 4$

$= 3x - 7$

III

Division to give answer $3x - 7$

$$x + 4 \,\overline{\big)\, 3x^2 + 5x - 28} \quad \overset{\displaystyle 3x - 7}{}$$

$$\underline{3x^2 + 12x}$$

$$-7x - 28$$

$$\underline{-7x - 28}$$

$$0$$

* $(3x+7)(x - 4)$ and continues is one blunder (B4), will also incur B5 **or** B6.
All other attempts to factorise apply B2, B3 and/or B4.

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Incorrect factors of $3x^2$ in method **I**
B3   Incorrect factors of -28 in method **I**
B4   Incorrect factors leading to an incorrect middle term in method **I**
B5   Fails to finish *i.e.* no cancellation in method **I**
B6   Incorrect cancellation
B7   Mathematical error, once if consistent

Slips (-1)
S1   Numerical errors to a maximum of -3

Attempts (3 marks)
A1   Some effort at factorising
A2   Sets up division
A3   Multiplies instead of dividing, with at least one correct term
A4   Finds guide number (-84) in method **II** and stops
A5   Quadratic with some correct substitution
A6   Sets up quadratic and identifies a,b or c
A7   Uses quadratic formula and stops at correct roots $(x = -4$ and $x = 7/3)$
A8   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   (   ) (   )
W3   Work of no merit

I

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$$\frac{3(4x+2) - 5(6-x)}{15} = -5$$

$3(4x + 2) - 5(6 - x) = 15 \,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

II

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$$\frac{(4x+2)(3)(5)}{5} - \frac{(6-x)(3)(5)}{3} = -5(3)(5)$$

$3(4x + 2) - 5(6 - x) = 15 \,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

III

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$3(4x + 2) - 5(6 - x) = (5)(3)\,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

* $x = -3$, by trial and error or similar, **fully verified** merits 10 marks

Blunders (-3)

B1   Correct answer no work shown. ✍

B2   Distribution error, once if consistent

B3   Incorrect common denominator or mishandles denominator

B4   Transposition error, once if consistent

B5   Mathematical error

B6   Mishandles numerator

B7   Combines unlike terms and continues

Slips (-1)
S1   Numerical errors to a maximum of -3

Attempts (3 marks)
A1   Common denominator and stops
A2   Oversimplified but some correct work
A3   Cross multiplies
A4   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   Adds or subtracts terms incorrectly *e.g.* $5+3$, or $4x+2\pm6-x$ etc.
W3   Work of no merit

A car park can accommodate cars and mini-buses.
On a particular day there were x cars and y mini-buses
in the car park, giving a total of 520 vehicles.
The parking area for a car is 7 m^2 and the parking area
for a mini-bus is 12 m^2.
On that day a total area of 3840 m^2 was occupied by cars and mini-buses.
(i)   Write down two equations to represent the above information.

(ii) ✍ Solve these equations to find the number of cars and the number of
         mini-buses in the car park on that day.
There is a flat rate charge per day for parking.
The flat rate for mini-buses is 3 times that for cars. On that day €3000 was taken in.

(iii) ✍ What is the flat rate for cars?

(c) (i) 10 (5,5) marks Att 2,2

$x + y$	=	520	**5m**
$7x + 12y$	=	3840	**5m**

* Two equations to mark in (c) (i)
* Each equation is marked separately
* Each equation is worth 5 marks, attempt 2

* Answer is sufficient for full marks (No ✍ in question)

Blunders (-3)
B1 Incorrect term

Attempts (2,2 marks)
A1 x or y or $7x$ or $12y$
A2 Effort at creating an equation equal to 520 or 3,840
A3 Any relevant step

(c) (ii) 5 marks Att 2

I

$x + y$ =   520 (-7)
$7x + 12y$ =   3840

$-7x - 7y$ = -3640
$\underline{7x + 12y = 3840}$
      $5y$ = 200
      y = $200 \div 5$
      y = 40

$x + y$ =   520
$x + 40$ =   520
x = $520 - 40$
x =   480
$x = 480$ and $y = 40$.

II

$x + y \quad = \quad 520 \quad (-12)$

$7x + 12y \quad = \quad 3840$

$-12x - 12y = -6240$

$\underline{7x \;+\; 12y = \quad 3840}$

$-5x \qquad = \;-2400$

$5x \qquad = \quad 2400$

$x \qquad = \quad 2400 \div 5$

$x \qquad = \quad 480$

$480 + y \qquad = \quad 520$

$y \qquad = \quad 520 - 480$

$y \qquad = \quad 40$

$x = 480 \quad$ and $\quad y = 40.$

III

$x = 520 - y$

$7(520 - y) + 12y = 3840$

$3640 - 7y + 12y = 3840$

$-7y + 12y = 3840 - 3640$

$5y = 200$

$y = 200 \div 5$

$y = 40$

$x + y \qquad = \quad 520$

$x + 40 \qquad = \quad 520$

$x \qquad = \quad 520 - 40$

$x \qquad = \quad 480$

$x = 480 \quad$ and $\quad y = 40.$

IV

$y = 520 - x$

$7x + 12(520 - x) = 3840$

$7x + 6240 - 12x = 3840$

$7x - 12x = 3840 - 6240$

$-5x = -2400$

$5x = 2400$

$x = 2400 \div 5$

$x = 480$

$480 + y \qquad = \quad 520$

$y \qquad = \quad 520 - 480$

$y \qquad = \quad 40$

$x = 480 \quad$ and $\quad y = 40$

*1   Equations may be also solved by substituting $y = \frac{3840 - 7x}{12}$ or $x = \frac{3840 - 12y}{7}$

*2   Accept candidate's equations from (c) (i) provided oversimplification does not occur

*3   Apply only one blunder in establishing the first equation in terms of x only or the first equation in terms of y only.

*4   Finding the second variable is subject to a maximum deduction of 3 marks

*5   Correct values of x and y without algebraic work, **both verified in both equations** merits full marks

*6   Correct values of x and y without algebraic work **not verified or not fully verified in both equations** merits attempt mark only

Blunders (-3)
B1   Finds one variable only
B2   Distribution error
B3   Mathematical error
B4   Incorrect substitution when finding second variable, but note M1
B5   Transposition error in solving first variable
B6   Transposition error in solving second variable
B7   Error(s) in establishing the first equation in terms of x $(-5x = -2400)$ only or the first equation in terms of y $(-5y = -200)$ only through elimination by cancellation **I** and **II**
B8   Error(s) in establishing the first equation in terms of x $(-5x = -2400)$ only or the first equation in terms of y $(5y = 200)$ only through elimination by substitution **III** and **IV**

Slips (-1)
S1   Numerical errors to a max of -3

Misreadings (-1)
M1   Misreads digits, providing it doesn't oversimplify

Attempts (2 marks)
A1   Any correct manipulation of either given equation and stops
A2   Some correct partial substitution and stops
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   Trial and error, but see *5 and *6 above
W3   Work of no merit

(c) (iii)   **5 marks**   **Att 2**

> $3x : x$
>
> $480x + 3x(40) = 3,000$
> $480x + 120x = 3,000$
> $600x = 3,000$
> $x = 3,000 \div 600$
> $x = 5$
>
> Flat rate for cars   =   €5

*   Accept candidate's answers from (c) (ii)

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Mathematical error
B3   Uses ratio Buses: Cars 1:3, but note M1

Slips (-1)
S1   Numerical errors to a maximum of −3

Misreadings (-1)
M1   Misreads values for buses and cars (*i.e.* uses 40 cars and 480 buses – note B3)

Attempts (2 marks)
A1   States ratio 3:1 and stops
A2   Some correct partial substitution and stops
A3 $3000 \div 4$ or 750 or $3 + 1$
A4   Some relevant step *e.g.* divides 3,000

Worthless (0)
W1   Incorrect answer no work shown
W2   Trial and error with incorrect value(s)
W3   Work of no merit

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 2,3,2
Part (c)	20 marks	Att 2,2,2,2

Part (a)	**10 marks**	**Att 3**

✎   Graph on the number line the solution set of

$$-2x + 1 > -7, \ x \in \mathbb{N}.$$

(a)	**10 marks**	**Att 3**

I

$-2x + 1 > -7$

$-2x > -7 - 1$

$-2x > -8$

$2x < 8$

$x < 4$

$x \in \{1,2,3\}$

II

$-2x + 1 > -7$

$1 + 7 > 2x$

$8 > 2x$

$4 > x$

$x < 4$

$x \in \{1,2,3\}$

*   Accept $x \in \{0, 1, 2, 3\}$ plotted
*   Graph must be based on candidate's inequality

Blunders (-3)

B1   Correct answer no work shown. ✎
B2   Mishandles inequality
B3   Fails to graph
B4   Transposition error
B5   Mathematical error
B6 $x \in R$ indicated
B7 $2x + 1 > -7$ solved to get $x > -4$ with correct graph

Slips (-1)
S1   Numerical errors to a max of −3
S2   Includes 4 on graph
S3   Each incorrect or missing number to a maximum of 3

Misreadings (-1)
M1   Includes equals in inequality

Attempts (3 marks)
A1   Tests any value in inequality and stops
A2   Draws any number line
A3   Any relevant step

Worthless (0)
W1   List given with no correct value
W2   Work of no merit

Part (b)   **20 (5,10,5) marks**   **Att 2,3,2**

(i)   Factorise $x^2 - 1$.

(ii)   ✍   Factorise fully $ax - 3 - a + 3x$.

(iii)   Factorise $6x^2 + x - 35$

(b) (i)   **5 marks**   **Att 2**

$x^2 - 1 =$	**Given**
$x^2 - 1^2 =$	**2m**
$(x - 1)(x + 1)$	**5m**

*   Answer is sufficient for full marks (No ✍ in question)
*   Accept also (with or without brackets) for full marks any of the following:
         $(x - 1)$ and $(x + 1)$ [The word 'and' is written down]
         $(x - 1)$ or $(x + 1)$ [The word 'or' is written down]
         $(x - 1), (x + 1)$ [a comma is used]
*   Quadratic equation method is subject to slips and blunders

Blunders (-3)
B1   Incorrect factors of x^2
B2   Incorrect factors of -1
B3 $(1 - x)(1 + x)$
B4   Answer left as roots $x = \pm 1$

Slips (-1)
S1 $(x - 1) + (x + 1)$
S2 $(x - 1) - (x + 1)$

Attempts (2 marks)
A1 $x^2 - 1^2$
A2   Correct factors of x^2 only
A3   Correct factors of 1 or -1 only
A4 $\pm x$ or ± 1
A5 $x^2 - 1 = x \times x - 1 \times 1$
A6   Difference of two squares mentioned
A7 $\sqrt{1}$
A8 $\sqrt{x^2}$
A9   Correct quadratic formula with some correct substitution

Worthless (0)
W1   Combines terms incorrectly and stops *e.g.* $-1\,x^2$
W2   Work of no merit

(b) (ii)   **10 marks**   **Att 3**

I

$ax - 3 - a + 3x$	**Given**
$ax - a + 3x - 3$	**3m**
$a(x-1) + 3(x-1)$	**7m**
$(a+3)(x-1)$	**10m**

II

$ax - 3 - a + 3x$	**Given**
$ax + 3x - a - 3$	**3m**
$x(a+3) - 1(a+3)$	**7m**
$(a+3)(x-1)$	**10m**

---

*   Accept also (with or without brackets) for full marks any of the following with work
       $(a+3)$ and $(x-1)$ [The word 'and' is written down]
       $(a+3)$ or $(x-1)$ [The word 'or' is written down]
       $(a+3)$, $(x-1)$ [A comma is used]

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Failure to complete last step *e.g.* stops at $a(x-1) + 3(x-1)$
B3   Error in factorising any pair of terms, apply once if consistent
B4   Incorrect last step *e.g.* $3a(x-1)$ or $(a+3)(-1x)$
B5   Incorrect common factor and continues *e.g.* $x(a+3) + 1(-a-3)$. B4 will also apply.

Slips (-1)
S1 $(a+3) + (x-1)$
S2 $(a+3) - (x-1)$

Attempts (3 marks)
A1   Pairing off matching terms, or indication of common factors and stops
A2   Correctly factorises any pair and stops
A3   Any relevant step

Worthless (0)
W1   (   )(   )
W2   Pairing of terms with nothing in common *e.g.* $ax - 3$ and no further work of merit
W3   Work of no merit

(b) (iii)   **5 marks**   **Att 2**

> **I**
> $6x^2 + x - 35$
> $(2x + 5)(3x - 7)$
>
> **II**
> $6x^2 + x - 35$
> $6x^2 - 14x + 15x - 35$
> $2x(3x - 7) + 5(3x - 7)$
> $(2x + 5)(3x - 7)$
>
> **III**
> $6x^2 + x - 35$
> $6x^2 + 15x - 14x - 35$
> $3x(2x + 5) - 7x(2x + 7)$
> $(2x + 5)(3x - 7)$

*   Answer is sufficient for full marks (No ✍ in question)

*   Quadratic may be used to solve $6x^2 + x - 35 = 0 \;\rightarrow\; x = -\dfrac{5}{2},\, x = \dfrac{7}{3}$ and continues

*   Accept also (with or without bracket) for full marks any of the following
       $(2x + 5)$ and $(3x - 7)$ [The word 'and' is written down]
       $(2x + 5)$ or $(3x - 7)$ [The word 'or' is written down]
       $(2x + 5),\, (3x - 7)$ [A comma is used]

Blunders (-3)
B1   Incorrect factors of $6x^2$
B2   Incorrect factors of -35
B3   Factors leading to an incorrect middle term
B4   Substitution error in quadratic
B5   Uses quadratic to get roots and stops

Slips (-1)
S1 $(2x + 5) + (3x - 7)$
S2 $(2x + 5) - (3x - 7)$

Attempts (2 marks)
A1   Some correct factors
A2   Identifies a, b or c for quadratic
A3   Quadratic with some correct substitution
A4   Any correct relevant step

Worthless (0)
W1   Incorrect answer no work shown but note attempts
W2   (   )(   )
W3   Quadratic formula only
W4   Work of no merit

> The new Lansdowne Road stadium has seating capacity for 200 journalists.
> It was decided initially that this seating would be in x rows of equal value.
> **(i)**   Write, in terms of x, the number of seats per row required to accommodate
>   the 200 journalists.
> During the construction it was decided to have 3 fewer rows to accommodate the 200
> journalists.
> **(ii)**   Write, in terms of x, the number of seats per row now required.
>   It was found that 15 extra seats per row were required compared to the initial
>   plan.
> **(iii)** ✍   Write an equation using the above information and solve for x.

(c) (i)   **5 marks**   **Att 2**

$$\frac{200}{x}$$

*   Answer is sufficient for full marks (No ✍ in question)

Misreadings (-1)
M1   Uses letter other than x

Blunders (-3)
B1   Inversion

Attempts (2 marks)
A1   Effort at forming expression using 200 and x

Worthless (0)
W1 x only or 200 or similar

(c) (ii)   **5 marks**   **Att 2**

$$\frac{200}{x-3}$$

*   Answer is sufficient for full marks (No ✍ in question)

Misreadings (-1)
M1   Uses letter other than x if not penalised already

Blunders (-3)
B1   Inversion, but do not penalise if already blundered in (c) (i)

B2 $\frac{200}{x+3}$

B3 $\frac{200}{x} \pm 3$ (Linear in (c)(iii) and subject to further penalty there)

Attempts (2 marks)
A1   Forms an incorrect expression with at least two of the following x, 3, 200

(c) (iii)   **10 (5,5) marks**   **Att 2,2**

Establish equation

$$\frac{200}{x-3} - \frac{200}{x} = 15 \qquad \textbf{2m}$$

$$\frac{200\,x - 200\,(x-3) = 15\,x\,(x-3)}{x\,(x-3)} \qquad \textbf{2m}$$

$$200\,x - 200\,x + 600 = 15x^2 - 45\,x \qquad \textbf{2m}$$

$$600 = 15x^2 - 45\,x \qquad \textbf{5m}$$
$$15x^2 - 45\,x - 600 = 0$$
$$x^2 - 3\,x - 40 = 0$$

Solve

$$(x-8)(x+5) = 0 \qquad \textbf{2m}$$
$$\rightarrow \quad 8 \text{ and} -5 \qquad \textbf{4m}$$
$$\text{Solution:} \quad x = 8 \qquad \textbf{5m}$$

---

*   Mark in two parts: 5 marks for **equation** and 5 marks for **solving**
*   Accept candidate's expressions from (c) (i) and (ii). Linear merits Att 2,2 at most

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Distribution error - apply each time but once if consistent
B3   Transposition error - apply each time but once if consistent
B4   Mathematical error in forming equation *e.g.* line 1
B5   Incorrect factors
B6   Correct factors and stops, will also incur S2
B7   Error in quadratic formula
B8   Each error in grouping terms or fails to group
B9   Error in establishing equation *e.g.* line 1

Slips (-1)
S1   Numerical errors to a maximum of 3
S2   Stops at $x = 8$, $x = -5$ or concludes $x = -5$

Attempts (2,2 marks)
A1   Linear equation merits attempt marks at most
A2   Trial and error merits attempt at most
A3   Any correct relevant step

Worthless (0)
W1   Incorrect answer no work shown
W2   (   )(   )
W3   Substitution of any number other than 8 or −5

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 3,3
Part (c)	20 marks	Att 2,3,2

| **Part (a)** | **10 marks** | **Att 3** |

> ✍ Given that $f(x) = 3x - 4$ and that $f(k) = 11$, find the value of k.

| **(a)** | **10 marks** | **Att 3** |

$$3k - 4 = 11 \qquad \textbf{3m}$$
$$3k \quad = 11 + 4 \; or \; 15 \qquad \textbf{4m}$$
$$k \quad = 15 \div 3 \qquad \textbf{7m}$$
$$k \quad = \quad 5 \qquad \textbf{10m}$$

*   Ignore notation (*e.g.* $x = 5$ or answer $= 5$ for full marks)
* $k = 5$ fully verified $= 10$ marks

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Transposition error
B3 $3k - 4 = 0$ and continues correctly
B4   Incorrect operation

Slips (-1)
S1   Numerical error to a maximum of 3

Attempts (3 marks)
A1   Fills in k and stops
A2   Attempt to divide by 3
A3   Tests values *e.g.* $k = 11$ to get answer of 29
A4   Any relevant step

Worthless (0)
W1   Incorrect answer with no work shown

| **Part (b)** | **20 (10,10) marks** | **Att 3,3** |

> Let f be the function $f : x \rightarrow 7x - x^2$.
>
> ✍   Draw the graph of f for $0 \le x \le 7, x \in \mathbb{R}$.

(b) Function *f*   **20 (10,10) marks**   **Att 3,3**
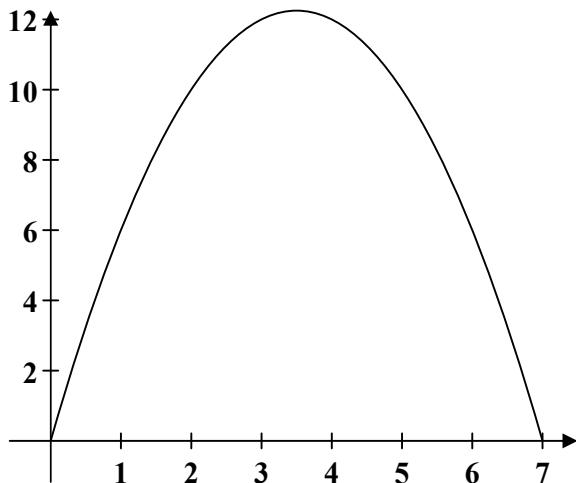
x	0	1	2	3	4	5	6	7
7x	0	7	14	21	28	35	42	49
−x²	0	−1	−4	−9	−16	−25	−36	−49

x	0	1	2	3	4	5	6	7
f (*x*)	0	6	10	12	12	10	6	0

f(x)	0	6	10	12	12	10	6	0

or

or

$f : x \rightarrow 7x - x^2.$
$f : x \rightarrow 7(0) - 0^2 = 0 - 0 = 0$
$f : x \rightarrow 7(1) - 1^2 = 7 - 1 = 6$
$f : x \rightarrow 7(2) - 2^2 = 14 - 4 = 10$
$f : x \rightarrow 7(3) - 3^2 = 21 - 9 = 12$
$f : x \rightarrow 7(4) - 4^2 = 28 - 16 = 12$
$f : x \rightarrow 7(5) - 5^2 = 35 - 25 = 10$
$f : x \rightarrow 7(6) - 6^2 = 42 - 36 = 6$
$f : x \rightarrow 7(7) - 7^2 = 49 - 49 = 0$

(0.0)   (1,6)   (2, 10)   (3, 12)   (4,12)   (5, 10)   (6, 6)   (7,10)

* Table is worth 10 marks, graph is worth 10 marks
* Middle lines of table do not have to be shown
* Candidates may choose not to use a table
* Points might not be listed, mark on position on graph

* Graph constitutes work in this question ✍

B1   Error in calculating $7x$, once if consistent
B2   Error in calculating $-x^2$, once if consistent, but note A1
B3   Error in calculating last line of table, once if consistent
B4   Each incorrect point without work
B5   Point plotted incorrectly, once if consistent
B6   Each missing point
B7   Axes scaled incorrectly, once only
B8   Reversed axes
B9   No curve between (3,12) and (4,12) on graph
B10   Points not joined, most probably incurs B9 also

Slips (-1)
S1   Numerical error to a maximum of −3

Attempts (3,3 marks)
A1   Error leading to a linear graph
A2   Some correct substitution
A3   Draws axes, with some indication of scaling

Part (c)   **20 (5,10,5) marks**   Att2,3,2

The formula for the height, y metres, of a golf ball above ground level x seconds after it is hit, is given by $7x - x^2$.
Use your graph from part **(b)**:

(i)   ✍   to find the maximum height reached by the golf ball

(ii)   ✍   to estimate the number of seconds the golf ball was more than 2 metres above the ground.

The graph below represents the flight of another golf ball.
The flight of the golf ball is given by the formula $ax - x^2$, $x \in \mathbb{R}$.
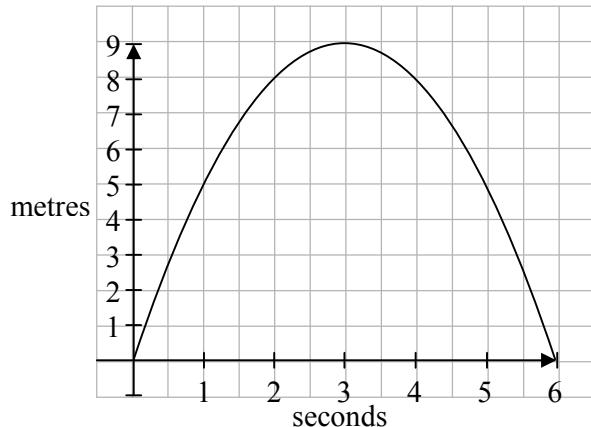
(iii)   ✍   Find the value of a.

(c) (i)   **5 marks**   **Att 2**

Maximum height → 12·25 m

*   Accept answer consistent with candidate's graph, tolerance ± ·2

Blunders (-3)

B1   Correct answer no indication on graph. ✍
B2   Maximum indicated on graph but no value given
B3   Outside of tolerance
B4   States x co-ordinate of maximum point
B5   Fails to use graph

Slips (-1)
S1   Accept maximum as point if y value is correct *i.e.* (3·5, 12·25)

Attempts (2 marks)
A1   Reads maximum from table
A2   Some relevant substitution in effort to find maximum
A3   Uses graph of (c) (iii) to find maximum of 9
A4 $x = 3· 5$ indicated on graph

Worthless (0)
W1   Incorrect answer no work, but note attempts

(c) (ii)   **10 marks**   **Att 3**

More than 2 metres above the ground→   6·7 − 0·3=   6·4 secs

*   Accept values from candidate's graph with a tolerance of ± ·2

Blunders (-3)
B1   No subtraction
B2   Value(s) not consistent with candidate's graph
B3   No indication on graph
B4   Indication on graph but no value given each time. B1 also applies
B5   Outside of tolerance, each time
B6   ·3 − 6·7 = **− 6·4** or candidate's equivalent

Slips (-1)
S1   Numerical error

Attempts (3 marks)
A1   Correctly solves $f(x) = 2$ by formula; graph not used
A2 $f(2)$ found (answer = 10, or candidate's equivalent)
A3   Uses graph of (c)(iii) to find answer (5·7 − ·3 = 5·4)

Worthless (0)
W1   Incorrect answer, no work shown

Fills in any of the following points (1,5) (2,8) (3,9) (4, 8) (5, 5) (6,0)
or any other correct points to solve equation $ax - x^2 = y$

I
e.g. (1, 5)
$a\,(1) - (1)^2 = 5$
$a - 1 = 5$
$a = 5 + 1$
Value of $a\ = 6$

II
$ax - x^2 = y$
$x(a - x) = y$
Fill in *e.g.* (1, 5)
$1(a - 1) = 5$
$a - 1 = 5$
$a = 5 + 1$
Value of $a\ = 6$

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Co-ordinates reversed when substituting
B3   Incorrect squaring
B4   Transposition error
B5   Substitutes for a instead of x
B6   Fails to finish
B7   Incorrect factors

Slips (-1)
S1   Numerical error to a maximum of -3

Attempts (2 marks)
A1   Some relevant substitution
A2 $ax - x^2 = \textbf{\textit{f(x)}}$ or $ax - x^2 = \textbf{y}$
A3   Writes down any point on the curve
A4   Attempt to factorise

Worthless (0)
W1   Incorrect answer no work shown
W2   No work of merit

QUESTION 6

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 3,2,2
Part (c)	20 marks	Att 3,2,2

Part (a)	10 marks	Att 3

✍ When $a = \dfrac{1}{4}$, find the value of $\dfrac{a+5}{3} - \dfrac{a+4}{2}$.

(a)	10 marks	Att 3

I

$$\frac{a+5}{3} - \frac{a+4}{2}$$

$$\frac{\frac{1}{4}+5}{3} - \frac{\frac{1}{4}+4}{2}$$

$$\frac{5\frac{1}{4}}{3} - \frac{4\frac{1}{4}}{2}$$

$$1\frac{3}{4} - 2\frac{1}{8}$$

$$= \frac{-3}{8}$$

II

$$\frac{a+5}{3} - \frac{a+4}{2}$$

$$\frac{2(a+5) - 3(a+4)}{6}$$

$$\frac{2a+10-3a-12}{6}$$

$$\frac{-a-2}{6}$$

$a = \frac{1}{4}$

$$\frac{-\frac{1}{4}-2}{6}$$

$$\frac{-2\frac{1}{4}}{6}$$

$$-\frac{3}{8}$$

* Accept answer in decimal format $(-\cdot 375)$ or equivalent fraction

Blunders (-3)

B1   Correct answer no work shown. ✍

B2   Incorrect denominator

B3   Mishandles denominator

B4   Mishandles numerator

B5   Mathematical error

B6   Distribution error, once if consistent

B7   Fails to combine like terms (if it affects final answer)

B8   Combines unlike terms

B9   Incomplete step (*e.g.* − 2¼/6 or similar)

Slips (-1)

S1   Numerical errors to a maximum of −3

Attempts (3 marks)

A1   Some correct substitution

A2   No denominator used

A3   Any correct relevant step

Worthless (0)

W1   Incorrect answer no work shown

W2   No work of merit

Part (b)   **20 (10,5,5) marks**   **Att 3,2,2**

> **(i)** ✍ Express in its simplest form:
> $$\frac{4}{x-1} - \frac{5}{x+2}.$$
>
> **(ii)** ✍ Hence, or otherwise, solve the equation:
> $$\frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{2},$$
> giving your answers correct to one decimal place.

(b)(i)   **10 marks**   **Att 3**

> $$\frac{4(x+2) - 5(x-1)}{(x-1)(x+2)}$$
>
> $$\frac{4x + 8 - 5x + 5}{(x-1)(x+2)}$$
>
> $$\frac{-x + 13}{(x-1)(x+2)}$$

*   Accept common denominator as $(x-1)(x+2)$. Penalise incorrect multiplication in (b) (ii)

Blunders (-3)

B1   Correct answer no work shown. ✍
B2   Incorrect denominator
B3   Mishandles denominator
B4   Mishandles numerator
B5   Mathematical error
B6   Distribution error, once if consistent
B7   Fails to combine like terms
B8   Combines unlike terms
B9   Reads as $\frac{4}{x-1} + \frac{5}{x+2}$. Continue to apply slips and blunders

Slips (-1)
S1   Numerical errors to a maximum of -3

Attempts (3 marks)
A1   Identifies common denominator and stops
A2   No denominator used
A3   Oversimplification
A4   Any relevant step

Worthless (0)
W1   Incorrect answer no work shown

W2   Adds or subtracts numerators and denominators *e.g.* $\frac{9}{2x+1}$ or $-\frac{1}{1}$ or $-\frac{1}{2x+1}$ *etc*

(b)(ii)   **10 (5,5) marks**   **Att 2,2**

$$\frac{-x+13}{(x-1)(x+2)} = \frac{3}{2}$$

$2(-x+13) = 3(x-1)(x+2)$

$-2x + 26 = (3x-3)(x+2)$

$-2x + 26 = 3x^2 + 3x - 6$

$3x^2 + 5x - 32 = 0$ **Equation 5m**

$$\frac{-5 \pm \sqrt{25 - 4(3)(-32)}}{2(3)}$$

$$\frac{-5 \pm \sqrt{25 + 384}}{6}$$

$$\frac{-5 \pm \sqrt{409}}{6}$$

$$\frac{-5 \pm 20 \cdot 22374842}{6}$$

$$\frac{-25 \cdot 22374842}{6} \quad \text{and} \quad \frac{15 \cdot 22374842}{6}$$

$x = -4 \cdot 2$ and $x = 2 \cdot 5$ **Solve 5m**

*   Mark in two parts: 5 marks for **equation** and 5 marks for **solving**

*   Accept candidate's expressions from (b)(i). Linear merits Att 2,2 at most

Blunders (-3)

B1   Correct answer no work shown. ✍

B2   Distribution error - apply each time but once if consistent

B3   Transposition error - apply each time but once if consistent

B4   Mathematical error in forming equation

B5   Incorrect denominator

B6   Mishandles denominator

B7   Mishandles numerator

B8   Fails to combine like terms

B9   Combines unlike terms

B10 Error in quadratic formula

B11 Error in application of quadratic formula

B12 Finds only one solution

Slips (-1)

S1   Numerical errors to a maximum of −3

S2   Fails to round or rounds incorrectly

Attempts (2,2 marks)

A1   Linear equation merits attempt marks at most

A2   Quadratic formula with some correct substitution

A3   Trial and error of correct solution(s) merits attempt at most

A4   Any relevant step

Worthless (0)

W1   Incorrect answer no work shown

W2   (   )(   )

W3   Substitution of any number other than 2·5 and − 4·2 or equivalent

W4   No work of merit

The diagram below shows part of the graphs of the functions
$f(x) = x^2 - 4x + 3$ and $g(x) = x + k.$
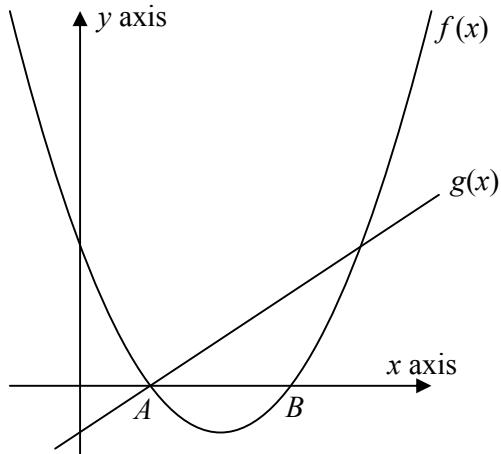
The graph of $f(x)$ cuts the x axis at A and B.
The graphs of $f(x)$ and $g(x)$ intersect at A.

(i)   ✍   Find the coordinates of A and the coordinates of B.

(ii)   ✍   Find the value of k.

(iii) ✍   Verify that $f(x)$ and $g(x)$ intersect also at the point $(4, 3)$.

(c) (i)   **10 marks**   **Att3**

	Solve $x^2 - 4x + 3 = 0$	**3m**
	$(x - 1)(x - 3) = 0$	**4m**
	$x = 1,\ x = 3$	**7m**
	$(1, 0)\ and\ (3, 0)$	**9m**
→	$A\ (1, 0)\quad B\ (3, 0)$	**10m**

*   If trial and error is used, must be fully verified for 10 marks

Blunders (-3)
B1   Correct answer no work shown. ✍
B2   Incorrect factors of x2
B3   Incorrect factors of +3
B4   Factors leading to an incorrect middle term
B5   Fails to find roots
B6   Fails to list coordinates with y = 0 included
B7   Uses quadratic and stops at roots
B8   (0,1) and (0,3) Reversed co-ordinates.

Slips (-1)
S1   Fails to specify A **or** B or incorrectly names A and B, apply once.
S2   After solving $x = 1$, $x = 3$ only states one point *i.e* (1,0) or (3,0). May also incur S1.

Attempts (3 marks)
A1 Some correct factors
A2 Identifies *a, b* or *c* for quadratic
A3 Quadratic with some correct substitution
A4 Finds where graph cuts y axis (0,3)
A5 *f*(*x*) = 0 or *g*(*x*) = 0 or (*x* , **0**)
A6 Substitution of 0,1 or 3
A7 Any correct relevant step
A8 Uses graph to read answer

Worthless (0)
W1 Incorrect answer no work shown

(c) (ii)   **5 marks**   **Att 2**

$$g(1) = 1 + k = 0 \qquad \textbf{2m}$$

$$\rightarrow \quad k = -1 \qquad \textbf{5m}$$

   *   Accept (4,3) or candidate's A co-ordinate from (c) (i)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Fails to let *g*(*x*) = 0
B3 Fails to let *x* = 1 (or candidate's equivalent value)
B4 Transposition error

Attempts (2 marks)
A1 Substitutes *x* = 1 (or candidate's equivalent) and stops
A2 *g*(*x*) = 0
A3 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown

(c) (iii)   **5 marks**   **Att 2**

I
Solve $\quad x^2 - 4x + 3 \ = \ x - 1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ **2m**
$\qquad\quad\ x^2 - 4x \ + 3 \ - x + 1 = 0$
$\qquad\quad\ x^2 - 5x \ + 4 \ = \ 0$
$\qquad\quad\ (\,x - 4)\,(x - 1) = 0$
$\qquad\quad\ x \ = 1$ and $x = 4$
$\qquad\quad\ x^2 - 4x \ + 3 \ = y \qquad\quad$ $or \ \ x - 1 = y$
$\qquad\quad\ (4)^2 - 4(4) + 3 = y \qquad\qquad 4 - 1 = y$
$\qquad\quad\ 16 - 16 + 3 = 3 = y \qquad\qquad 3 = y$
$\qquad\qquad\quad$ ➔ $\quad$ point (4, 3) $\qquad\qquad\qquad\qquad$ **5m**
II
Substitute $x = 4$ into $f(x) \ = x^2 - 4x \ + 3$ **and** $g(x) = x - 1$
$\qquad\quad\ f(x) \ = x^2 - 4x \ + 3$
$\qquad\quad\ f(x) \ = (4)^2 - 4(4) \ + 3 \qquad\qquad\qquad\qquad$ **2m**
$\qquad\quad\ f(x) \ = 16 \ - 16 \ + 3$
$\qquad\quad\ f(x) \ = 3$
$\qquad\quad\ (4,3)$
$\qquad\quad\ g(x) = x - 1$
$\qquad\quad\ g(x) = 4 - 1$
$\qquad\quad\ g(x) = 3$
$\qquad\quad\ (4,3) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ **4m**
$\qquad\quad\ (4,3)$ on both lines → point of intersection $\qquad\qquad$ **5m**

*   Accept $g(x) = \ x + k$, based on candidate's k value from (c) (ii)

Blunders (-3)
B1   Fails to equate $f(x)$ and $g(x)$ in method **I**
B2   Incorrect squaring
B3   Transposition error
B4   Does not substitute into second function
B5   Fills in $x = 1$
B6   Fails to finish

Slips (-1)
S1   Does not conclude in method **II**
S2   Numerical errors to a maximum of 3

Attempts (2 marks)
A1   Some relevant substitution
A2   Linear equation attempt at most in Method I

Worthless (0)
W1   Incorrect answer no work shown
W2 $x - 1 = 0$
W3 $x^2 - 4x + 3 \ = 0$
W4   Work of no merit

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded *down*.
(e.g. 198 marks × 5% = 9.9 ⇒ bonus = 9 marks.)

If the mark awarded is above 225, the following table applies:

Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)	Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0

JUNIOR CERTIFICATE EXAMINATION

2011

MARKING SCHEME

MATHEMATICS
HIGHER LEVEL
PAPER 2

MARKING SCHEME
JUNIOR CERTIFICATE EXAMINATION 2011
MATHEMATICS -HIGHER LEVEL - PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
    *   Blunders - mathematical errors/omissions   (-3)
    *   Slips- numerical errors   (-1)
    *   Misreadings (provided task is not oversimplified)   (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
    *   any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
    *   if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    *   a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	20 (10,5,5)marks	Att (3,2,2)
Part (c)	20 (5,5,10) marks	Att (2,2,3)

Part (a)	10 marks	Att 3

The diagram shows two pulley wheels of equal size, connected by a drive belt.
The radius of each wheel is 7 cm and the distance between the centres is 28 cm.

✍ Calculate the length of the belt.
Give your answer correct to the nearest whole number.
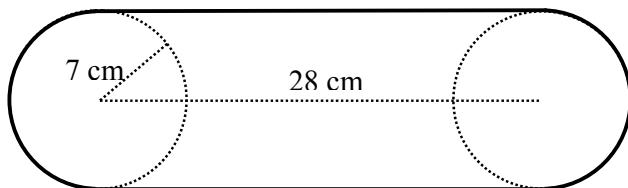
(a)		10 marks				Att 3
Semicircular lengths	=	$2\pi r$	=	14π	Step 1	
Straight lengths	=	2(28)	or	56	Step 2	
Total length	=	$14\pi + 56$				
	=	99·98				
	=	100 cm			Step 3	

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Incorrect *r*
B5   Failure to add
B6   Value of π which affects the accuracy of the answer

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)
A1   Correct perimeter formula - $2\pi r + 2l$
A2   Correct formula with some correct substitution
A3   Any relevant use of 28

Worthless (0)
W1   Area of rectangle and/or disc

Part (b)   **20 (10,5,5) marks**   **Att (3,2,2)**

The diagram shows a solid cylinder of diameter
54 cm and of height 70 cm.
A cone, of the same diameter and height as the
cylinder,is cut from inside the cylinder.

(i)   ✍   Calculate the volume of the cylinder.
            Give your answer in terms of π.

(ii)   ✍   Calculate the volume of the cone.
            Give your answer in terms of π.

(iii) ✍   What fraction of the cylinder
            remains after the cone is removed?

70 cm

54 cm
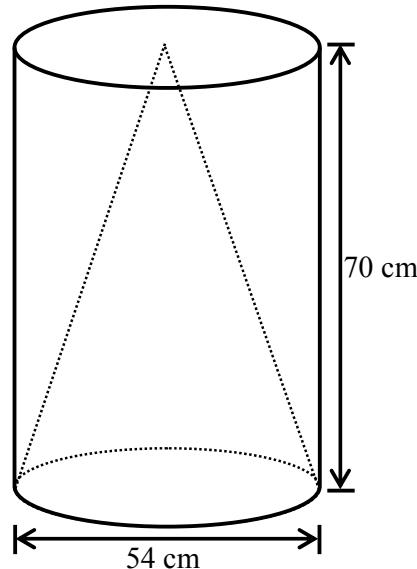
(b) (i)   **10 marks**   **Att 3**

Volume of cylinder $=$ $\pi r^2 h$

                      $=$ $\pi(27)^2(70)$

                      $=$ 51030π cm^3

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Incorrect r
B5   Incorrect h
B6   Incorrect squaring

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not in terms of π

Attempts (3 marks)
A1 $r = 27$
A2   Indication that radius length is half diameter length
A3   Correct formula with some correct substitution

Worthless (0)
W1   Surface area formula for cylinder

(b) (ii)   **5 marks**   **Att 2**

Volume of cone $\quad=\quad \dfrac{1}{3}\pi r^2 h$
$\qquad\qquad\qquad\quad=\quad \dfrac{1}{3}\pi(27)^2(70)$
$\qquad\qquad\qquad\quad=\quad 17010\pi \text{ cm}^3$
or
Volume of cone $\quad=\quad \dfrac{1}{3}(51030\pi)$
$\qquad\qquad\qquad\quad=\quad 17010\pi \text{ cm}^3$

*   Accept candidate's answer from (b) (i)

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Incorrect r
B5   Incorrect h
B6   Incorrect squaring

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not in terms of π

Attempts (2 marks)
A1 $r = 27$
A2   Indication that radius length is half diameter length
A3   Indication that volume of the cone is $\dfrac{1}{3}$ (volume of the cylinder)
A4   Correct formula with some correct substitution

Worthless (0)
W1   Surface area formula for cone

(b) (iii)   **5 marks**   **Att 2**

Remainder	$=$	$51030\pi - 17010\pi$
	$=$	34020π
Fraction	$=$	$\dfrac{34020\pi}{51030\pi}$
	$=$	$\dfrac{34020}{51030}$ or $\dfrac{2}{3}$

or

$$1 - \frac{1}{3} \quad = \quad \frac{2}{3}$$

*   Accept candidate's answer from (b) (i) and (ii)

Blunders (-3)

B1   Correct answer without work shown (✍)

B2 $\dfrac{\text{Volume of cone}}{\text{Volume of cylinder}}$

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2 π not cancelled
S3   Decimal answer

Attempts (2 marks)
A1   Effort at Remainder

The diagram, not to scale, represents a shot-put zone in an athletics stadium.
The area of *CDE* is a quarter of the area of a disc of centre *C* and of radius 100 m.

(i)   ✍   Calculate the area of *CDE*,
correct to two decimal places.

The shot-put zone consists of a throwing zone
and a landing zone.
The throwing zone (shaded) is a disc of centre *C*
and of radius 1 m.

(ii)   ✍   Calculate the area of the throwing zone,
correct to two decimal places.

The landing zone is the unshaded area *ABDE*,
which is part of *CDE*.

(iii)   ✍   Calculate the total area of the shot-put zone,
correct to two decimal places.
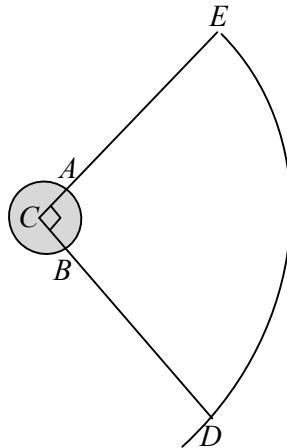
(c) (i)   **5 marks**   **Att 2**

$$\text{Area of } CDE \quad = \quad \frac{1}{4}\pi r^2 \quad = \quad \frac{1}{4}\pi(100)^2$$
$$= \quad 2500\pi$$
$$= \quad 7853{\cdot}9816$$
$$= \quad 7853{\cdot}98 \text{ m}^2$$

*   Value of π used, other than value of π from calculator, giving an answer in the range
7850 - 7857·14 incurs -1

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Value of π which affects the accuracy of the answer
B5   Answer in terms of π
B6   Incorrect *r*
B7   Incorrect squaring

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (2 marks)
A1   Correct formula with some correct substitution

Worthless (0)
W1   Length formula

(c) (ii)   **5 marks**   **Att 2**

$$\text{Area of throwing zone} \quad = \quad \pi r^2 \quad = \quad \pi(1)^2$$
$$= \quad 1\pi$$
$$= \quad 3{\cdot}1416$$
$$= \quad 3{\cdot}14 \text{ m}^2$$

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Value of π which affects the accuracy of the answer
B5   Answer in terms of π
B6   Incorrect r
B7   Incorrect squaring

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (2 marks)
A1   Correct formula with some correct substitution

Worthless (0)
W1   Length formula

(c) (iii)   **10 marks**   **Att 3**

$$\text{Area of the shot-put zone} \quad = \quad \frac{3}{4}(\text{ Area of throwing zone}) + \text{Area } CDE$$

$$\frac{3}{4}(\text{ Area of throwing zone}) \quad = \quad \frac{3}{4}(3{\cdot}14) \quad = \quad 2{\cdot}355$$

$$\text{Area of shot-put zone} \quad = \quad 2{\cdot}355 + 7853{\cdot}98$$
$$= \quad 7856{\cdot}335$$
$$= \quad 7856{\cdot}34$$

or

$$\frac{3}{4}(\text{ Area of throwing zone}) \quad = \quad 0{\cdot}75\pi$$

$$\text{Area of shot put zone} \quad = \quad 0{\cdot}75\pi + 2500\pi \quad = \quad 2500{\cdot}75\pi$$
$$= \quad 7856{\cdot}337828$$
$$= \quad 7856{\cdot}34 \text{ m}^2$$

or

$\frac{1}{4}$ (Area of throwing zone) $\quad = \quad \frac{1}{4}(3 \cdot 14) \quad = \quad 0 \cdot 785$

Area of shot put zone $\quad\quad = \quad 7853 \cdot 98 + 3 \cdot 14 - 0 \cdot 785$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = \quad 7856 \cdot 335$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = \quad 7856 \cdot 34$

or

$\frac{1}{4}$ (Area of throwing zone) $\quad = \quad 0 \cdot 25\pi$

Area of shot put zone $\quad\quad = \quad 2500\pi + 1\pi - 0 \cdot 25\pi \quad = \quad 2500 \cdot 75\pi$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = \quad 7856 \cdot 337828$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = \quad 7856 \cdot 34 \text{ m}^2$

*   Accept candidate's answers from (c) (i) and (ii)

Blunders (-3)

B1   Correct answer without work shown (✎)

B2   Area $CDE - \frac{1}{4}$ (Area of throwing zone)

B3   Area CDE + Area of throwing zone

B4   Value of π which affects the accuracy of the answer

B5   Answer in terms of π

Slips (-1)

S1   Arithmetic slips to a maximum of *(-3)*

S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)

A1   Indication of $\frac{3}{4}$ or $\frac{1}{4}$ of area of throwing zone

A2   Correct formula with some correct substitution

QUESTION 2

Part (a)	10 marks	Att 3
Part (b)	20 (5,10,5) marks	Att (2,3,2)
Part (c)	20 (10,10) marks	Att (3,3)

Part (a)	10 marks	Att 3

$X(-3, 1)$ and $Y(4, -2)$ are two points.

✎ Find the length of the line segment $[XY]$.
Give your answer in surd form.

(a)	10 marks	Att 3

$$|XY| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - (-3))^2 + (-2 - 1)^2} \qquad \text{Step 1}$$

$$= \sqrt{(7)^2 + (-3)^2} \qquad \text{Step 2}$$

$$= \sqrt{49 + 9}$$

$$= \sqrt{58} \qquad \text{Step 3}$$

Blunders (-3)

B1 Correct answer without work shown (✎)
B2 Incorrect relevant formula
B3 Both x and y switched in substitution
B4 Incorrect squaring
B5 Error in signs
B6 Answer not in surd form

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 One incorrect substitution for x or y

Attempts (3 marks)
A1 Correct formula with some correct substitution
A2 Attempt at difference of x values and/or difference of y values

Worthless (0)
W1 Incorrect formula with or without substitution

Part (b)   **20 (5,10,5) marks**   **Att (2,3,2)**

The diagram shows the gable end of a house.
The total height is 7 m.
The height to roof level is 4 m, i.e. $|AE| = 4$ m.
A is the point (1, 0).
B is the point (9, 0).
(i)   Write down the coordinates of the points *C*, *D* and *E*.

(ii)   ✍   Find the slope of the rafter [*ED*].

(iii) ✍   Find the area of the gable.
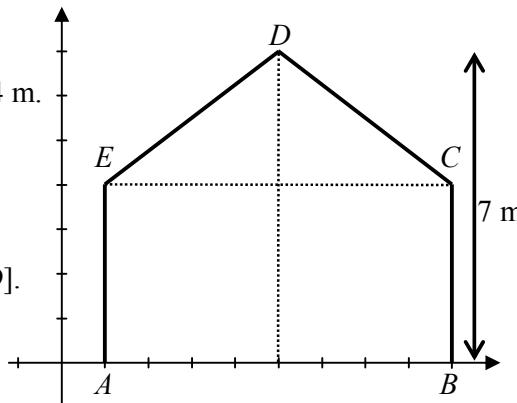
(b) (i)   **5 marks**   **Att 2**

C is the point (9, 4)   *D* is the point (5, 7)   *E* is the point (1, 4).

Blunders (-3)
B1   Each incorrect point
B2   Both *x* and *y* switched

Attempts (2 marks)
A1   One correct *x* or *y* value

Worthless (0)
W1   No correct *x* or *y* value

(b) (ii)   **10 marks**   **Att 3**

Slope [ED]	$=$	$\dfrac{y_2 - y_1}{x_2 - x_1}$	**or**	Slope $=$	$\tan \angle DEC$
	$=$	$\dfrac{7-4}{5-1}$		$=$	$\dfrac{\text{opposite}}{\text{adjacent}}$
	$=$	$\dfrac{3}{4}$		$=$	$\dfrac{3}{4}$

* Accept candidate's answers from (b) (i)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Both *x* and *y* switched in substitution
B4 Correct substitution but answer not simplified
B5 Incorrect tan ratio

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 One incorrect substitution for *x* or *y*

Attempts (3 marks)
A1 Correct formula with some correct substitution
A2 Attempt at difference of *x* values and/or difference of *y* values

Worthless (0)
W1 Incorrect formula with or without substitution

(b) (iii)   **5 marks**   **Att 2**

Area *ABCE*	=	4×8	=	32	
Area *CDE*	=	$\frac{1}{2}(8 \times 3)$	=	12	
Area of gable	=	32 + 12			
	=	44 m^2			

or

Area of large rectangle	=	8×7	=	56	
Area of two missing triangles	=	$2\left(\frac{1}{2}\right)(4 \times 3)$	=	12	
Area of gable	=	56 − 12			
	=	44 m^2			

*   Accept candidate's answers from (b) (i)
*   Accept area of triangle for full marks

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect substitution into correct formula
B4   Incorrect perpendicular height
B5   Incorrect base
B6   Incorrect length
B7   Incorrect width
B8   Area of rectangle only

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)
A1   Correct triangle area formula with some correct substitution
A2   Correct rectangle area formula with or without substitution

Worthless (0)
W1   Perimeter formula

Part (c)   **20 (10,10) marks**   **Att (3,3)**

> The line k passes through the point $P(3, 2)$.
> k is perpendicular to the line l: $2x + 3y = -1$.
>
> (i)   ✍   Find the equation of k.
>
> (ii)   ✍   Find the coordinates of the image of P by an axial symmetry in l.

(c) (i)   **10 marks**   **Att 3**

Slope of l $= -\dfrac{2}{3}$ Step 1

∴   Slope of k $= \dfrac{3}{2}$ Step 2

Equation of k: $y - y_1 = m(x - x_1)$

$y - 2 = \dfrac{3}{2}(x - 3)$

$2(y - 2) = 3(x - 3)$

$3x - 2y - 5 = 0$ Step 3

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect relevant formula
B3   Incorrect slope
B4   Both x and y switched in substitution

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   One incorrect substitution for x or y

Attempts (3 marks)
A1   Correct formula with some correct substitution
A2   Indication that the product of the slopes of perpendicular lines is -1

Worthless (0)
W1   Incorrect formula with or without substitution

(c) (ii)   **10 marks**   **Att 3**

$2l$: $4x + 6y = -2$

$3k$: $9x - 6y = 15$

$2l + 3k$: $13x = 13$

$\Rightarrow x = 1$

Substitution for x into l: $2(1) + 3y = -1$

$\Rightarrow 3y = -3$

$\Rightarrow y = -1$

The point of intersection of l and k: $(1, -1)$

$P(3, 2) \rightarrow (1, -1)$ $x: -2, y: -3$

The image of P by axial symmetry in l is $(-1, -4)$

---

*   Accept candidate's answer from (c) (i)

*   Accept $(1, -1) \in l$ and $(1, -1) \in k$ verified in each case

*   Blunders in simplifying k to a maximum of *(-3)*

Blunders (-3)

B1   Correct answer without work shown (✍)

B2   Error in transposition

B3   Wrong translation

B4   One co-ordinate of image only

Slips (-1)

S1   Arithmetic slips to a maximum of *(-3)*

Misreadings (-1)

M1   One value found for point of intersection and incorrectly substituted

Attempts (3 marks)

A1   Correct graphical solution

A2   Effort at translation using a point on one of the lines and (3, 2)

A3   Effort at finding the point of intersection of l and k

QUESTION 3

Part (a)	10 marks	Att 3
Part (b)	20 (10,10)marks	Att (3,3)
Part (c)	20 (10,10)marks	Att (3,3)

Part (a)	10 marks	Att 3

In the diagram, $|\angle DBC| = 50°$.

✎ Find the value of x.
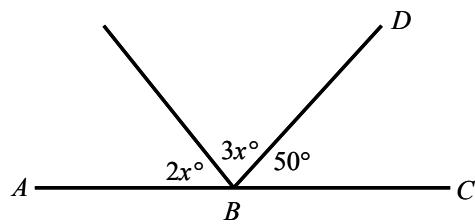
(a)	10 marks	Att 3

$$2x + 3x + 50° = 180°$$
$$5x = 130°$$
$$x = 26°$$

* Accept work on diagram

Blunders (-3)

B1 Correct answer without work shown (✎)
B2 Sum of angles ≠ 180° stated
B3 Error in transposition

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)
A1 Indication that a straight angle = 180°
A2 Mention of 5x or 130°

Worthless (0)
W1 Diagram from examination either partially or fully drawn
W2 $2x = 50°$
W3 $3x = 90°$

(i)   ✍   Prove that if two sides of a triangle are equal in measure, then the angles opposite these sides are equal in measure.

(ii)   The triangle ABC is isosceles with $|AC| = |BC|$.
The triangle *ADC* is also isosceles with $|AD| = |CD|$.
$|\angle ACB| = 40°$.

     ✍   Find $|\angle DAB|$ and $|\angle ADB|$.
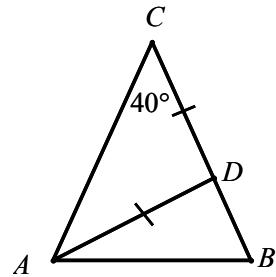
(b) (i)   **10 marks**   **Att 3**
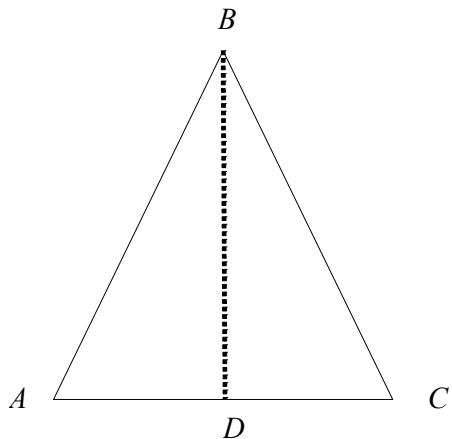
Given:	A triangle *ABC*, with $|AB|=|BC|$
To Prove:	$\left|\angle BAC\right| = \left|\angle BCA\right|$
Construction:	Draw the line segment [*BD*] which bisects the angle at *B*. Step 1
Proof:	Consider ΔBAD and ΔBCD.

$$\begin{array}{lll}
\left|AB\right| = \left|BC\right| & \text{given} & \\
\left|\angle ABD\right| = \left|\angle CBD\right| & \text{construction} & \\
\left|BD\right| = \left|BD\right| & \text{common side} & \text{Step 2} \\
\therefore \quad \Delta BAD \equiv \Delta BCD & \text{S.A.S.} & \\
\Rightarrow \quad \left|\angle BAC\right| = \left|\angle BCA\right| & \text{(corresponding angles)} & \text{Step 3}
\end{array}$$

or
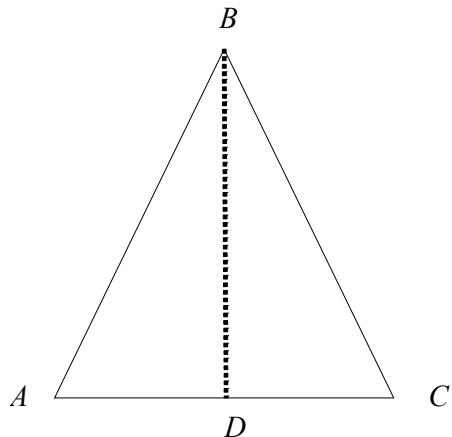
Given: A triangle *ABC*, with |*AB*|=|*BC*|

To Prove: $|\angle BAC| = |\angle BCA|$

Construction: Draw the line segment [*BD*], where *D* is the midpoint of [*AC*]   Step 1

Proof: Consider Δ*BAD* and Δ*BCD*.

$$|AB| = |BC|$$ given
$$|AD| = |CD|$$ construction
$$|BD| = |BD|$$ common side   Step 2
$$\therefore \Delta BAD \equiv \Delta BCD$$ S.S.S.
$$\therefore |\angle BAC| = |\angle BCA|$$ (corresponding angles)   Step 3

or
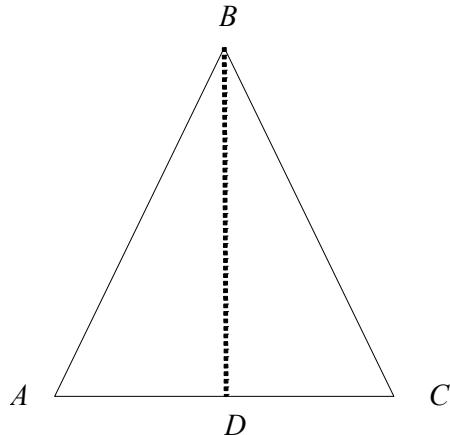
Given: A triangle ABC, with $|AB|=|BC|$

To Prove: $|\angle BAC| = |\angle BCA|$

Construction: Draw the line segment $[BD]$, where $[BD] \perp [AC]$ Step 1

Proof: Consider ΔBAD and ΔBCD.

                   $|AB| = |BC|$ given

                   $|\angle BDA| = |\angle BDC|$ both 90°

                   $|BD| = |BD|$ common side   Step 2

              $\therefore \Delta BAD \equiv \Delta BCD$ R.H.S.

             $\therefore |\angle BAC| = |\angle BCA|$ (corresponding angles)   Step 3

*   Some steps may be indicated on diagram

Blunders (-3)
B1   Each step incorrect or omitted
B2   Each step incomplete

Attempts (3 marks)
A1 $[BD]$ shown in diagram with or without labelling
A2   Triangle drawn with equal sides indicated

Worthless (0)
W1   Unlabelled triangle drawn
W2   No diagram or no valid diagram
W3   Wrong theorem

(b) (ii)   **10 marks**   **Att 3**

$$|\angle CAB| = \frac{1}{2}(180° - 40°)$$

or

$$|\angle CAD| = 40°$$
$$|\angle CDA| = 100°$$
$$|\angle ADB| = 180° - 100°$$
$$= 80°$$

$$= 70°$$

$$|\angle CAD| = 40°$$

$$|\angle DBA| = 70°$$

$$|\angle DAB| = 70° - 40°$$

$$|\angle DAB| = 180° - (|\angle DBA| + |\angle BDA|)$$

$$= 30°$$

$$= 180° - (70° + 80°)$$

$$= 30°$$

$$|\angle ADB| = |\angle CAD| + |\angle ACD|$$
$$= 40° + 40°$$
$$= 80°$$

*   Accept work on diagram

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Second angle not found
B3   Sum of angles in a triangle $\neq 180°$

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)
A1   Indication that the sum of the angles in a triangle $= 180°$
A2   Indication that angles opposite equal sides are equal in measure
A3   Indication that an exterior angle is equal to the sum of the two interior opposite angles
A4 $|\angle DAC| = 40°$
A5 $|AC| = |BC|$ indicated on diagram

Worthless (0)
W1   Diagram from examination paper either partially or fully drawn
W2 $|\angle DAB| = 40°$
W3 $|\angle ADC| = 90°$

*AB* is parallel to *CD*. *BC* and *AD* intersect at the point *E*.

(i)   ✎   Prove that the triangles *ABE* and *CDE* are equiangular.

|*AB*| = 12 cm, |*BE*| = 8 cm and |*CD*| = 7 cm.

(ii)   ✎   Find |*EC*| correct to one decimal place.
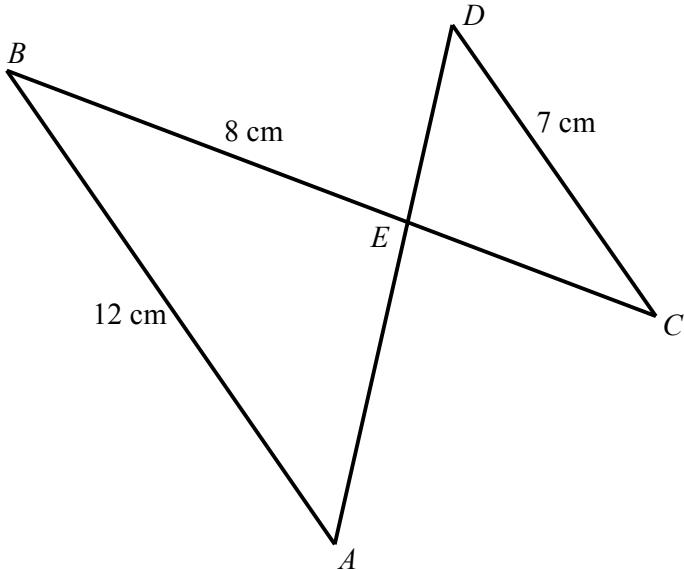
(c) (i)   **10 marks**   **Att 3**

|∠*ABE*|	=	|∠*DCE*|	(alternate angles)	Step 1
|∠*BAE*|	=	|∠*CDE*|	(alternate angles)	Step 2
|∠*AEB*|	=	|∠*CED*|	(vertically opposite angles)	
Therefore, the triangles *ABE* and *CDE* are equiangular.				Step 3

*   Some steps may be indicated on diagram

Blunders (-3)
B1   Each step incorrect or omitted

Attempts (3 marks)
A1   Stated that alternate angles are equal if lines are parallel
A2   Indication that the sum of the angles in a triangle = 180°
A3   Stated that vertically opposite angles are equal
A4   *AB* parallel to *CD* indicated on diagram

Worthless (0)
W1   Diagram from examination paper either partially or fully drawn

(c) (ii)   **10 marks**   **Att 3**

$$\frac{|EC|}{|BE|} = \frac{|CD|}{|AB|}$$

$$\frac{|EC|}{8} = \frac{7}{12}$$

$$|EC| = \frac{8 \times 7}{12}$$

$$|EC| = 4\cdot666$$

$$= 4\cdot7 \text{ cm}$$

*   Some steps may be indicated on diagram

Blunders (-3)

B1   Correct answer without work shown (✍)

B2 $\dfrac{|EC|}{|BE|} = \dfrac{|AB|}{|CD|}$ or similar incorrect ratio

B3   Error in transposition

Slips (-1)

S1   Arithmetic errors to a maximum of *(-3)*

S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)

A1   One correct relevant ratio

A2   Stated that corresponding sides are proportional


Worthless (0)

W1   Diagram from examination paper either partially or fully drawn

QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 (10,10) marks	Att (3,3)

Part (a)	10 marks	Att 3

X, *Y* and *Z* are points on a circle with centre *C*.
$|\angle YXZ| = 25°$.

✐   Find $|\angle XZY|$.
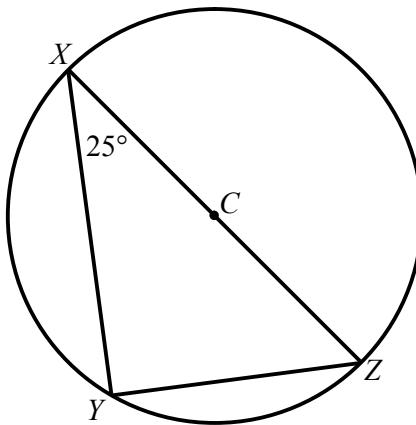
(a)	10 marks	Att 3

$$|\angle XZY| \quad = \quad 90° - 25°$$
$$= \quad 65°$$

*   Some steps may be indicated on diagram

Blunders (-3)

B1   Correct answer without work shown (✐)
B2   Sum of angles in a triangle $\neq 180°$
B3 $|\angle XYZ| \neq 90°$

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)
A1   Indication that the sum of angles in a triangle = 180°
A2   Indication that the angle in a semi-circle = 90°
A3   Identification of a radius

Worthless (0)
W1 $|\angle XZY| = 25°$
W2   Diagram from examination paper either partially or fully drawn

✍   Prove that the measure of the angle at the centre of the circle is twice the measure of the angle at the circumference, standing on the same arc.

(b)   **20 marks**   **Att 7**
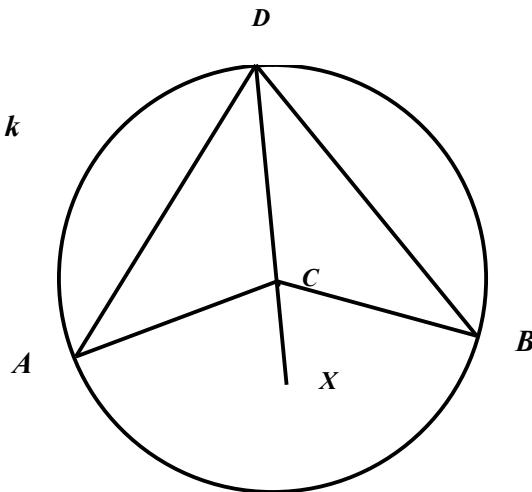
Given: Circle k with centre C and three points A, B and D on k.

To Prove: $|\angle ACB| = 2\,|\angle ADB|$

Construction: Join D to C and produce to X.   Step 1

Proof: $|AC| = |CD|$ both radius length

                   $\Rightarrow |\angle CAD| = |\angle ADC|$ Step 2

           $|\angle ACX| = |\angle CAD| + |\angle ADC|$ exterior angle equals the sum of
                                          the two interior opposite angles
                                                        Step 3

                   $\Rightarrow |\angle ACX| = 2\,|\angle ADC|$ Step 4

            Similarly, $|\angle BCX| = 2\,|\angle CDB|$ Step 5

        $\therefore |\angle ACX| + |\angle BCX| = 2\,|\angle CDA| + 2\,|\angle CDB|$ Step 6

               $\therefore |\angle ACB| = 2\,|\angle ADB|$ Step 7

*   Some steps may be indicated on diagram

Blunders (-3)
B1   Each step incorrect or incomplete
B2   Each step omitted

Attempts (7 marks)
A1   Diagram with angle at centre or angle at circumference drawn

*Worthless (0)*
W1   No diagram or no valid diagram
W2   Wrong theorem

The quadrilateral *ABDE* has diagonals [*AD*] and [*BE*] intersecting at *C*.
C is the midpoint of both [*AD*] and [*BE*].
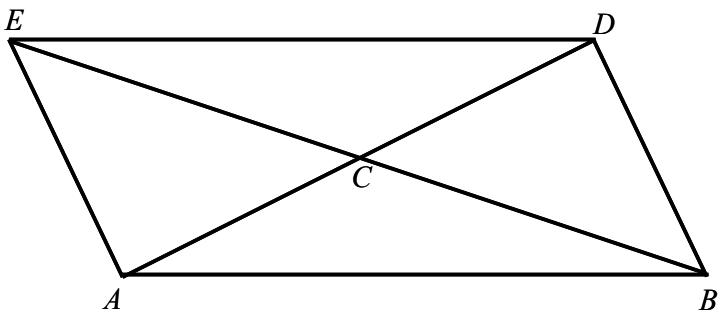
(i)   ✍   Prove that $\triangle ECD$ is congruent to $\triangle ACB$.

(ii)   ✍   Hence, prove that *ABDE* is a parallelogram.

(c) (i)   **10 marks**   **Att 3**

$\lvert \angle ECD \rvert$	$=$	$\lvert \angle ACB \rvert$	(vertically opposite angles)
$\lvert CE \rvert$	$=$	$\lvert BC \rvert$	(given)
$\lvert AC \rvert$	$=$	$\lvert CD \rvert$	(given)
Therefore, ΔECD is congruent to ΔACB			S.A.S.

*   Some work may be indicated on diagram

Blunders (-3)
B1   Each line in solution incorrect or omitted
B2   Reason for congruency not given

Misreadings (-1)
M1 ΔACE is congruent to ΔBCD proven

Attempts (3 marks)
A1   Both triangles indicated or redrawn
A2   Indication of some knowledge of congruent triangles

Worthless (0)
W1   Diagram from examination paper either partially or fully drawn

ΔECD is congruent to ΔACB

$\therefore$ $|\angle CDE| = |\angle BAC| / |\angle CED| = |\angle ABC|$ (corresponding angles)

$\therefore$ ED is parallel to AB Step 1

ΔACE is congruent to ΔBCD (S.A.S.)

$\therefore$ $|\angle AEC| = |\angle CBD| / |\angle CAE| = |\angle BDC|$ (corresponding angles)   Step 2

$\therefore$ AE is parallel to BD

$\therefore$ $ABDE$ is a parallelogram   (two pairs of opposite sides parallel)   Step 3

or

ΔECD is congruent to ΔACB

$\therefore$ $|ED| = |AB|$ (corresponding sides)   Step 1

   and $|\angle CDE| = |\angle BAC| / |\angle CED| = |\angle ABC|$ (corresponding angles)   Step 2

$\therefore$ ED is parallel to AB

$\therefore$ $ABDE$ is a parallelogram   (one pair of opposite sides equal and parallel) Step 3

or

ΔACE is congruent to ΔBCD (S.A.S.)

$\therefore$ $|AE| = |BD|$ (corresponding sides)   Step 1

   and $|\angle AEC| = |\angle CBD| / |\angle CAE| = |\angle BDC|$ (corresponding angles)   Step 2

$\therefore$ AE is parallel to BD

$\therefore$ $ABDE$ is a parallelogram   (one pair of opposite sides equal and parallel) Step 3

or

ΔECD is congruent to ΔACB

$\therefore$ $|ED| = |AB|$ (corresponding sides)   Step 1

ΔACE is congruent to ΔBCD (S.A.S.)   Step 2

$\therefore$ $|AE| = |BD|$ (corresponding sides)

$\therefore$ $ABDE$ is a parallelogram   (two pairs of opposite sides equal)   Step 3

or

ΔECD is congruent to ΔACB

$\therefore$ $|\angle CDE| = |\angle BAC|$ and $|\angle CED| = |\angle ABC|$ (corresponding angles)   Step 1

ΔACE is congruent to ΔBCD (S.A.S.)

$\therefore$ $|\angle AEC| = |\angle CBD|$ and $|\angle CAE| = |\angle BDC|$ (corresponding angles)   Step 2

$\therefore$ $|\angle CDE| + |\angle BDC| = |\angle BAC| + |\angle CAE|$

   and $|\angle CED| + |\angle AEC| = |\angle ABC| + |\angle CBD|$

   i.e. $|\angle BDE| = |\angle BAE|$ and $|\angle AEB| = |\angle ABD|$

$\therefore$ $ABDE$ is a parallelogram   (two pairs of opposite angles equal)   Step 3

---

*   Some steps may be indicated on diagram

Blunders (-3)
B1   Each step incorrect or omitted

Attempts (3 marks)
A1   Stated that opposite sides or opposite angles of a parallelogram are equal or opposite
     sides are parallel
A2   Indication that ΔACE is congruent to ΔBCD
A3   Indication that the diagonals of a parallelogram bisect each other
A4   Use of symmetry

Worthless (0)
W1   Diagram from examination paper either partially or fully drawn

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 (10,10)marks	Att (3,3)
Part (c)	20 (15,5) marks	Att (5,2)

Part (a)   **10 marks**   **Att 3**

ABC is a right angled triangle.
$|\angle ACB| = 60°$ and $|AC| = 10$ cm.

✎ Calculate the length of [*AB*],
correct to two decimal places.

A

10 cm

60°

B   *C*
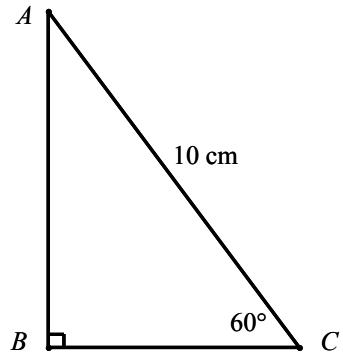
(a)   **10 marks**   **Att 3**

$$\sin 60^{\circ} = \frac{|AB|}{|AC|} = \frac{|AB|}{10}$$

or

$$|AB| = 10\sin 60^{\circ}$$
$$= 8\cdot660254$$
$$= 8\cdot66$$

or

$$\frac{|AB|}{\sin 60^{\circ}} = \frac{|AC|}{\sin 90^{\circ}}$$
$$\frac{|AB|}{\sin 60^{\circ}} = \frac{10}{\sin 90^{\circ}}$$

$$|AB| = \frac{10\sin 60^{\circ}}{\sin 90^{\circ}}$$
$$= 8\cdot660254$$
$$= 8\cdot66 \text{ cm}$$

$$\cos 30^{\circ} = \frac{|AB|}{|AC|} = \frac{|AB|}{10}$$

$$|AB| = 10\cos 30^{\circ}$$
$$= 8\cdot660254$$
$$= 8\cdot66$$

or

$$\cos 60^{\circ} = \frac{|BC|}{10}$$
$$|BC| = 10\cos 60^{\circ}$$
$$= 5$$

$$|AC|^{2} = |AB|^{2} + |BC|^{2}$$
$$10^{2} = |AB|^{2} + 5^{2}$$
$$|AB|^{2} = 100 - 25$$
$$= 75$$
$$|AB| = \sqrt{75}$$
$$= 8\cdot660254$$
$$= 8\cdot66 \text{ cm}$$

Blunders (-3)

B1   Correct answer without work shown (✎)
B2   Incorrect ratio for sin/cos function
B3   Incorrect ratio for Sine Rule
B4   Error in transposition
B5   Calculator in incorrect mode
B6   Early rounding which affects the accuracy of the answer
B7   Sum of angles in a triangle $\neq 180°$
B8   Incorrect squaring
B9   Error in Pythagoras' Theorem
B10 Error in square root

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)

A1 $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ or $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

A2 $\sin 60°$ or $\cos 30°$ or $\cos 60°$ or $\sin 30°$

A3 $\dfrac{|AB|}{10}$ or $\dfrac{|BC|}{10}$

A4   Sine Rule with some correct substitution

A5   Mention of $30°$

A6   Indication that the sum of the angles in a triangle $= 180°$

A7   Effort at Pythagoras' Theorem

Worthless (0)

W1 $\dfrac{|AB|}{60} = \dfrac{10}{90}$

W2 $\dfrac{10}{60}$ or $\dfrac{60}{10}$

W3   Diagram from examination paper either partially or fully drawn

Part (b)   **20 (10,10) marks**   **Att (3,3)**

In the diagram *MNO* is a triangle with [*OP*] perpendicular to [*MN*]. |*MP*| = 10 cm, |*ON*| = 30 cm and |∠*PMO*| = 65°.
Calculate

(i) ✎ |*OP*|, correct to one decimal place

(ii) ✎ |∠*MON*|, correct to one decimal place.

O

30 cm

65°

M 10 cm P N
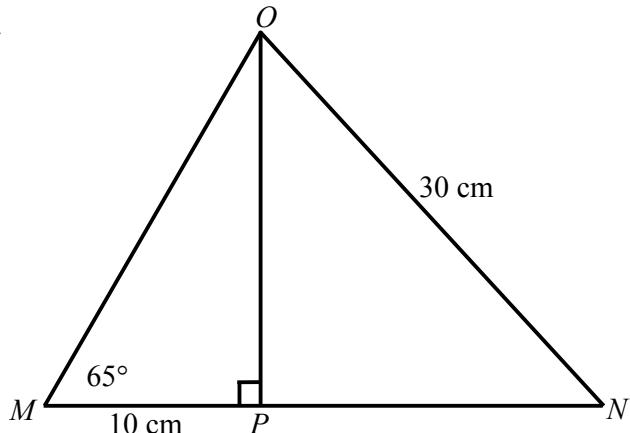
(b) (i)   **10 marks**   **Att 3**

$$\tan 65° = \frac{|OP|}{|MP|} = \frac{|OP|}{10} \qquad \textbf{or} \qquad \tan 25° = \frac{|MP|}{|OP|} = \frac{10}{|OP|}$$

$$|OP| = 10\tan 65° \qquad\qquad\qquad\qquad |OP| = \frac{10}{\tan 25°}$$

$$= 21·445 \qquad\qquad\qquad\qquad\qquad\qquad = 21·445$$

$$= 21·4 \text{ cm} \qquad\qquad\qquad\qquad\qquad = 21·4 \text{ cm}$$

or   **or**

$$\cos 65° = \frac{|MP|}{|OM|} = \frac{10}{|OM|}$$

$$|OM| = \frac{10}{\cos 65°}$$

$$= 23·662$$

$$\frac{|OP|}{\sin 65°} = \frac{10}{\sin 25°}$$

$$|OM|^2 = |OP|^2 + |MP|^2$$

$$|OP| = \frac{10\sin 65°}{\sin 25°}$$

$$23·662^2 = |OP|^2 + 10^2$$

$$= 21·445 \qquad\qquad\qquad |OP|^2 = 559·89 - 100$$

$$= 21·4 \text{ cm} \qquad\qquad\qquad = 459·89$$

$$|OP| = \sqrt{459·89}$$

$$= 21·445$$

$$= 21·4 \text{ cm}$$

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect ratio for tan/cos function
B3   Incorrect ratio for Sine Rule
B4   Error in transposition
B5   Calculator in incorrect mode
B6   Early rounding which affects the accuracy of the answer
B7   Sum of angles in a triangle $\neq 180^\circ$
B8   Incorrect squaring
B9   Error in Pythagoras' Theorem
B10 Error in square root

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)

A1 $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$ or $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

A2 $\tan 65^\circ$ or $\tan 25^\circ$ or $\cos 65^\circ$

A3 $\dfrac{|OP|}{10}$ or $\dfrac{10}{|OP|}$ or $\dfrac{10}{|OM|}$

A4   Sine Rule with some correct substitution
A5   Mention of 25°
A6   Indication that the sum of the angles in a triangle $= 180^\circ$
A7   Effort at Pythagoras' Theorem

Worthless (0)

W1 $\dfrac{|OP|}{65} = \dfrac{10}{90}$

W2 $\dfrac{10}{65}$ or $\dfrac{65}{10}$

W3   Diagram from examination paper either partially or fully drawn

$$|\angle MOP| \quad = \quad 180^\circ - (65^\circ + 90^\circ) \quad = \quad 25^\circ$$

$$\cos \angle NOP \quad = \quad \frac{|OP|}{|ON|} \quad = \quad \frac{21\cdot4}{30}$$

$$\therefore \quad |\angle NOP| \quad = \quad 44\cdot49322423^\circ$$

$$|\angle MON| \quad = \quad |\angle MOP| + |\angle NOP|$$

$$= \quad 25^\circ + 44\cdot49322423^\circ$$

$$= \quad 69\cdot49322423^\circ$$

$$= \quad 69\cdot5^\circ$$

or

$$\sin \angle PNO \quad = \quad \frac{|OP|}{|ON|} \quad = \quad \frac{21\cdot4}{30}$$

$$\therefore \quad |\angle PNO| \quad = \quad 45\cdot50677577^\circ$$

$$|\angle MON| \quad = \quad 180^\circ - (45\cdot50677577^\circ + 65^\circ)$$

$$= \quad 69\cdot49322423^\circ$$

$$= \quad 69\cdot5^\circ$$

or

$$\frac{\sin \angle PNO}{21\cdot4} \quad = \quad \frac{\sin 90^\circ}{30}$$

$$\sin \angle PNO \quad = \quad \frac{21\cdot4 \sin 90^\circ}{30}$$

$$|\angle PNO| \quad = \quad 45\cdot50677577^\circ$$

$$|\angle MON| \quad = \quad 180^\circ - (45\cdot50677577^\circ + 65^\circ)$$

$$= \quad 69\cdot49322423^\circ$$

$$= \quad 69\cdot5^\circ$$

or

$$30^2 = 21\cdot4^2 + |PN|^2$$

$$900 = 457\cdot96 + |PN|^2$$

$$442\cdot04 = |PN|^2$$

$$|PN| = \sqrt{442\cdot04} = 21\cdot0247$$

$\sin \angle NOP = \dfrac{|PN|}{|ON|} = \dfrac{21\cdot0247}{30}$	$\dfrac{\sin \angle NOP}{21\cdot0247} = \dfrac{\sin 90^\circ}{30}$
$|\angle NOP| = 44\cdot49322423^\circ$	$\sin \angle NOP = \dfrac{21\cdot0247 \sin 90^\circ}{30}$
$|\angle MON| = |\angle MOP| + |\angle NOP|$	$|\angle NOP| = 44\cdot49322423^\circ$
$\quad = \ 25^\circ + 44\cdot49322423^\circ$	$|\angle MON| = |\angle MOP| + |\angle NOP|$
$\quad = \ 69\cdot49322423^\circ$	$\quad = \ 25^\circ + 44\cdot49322423^\circ$
$\quad = \ 69\cdot5^\circ$	$\quad = \ 69\cdot49322423^\circ$
	$\quad = \ 69\cdot5^\circ$

*   Accept candidate's answer from (b) (i)

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   Incorrect ratio for trigonometric function
B3   Incorrect ratio for Sine Rule
B4   Error in transposition
B5   Calculator in incorrect mode
B6   Early rounding which affects the accuracy of the answer
B7   Failure to add/subtract
B8   Error in Pythagoras' Theorem
B9   Incorrect squaring
B10 Error in square root
B11 Sum of angles in a triangle $\neq 180^{\circ}$

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)
A1 $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ or $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
A2   Indication that the sum of the angles in a triangle $= 180^{\circ}$
A3   Effort at Pythagoras' Theorem
A4   Sine Rule with some correct substitution
A5   Mention of 25°

Worthless (0)
W1   Diagram from examination paper either partially or fully drawn

A boat sails due east from the base *A* of a 30 m high lighthouse, [*AD*].
At the point *B* the angle of depression of the boat from the top of the lighthouse is 68°.
Ten seconds later the boat is at the point *C* and the angle of depression is now 33°.

(i) ✍ Find |*BC*|, the distance the boat has travelled in this time.

(ii) ✍ Calculate the average speed at which the boat is sailing between *B* and *C*. Give your answer in metres per second, correct to one decimal place.

30 m

D

A   *B*   *C*
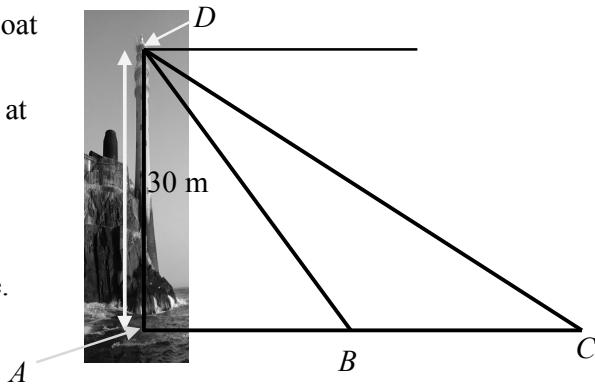
(c) (i)   **15 marks**   **Att 5**

$$\sin 68^\circ \quad = \quad \frac{|AD|}{|BD|} = \frac{30}{|BD|} \quad \text{(may also use } \cos 22^\circ\text{)}$$

$$|BD| \quad = \quad \frac{30}{\sin 68^\circ} \qquad \qquad \text{Step 1}$$

$$= \quad 32{\cdot}3560 \qquad \qquad \text{Step 2}$$

$$|\angle BDC| \quad = \quad 68^\circ - 33^\circ \quad = \quad 35^\circ \qquad \text{Step 3}$$

$$\frac{|BC|}{\sin \angle BDC} \quad = \quad \frac{|BD|}{\sin \angle BCD}$$

$$\frac{|BC|}{\sin 35^\circ} \quad = \quad \frac{32 \cdot 3560}{\sin 33^\circ} \qquad \qquad \text{Step 4}$$

$$|BC| \quad = \quad \frac{32 \cdot 3560 \sin 35^\circ}{\sin 33^\circ}$$

$$= \quad 34{\cdot}075 \text{ m} \qquad \qquad \text{Step 5}$$

$$\tan 68^{\circ} \quad = \quad \frac{|AD|}{|AB|} \quad = \quad \frac{30}{|AB|}$$

$$|AB| \quad = \quad \frac{30}{\tan 68^{\circ}} \qquad\qquad \text{Step 1}$$

$$= \quad 12{\cdot}1207869 \qquad\qquad \text{Step 2}$$

$$\tan 33^{\circ} \quad = \quad \frac{|AD|}{|AC|} \quad = \quad \frac{30}{|AC|}$$

$$|AC| \quad = \quad \frac{30}{\tan 33^{\circ}} \qquad\qquad \text{Step 3}$$

$$= \quad 46{\cdot}19594891 \qquad\qquad \text{Step 4}$$

$$|BC| \quad = \quad 46{\cdot}19594891 - 12{\cdot}1207869$$

$$= \quad 34{\cdot}07516201 \text{ m} \qquad\qquad \text{Step 5}$$

* Some steps may be indicated on diagram

Blunders (-3)

B1 Correct answer without work shown (✎)
B2 Each step incorrect or incomplete
B3 Incorrect ratio for trigonometric function
B4 Incorrect ratio for Sine Rule
B5 Calculator in incorrect mode
B6 Error in transposition
B7 $|\angle ADB| = 68^{\circ}$
B8 $|\angle ADC| = 33^{\circ}$
B9 Sum of angles in a triangle $\neq 180^{\circ}$

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (5 marks)
A1 $\sin A = \dfrac{\text{oppposite}}{\text{hypotenuse}}$ or $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$ or $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
A2 Sine Rule with some correct substitution
A3 $\sin 68^{\circ}$ or $\tan 68^{\circ}$ or $\tan 33^{\circ}$ or $\cos 22^{\circ}$
A4 Mention of 35° or 22° or 112° or 57°
A5 $\dfrac{30}{|BD|}$ or $\dfrac{30}{|AB|}$
A6 Indication that the sum of the angles in a triangle $= 180^{\circ}$
A7 68° and/or 33° correctly indicated on diagram

Worthless (0)
W1 Diagram from examination paper either partially or fully drawn
W2 ΔBCD treated as right-angled

(c) (ii)   **5 marks**   **Att 2**

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{34 \cdot 075}{10}$$

$$= 3 \cdot 4075$$

$$= 3 \cdot 4 \text{ m/sec}$$

*   Accept candidate's answer from (c) (i)

Blunders (-3)

B1   Correct answer without work shown (✐)

B2   Incorrect relevant formula

Slips (-1)

S1   Arithmetic slips to a maximum of *(-3)*

S2   Answer not rounded or incorrectly rounded

Attempts (2 marks)

A1   Correct formula with or without substitution

QUESTION 6

Part (a)	10 marks	Att 3
Part (b)	20 (5,10,5)marks	Att (2,3,2)
Part (c)	20 (5,10,5) marks	Att (2,3,2)

Part (a)   **10 marks**   **Att 3**

The mean of 7, 2, x, 15 and 5 is 9.

✏   Find the value of x.

(a)   **10 marks**   **Att 3**

$$\frac{7+2+x+15+5}{5} = 9$$

$$\frac{x+29}{5} = 9$$

$$x+29 = 45$$

$$x = 16$$

Blunders (-3)

B1   Correct answer without work shown (✏)

B2   Incorrect denominator

B3   Omission of a value

B4   Error in transposition

B5   29x in numerator

Slips (-1)

S1   Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)

A1   Addition of some or all of the numbers

A2   Indication of division by 5

A3 $\dfrac{7+2+x+15+5}{5}$

A4 $\dfrac{7+2+x+15+5+9}{6}$

Part (b)   **20 (5,10,5) marks**   **Att (2,3,2)**

The results obtained by 200 students in an examination are recorded in the following grouped frequency distribution.

Mark	0 – 20	20 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 100
Number of students	20	36	36	52	30	14	12

[Note: 20 – 40 means 20 or more but less than 40, etc.]

(i) ✍ Draw a cumulative frequency table.

(ii) ✍ Use your cumulative frequency table to construct the ogive.

(iii) ✍ If 50% of the students passed, use your ogive to estimate the pass mark.

(b) (i)   **5 marks**   **Att 2**

Mark	< 20	< 40	< 50	< 60	< 70	< 80	< 100
Number of students	20	56	92	144	174	188	200

Blunders (-3)
B1   Number omitted (sum ≠ 200)
B2   Subtraction of values

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Each incorrect entry

Attempts (2 marks)
A1   One value correctly filled into table
A2   Indication of addition of frequencies

Worthless (0)
W1   Table copied from examination paper

(b) (ii)   **10 marks**   **Att 3**
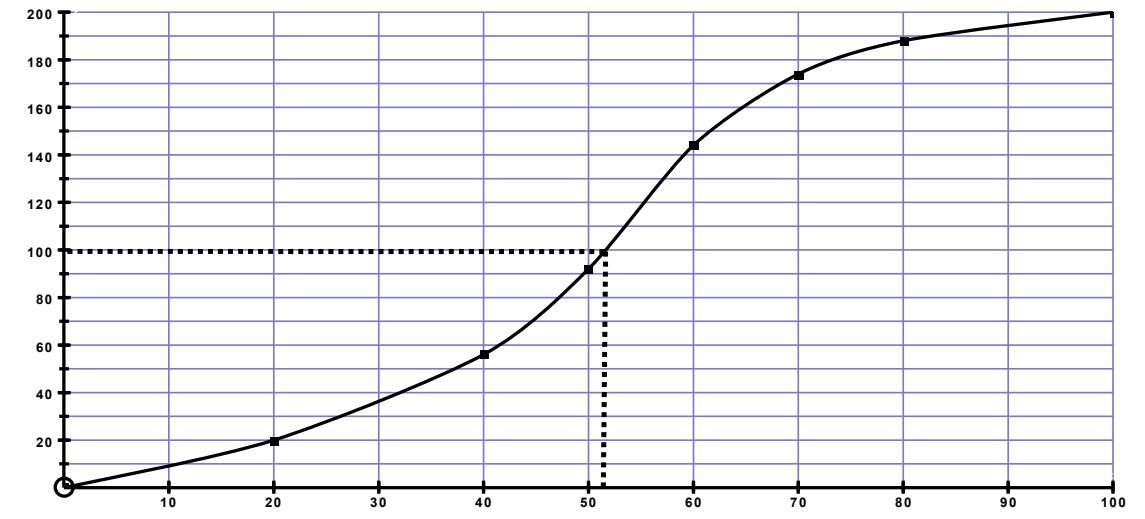
*   Accept candidate's perpendicular axes

Blunders (-3)
B1   Scale not uniform
B2   Points plotted but not joined
B3   Graph not drawn from origin
B4   Consistent error in plotting points

Slips (-1)
S1   Each point incorrectly plotted
S2   Each point omitted
S3   Points joined with straight lines

Attempts (3 marks)
A1   Graph from frequency table drawn
A2   Scaled axes drawn

Worthless (0)
W1   Unscaled axes drawn

Pass mark:	52

*   Accept answer consistent with candidate's work in (b) (ii)

*   Accept a tolerance of ± 2

Blunders (-3)

B1   Correct answer without work shown ($\mathscr{A}$)

B2   Line drawn from incorrect starting point on correct axis

Slips (-1)

S1   Incorrect reading or no reading from graph with correct work shown

Attempts (2 marks)

A1   Horizontal line from 100

A2   Indication of use of 100

A3   Horizontal axis used for 50%

Worthless (0)

W1   Answer from trend graph

Part (c)   **20 (5,10,5) marks**   **Att (2,3,2)**

130 people were surveyed as they were leaving a shop to see how much they had just spent in the shop. The results are recorded in the following table.

| Amount spent (€)	0 – 20	20 – 30	30 – 40	40 – 50	50 – 100
No. of people	60	10	5	25	30

[Note: 20 – 30 means 20 or more but less than 30, etc.]

(i) $\mathscr{A}$ Draw a histogram to illustrate the data in the above table.

(ii) $\mathscr{A}$ Taking mid-interval values, calculate the mean amount of money spent in the shop. Give your answer correct to the nearest euro.

(iii) $\mathscr{A}$ What is the maximum number of people who could have spent less than the mean?

(c) (i)   **5 marks**   **Att 2**
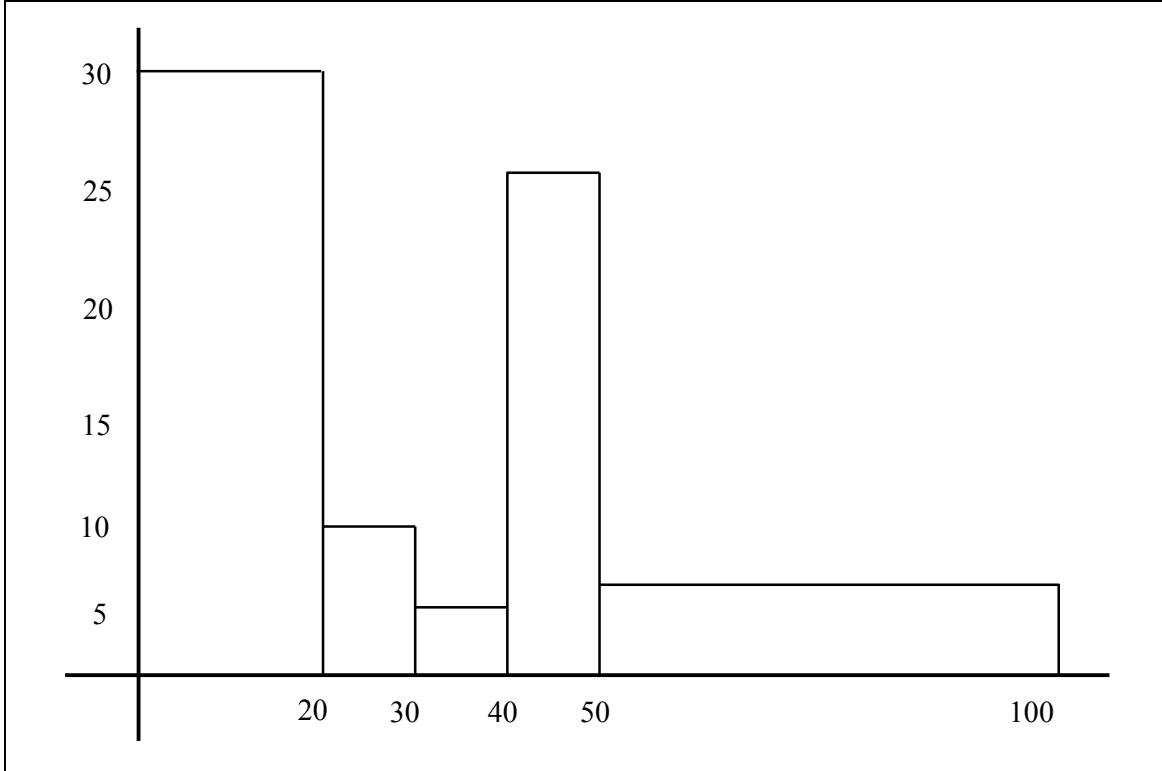
*   Accept candidate's perpendicular axes

Blunders (-3)
B1   Scale not uniform on each axis
B2   Incorrect width of rectangle
B3   Incorrect height of rectangle
B4   Rectangle omitted

Attempts (2 marks)
A1   Graph from frequency table drawn
A2   Axes drawn

Worthless (0)
W1   Table from examination paper copied

(c) (ii)   **10 marks**   **Att 3**

$$\text{Mean} = \frac{60 \times 10 + 10 \times 25 + 5 \times 35 + 25 \times 45 + 30 \times 75}{130}$$

$$= \frac{600 + 250 + 175 + 1125 + 2250}{130}$$

$$= \frac{4400}{130}$$

$$= 33 \cdot 846$$

$$= \text{€}34$$

Blunders (-3)

B1   Correct answer without work shown ($\cancel{\swarrow}$)
B2   Consistent incorrect mid-interval values
B3   Division by 5
B4   Division by sum of mid interval values
B5   Mid-interval values added to frequencies instead of multiplied

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*
S2   Answer not rounded or incorrectly rounded

Attempts (3 marks)
A1   One correct multiplication in numerator
A2   Indication of division by 130
A3   One correct mid-interval value
A4   Sum of mid-interval values divided by 130 or 5

Worthless (0)
W1   Sum of frequencies divided by 5

(c) (iii)   **5 marks**   **Att 2**

The maximum less than the mean: $60 + 10 + 5$ $=$ 75

*   Accept candidate's answer from (c) (ii)

Blunders (-3)

B1   Correct answer without work shown (✍)
B2   One value omitted

Slips (-1)
S1   Arithmetic slips to a maximum of *(-3)*

Misreadings (-1)
M1   Maximum greater than the mean found

Attempts (2 marks)
A1   One correct value