Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION

2009

MARKING SCHEME

MATHEMATICS
HIGHER LEVEL
PAPER 2

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GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips- numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(10,5,5) marks**	**Att (3,2,2)**
Part (c)	**20(10,10) marks**	**Att (3,3)**

Part (a) **10 marks** **Att 3**

✍ Find the total surface area of a solid hemisphere of diameter 14 cm.
Give your answer correct to the nearest whole number.

(a) **10 marks** **Att 3**

Total surface area $= 2\pi r^2 + \pi r^2 = 3\pi r^2$
$= 3\pi(7)^2$
$= 147\pi$
$= 461.8141201$
$= 462 \text{ cm}^2$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Total surface area $= 2\pi r^2$
B3 Total surface area $= 4\pi r^2$
B4 $r = 14$
B5 Value of π used which affects the accuracy of the answer
B6 Incorrect relevant area formula
B7 Answer in terms of π
B8 Incorrect squaring

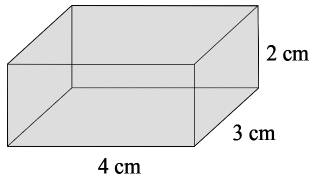
Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded off or incorrectly rounded

Attempts (3 marks)
A1 Total surface area $= 2\pi r^2 + \pi r^2$ or $2\pi r^2$ or πr^2
A2 Correct r indicated

Worthless (0)
W1 Volume of hemisphere
W2 $2\pi r$

Part (b) **20 (10,5,5) marks** **Att (3,2,2)**

A jeweller buys a rectangular block of gold
of length 4 cm, width 3 cm and height 2 cm.
1 cm^3 of gold costs €400.



(i) ✍ Calculate the cost of the block of gold.

The jeweller needs 250 mm^3 of gold to make a gold ring.

(ii) ✍ How many rings can be made from the block?

Each ring is sold for €120.

(iii) ✍ Calculate the amount of profit the jeweller makes on each ring.

(b) (i) **10 marks** **Att 3**

Volume of block = 4 × 3 × 2
= 24 cm^3
Cost = 24 × €400
= €9 600

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)

A1 Volume = length × breadth × height
A2 Indication of multiplication by 400

Worthless (0)

W1 Surface area formula

(b) (ii) **5 marks** **Att 2**

$24\text{cm}^3 = 24000\text{mm}^3$

$$\text{Number of rings} = \frac{24000}{250} = 96$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 $1\text{cm}^3 \neq 1000\text{mm}^3$
B3 Incorrect operation

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)
A1 1cm = 10mm
A2 Indication of division by 250

Worthless (0)
W1 Answer from (b) (i) divided by 120

(b) (iii) **5 marks** **Att 2**

Total Selling Price = €120 × 96 = €11 520
Total Profit = €11 520 − €9 600
= €1 920

$$\text{Profit on each ring} = \frac{1920}{96} = €20$$

or

Cost per ring = €9 600 ÷ 96 = €100
Profit on each ring = €120 − €100
= €20

* Accept candidate's answers from parts (i) and (ii)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Profit not calculated
B3 Incorrect operation
B4 Answers from part (i) and/or part (ii) not used

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
Attempts (2 marks)
A1 Profit = selling price – cost price
A2 Works with answer from (b) (i) or (b) (ii)

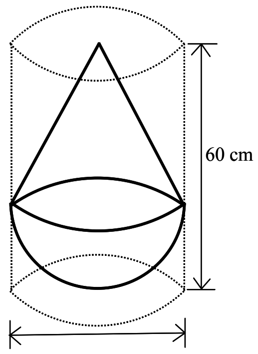
Part (c) **20 (10,10) marks** **Att (3,3)**

A float in the shape of a cone on top of a hemisphere is made from solid rubber.
The diameter of the hemisphere is 30 cm and the height of the float is 60 cm.

(i) ✍ Find the volume of the float in terms of π.

The float is cut from a solid rubber cylinder of diameter 30 cm and height 60 cm.

(ii) ✍ Express the volume of rubber used in the float as a percentage of the volume of the cylinder.
Give your answer correct to the nearest whole number.



(c) (i) **10 marks** **Att 3**

Volume = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ ....Step 1

$= \frac{1}{3}\pi(15)^2(45) + \frac{2}{3}\pi(15)^3$ Step2

$= 3375\pi + 2250\pi$ or 5625π ....Step 3

* Accept $\frac{2}{8}\pi r^3$ or $\frac{1}{2}\left(\frac{4}{8}\pi r^3\right)$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Each step incorrect
B3 Incorrect substitution into correct formula
B4 Incorrect relevant volume formula
B5 Incorrect h
B6 $r \neq 15$
B7 Incorrect squaring or cubing

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not in terms of π
S3 Fails to indicate addition

Attempts (3 marks)

A1 $r = 15$
A2 $h = 45$
A3 Volume of the hemisphere = $\frac{2}{3}\pi r^3$
A4 Volume of cone formula correct with some substitution
A5 Volume of the cone or hemisphere found only
A6 Effort at calculating the volume of either the cone or the hemisphere

Worthless (0)

W1 Area formula for both the cone and the hemisphere

(c) (ii) **10 marks** **Att 3**

Volume of cylindrical shape = $\pi r^2 h$

$= \pi(15)^2(60) = 13500\pi$............Step 1

Percentage $= \dfrac{5625\pi}{13500\pi}$ or equivalent ratio....................Step 2

$= \dfrac{5625\pi}{13500\pi} \times 100\% = 41\tfrac{2}{3}\% = 42\%$ ....Step 3

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 $r \neq 15$
B3 $h \neq 60$
B4 Incorrect squaring
B5 Incorrect relevant volume formula
B6 Incorrect substitution into correct formula
B7 Ratio inverted
B8 Percentage not found

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded off or incorrectly rounded

Attempts (3 marks)

A1 Correct formula with some substitution
A2 $r = 15$
A3 Use of 100
A4 volume float ÷ volume cylinder

Worthless (0)

W1 Area formula

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**15 marks**	**Att 5**
Part (c)	**25(5,10,5,5) marks**	**Att (2,3,2,2)**

Part (a) **10 marks** **Att 3**

a (-2, -1) and *b* (5, -4) are the end points of the diameter of a circle.

✍ Find the coordinates of the centre of the circle.

(a) **10 marks** **Att 3**

Centre = midpoint$[ab]$

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-2+5}{2}, \frac{-1-4}{2} \right)$$

$$= \left(\frac{3}{2}, -\frac{5}{2} \right)$$

or

$$a \to b \quad (-2,-1) \to (5,-4)$$

$$x : +7, y : -3$$

$$\Rightarrow a \to \text{centre} \quad x : +\frac{7}{2}, y : -\frac{3}{2}$$

$$\therefore \text{centre} = \left(-2 + \frac{7}{2}, -1 - \frac{3}{2} \right)$$

$$= \left(\frac{3}{2}, -\frac{5}{2} \right)$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Both *x* and *y* switched in substitution
B4 Correct substitution but midpoint not found
B5 Error in translation

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*
S2 One incorrect substitution

Attempts (3 marks)

A1 Correct formula with or without some substitution
A2 Effort at translation
A3 Graphical solution or effort at graphical solution

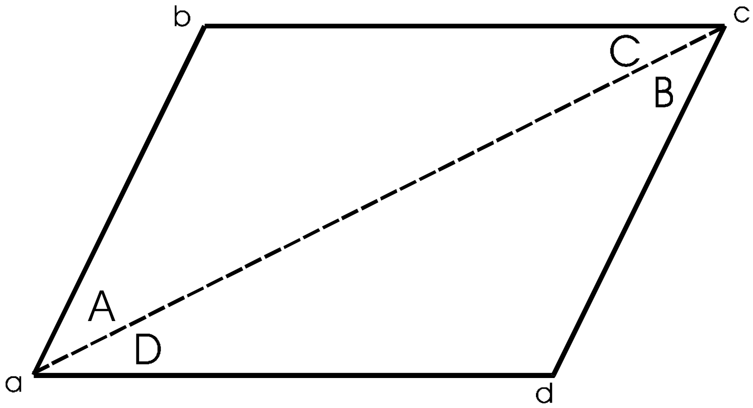
Worthless (0)

W1 Wrong formula with or without substitution

Part (b) **15 marks** **Att 5**

✍ Prove that the opposite sides and opposite angles of a parallelogram are respectively equal in measure.

(b) (i) **15 marks** **Att 5**



Given: Parallelogram *abcd*.

To Prove: |*ab*|=|*dc*| and |*bc*|=|*ad*|
|∠ *abc*|=|∠ *adc*| and |∠ *bad*|=|∠ *bcd*|.......................Step 1

Construction: Join *a* to *c*.. Step 2
Mark in angles A, B, C and D

Proof |∠ A|=|∠ B| (alternate angles)
|∠ C|=|∠ D| (alternate angles)
|*ac*|=|*ac*| (common side)..Step 3
∴ Δ*abc* ≡ Δ*adc* (A.S.A.)... Step 4
∴ |*ab*|=|*dc*| and |*bc*|=|*ad*| [corresponding sides]
|∠ *abc*|=|∠ *adc*| [corresponding angles]
|∠ *bad*|=|∠ A|+|∠ D|=|∠ B|+|∠ C|=|∠ *bcd*|.........Step 5

* Some steps may be indicated on candidate's diagram
* Accept other valid proofs
* Must have one reason in Step 3 and A.S.A. in Step 4
* Accept one pair of sides and one pair of angles for Step 1

Blunders (-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (5 marks)
A1 Parallelogram drawn with diagonal indicated
A2 Parallelogram drawn with sides or angles to be proven equal indicated

Worthless (0)
W1 Wrong Theorem
W2 Parallelogram drawn and stops
W3 No diagram

Part (c) **25 (5,10,5,5) marks** **Att (2,3,2,2)**

(i)	✍	Verify that the points (3, 0) and (0, -2) are on the line L: $2x - 3y = 6$.
(ii)	✍	Find the equation of the line K through (3, 0) which is perpendicular to L.
(iii)	✍	Show the lines L and K on a coordinate diagram on graph paper.
(iv)	✍	Find the area of the triangle formed by the lines L and K and the y axis.

(c) (i) **5 marks** **Att 2**

$(3,0): 2(3) - 3(0) = 6$
$6 - 0 \quad = 6 \quad \text{yes}$

$(0,-2): 2(0) - 3(-2) = 6$
$0 + 6 \quad = 6 \quad \text{yes}$

or

Slope of $L: \dfrac{-2-0}{0-3} = \dfrac{2}{3}$

The equation of $L: y - 0 = \dfrac{2}{3}(x-3)$

$3y = 2x - 6$

$2x - 3y = 6$

Thus, verified.

or

Let $x = 3: 2(3) - 3y = 6$
$\Rightarrow 6 - 3y = 6$
$\Rightarrow -3y = 0$
$\Rightarrow y = 0$

Let $y = -2: 2x - 3(-2) = 6$
$\Rightarrow 2x + 6 = 6$
$\Rightarrow 2x = 0$
$\Rightarrow x = 0$

Thus, verified.

or

Cuts x-axis: $y = 0$
$\Rightarrow 2x - 3(0) = 6$
$\Rightarrow 2x - 0 = 6$
$\Rightarrow 2x = 6$
$\Rightarrow x = 3$

Cuts y-axis: $x = 0$
$\Rightarrow 2(0) - 3y = 6$
$\Rightarrow 0 - 3y = 6$
$\Rightarrow -3y = 6$
$\Rightarrow y = -2$

$\therefore$ (3,0) and (0,–2) are points on the line

Blunders (-3)
B1 Both x and y switched in substitution
B2 Incorrect relevant formula
B3 Transposition error

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)
A1 Correct formula with or without substitution
A2 Effort at substitution
A3 At least two points on L correctly plotted

(c) (ii) **10 marks** **Att 3**

slope $L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{0-3} = \frac{2}{3}$ or $y = mx + c$ method or $m = -\frac{a}{b}$ method

$\Rightarrow$ slope $K = -\frac{3}{2}$

The equation of K : $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{3}{2}(x-3)$$

or $2y = -3x + 9$

or $3x + 2y - 9 = 0$

* Accept candidate's slope from part (i)

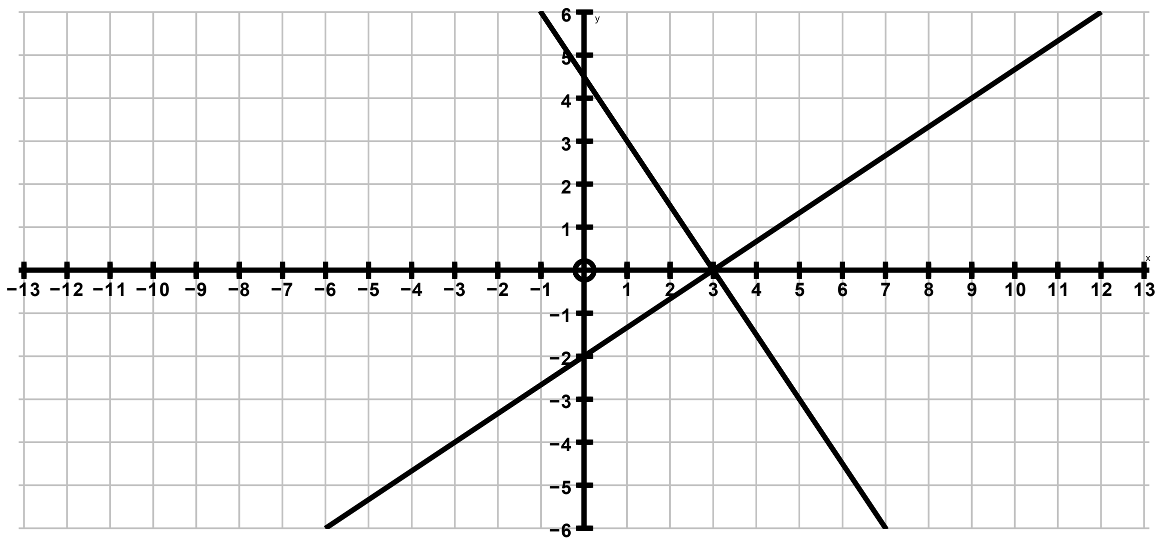
Blunders (-3)
B1 Incorrect relevant formula
B2 Both x and y switched in substitution
B3 Incorrect perpendicular slope

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 One incorrect substitution

Misreadings (-1)
M1 (0,−2) used in equation formula

Attempts (3 marks)
A1 Correct slope formula and/or line formula with or without substitution
A2 Indication that the product of perpendicular slopes is -1

(c) (iii) **5 marks** **Att 2**



- * Accept candidate's perpendicular axes
- * Accept any other point on *K*
- * If second point on *K* not found accept perpendicular angle within a tolerance of $\pm 10^{o}$
- * Accept candidate's equation of *K* from part (ii)

Blunders (-3)
B1 Scale not uniform
B2 One line only sketched

Slips (-1)
S1 Not on graph paper

Attempts (2 marks)
A1 One point only plotted
A2 Scaled axes drawn and stops
A3 Effort at finding a point on either *L* or *K*

(c) (iv) **5 marks** **Att 2**

Area of triangle = $\frac{1}{2}(6\cdot 5)(3) = 9\cdot 75$ units^2

- * Accept any valid method
- * Accept values consistent with candidate's graph, within a tolerance of ±0·2

Blunders (-3)
B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect base
B4 Incorrect height
B5 Sum of areas of two smaller triangles not equal to area of required triangle

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Sum of areas of smaller triangles not found

Attempts (2 marks)
A1 Correct formula with or without substitution (applies to coordinate geometry formula)

QUESTION 3

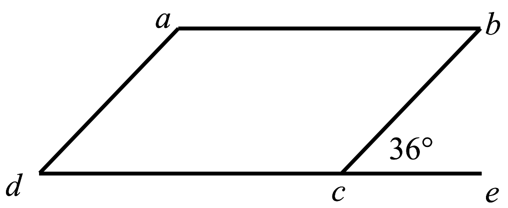
Part (a)	**20(15,5) marks**	**Att (5,2)**
Part(b)	**20(10,10) marks**	**Att (3,3)**
Part (c)	**10(5,5) marks**	**Att (2,2)**

Part (a) **20(15,5)marks** **Att 5,2**

abcd is a parallelogram with [*dc*] produced to *e* and $|\angle bce| = 36°$, as shown.

Find **(i)** $|\angle abc|$,

(ii) ✍ $|\angle bad|$.



*Accept (i) and (ii) in any order

(a)(i) **15 marks** **Att 5**

$|\angle abc| = |\angle bce| = 36^{\circ}$ [alternate angles]

or

$|\angle bcd| = 180^{\circ} - 36^{\circ} = 144^{\circ} \Rightarrow |\angle abc| = 36^{\circ}$ [interior angles]

* Some steps may be indicated on candidate's diagram

* Accept correct answer without work shown

Blunders (-3)

B1 $|\angle bcd| + |\angle bce| \neq 180^{\circ}$

B2 Sum of the angles in a parallelogram ≠ 360°

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (5 marks)

A1 Diagram from examination paper drawn with equal angles indicated

A2 States that alternate angles are equal

A3 Indication that a straight angle = 180°

A4 Indication that the sum of the angles in a parallelogram = 360°

Worthless (0)

W1 Diagram from examination paper either partially or fully drawn

(a)(ii) **5 marks** **Att 2**

$\left|\angle bad\right| = 180° - 36° = 144°$

* Some steps may be indicated on candidate's diagram
* Accept candidate's answer from part (i)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Sum of the angles in a parallelogram $\neq 360^{o}$
B3 Opposite angles in a parallelogram = 180^{o}
B4 $|\angle bcd| + |\angle bce| \neq 180^{o}$

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)
A1 Diagram from examination paper drawn with equal angles indicated
A2 Indication that the opposite angles in a parallelogram are equal
A3 Indication that the sum of the angles in a parallelogram = 360^{o}
A4 $|\angle bcd| = 144^{o}$
A5 $|\angle bad| = 360^{o} - 2(36^{o})$

Worthless (0)
W1 Diagram from examination paper either partially or fully drawn

Part (b) **20 (10,10) marks** **Att (3,3)**

(i) ✍ Show how to construct the circumcircle of a triangle.
All construction lines must be clearly shown.

(ii) ✍ Each of the three figures labelled *A*, *B* and *C* shown below is the image of the figure *X* under a transformation. For each of *A*, *B* and *C*, state what the transformation is (translation, central symmetry, axial symmetry or rotation) and in the case of a rotation, state the angle.

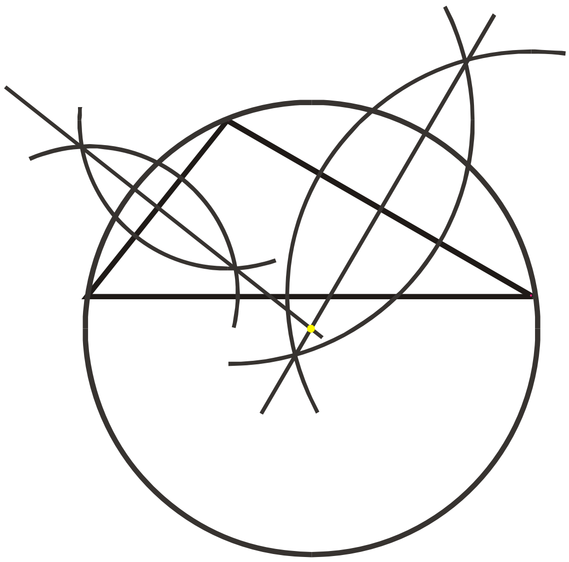
X

A

B

C

(b) (i) **10 marks** **Att 3**



One side bisected................Step 1
Second side bisected...........Step 2
Circle drawn.......................Step 3

- * Accept constructions with tolerance of 2mm
- * 4 marks: one side correctly bisected
- * 7 marks: two sides correctly bisected

Blunders (-3)
B1 Circumcentre indicated but circumcircle not drawn
B2 Vertex not on circle (outside tolerance) once only

Attempts (3 marks)
A1 Effort at bisecting any side
A2 Incircle drawn with construction lines shown
A3 Triangle and circumcircle drawn with no construction lines shown

Worthless (0)
W1 Incircle with no construction lines

(b) (ii) **10 marks** **Att 3**

A	Rotation of 90° [anti-clockwise] or Rotation of 270° [clockwise]
B	Axial symmetry
C	Translation or Rotation of 360°

* Accept angle of rotation without reference to anti-clockwise or clockwise
* One correct transformation: 4 marks
* Two correct transformations: 7 marks
* Three correct transformations: 10 marks

Slips (-1)
S1 No angle or incorrect angle of rotation

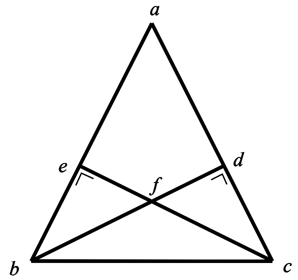
Attempts (3 marks)
A1 Effort at drawing the figure *X* under one of the given transformations

Part (c) **10 (5,5) marks** **Att (2,2)**

The triangle *abc* is an isosceles triangle, with $|ab| = |ac|$ and $|\angle bec| = |\angle cdb| = 90°$.
The lines *ec* and *bd* intersect at f.

(i) ✍ Prove $|\angle dbc| = |\angle ecb|$.

(ii) ✍ Prove $|ef| = |fd|$.



(c) (i) **5 marks** **Att 2**

[Consider Δbef and Δcdf]
$|\angle bfe| = |\angle cfd|$ [vertically opposite angles]
$|\angle bef| = |\angle cdf|$ [given]..Step 1
$\therefore |\angle ebf| = |\angle dcf|$ [3rd pair of angles]....................Step 2
$|\angle abc| = |\angle acb|$ [Δabc is isosceles]
$\Rightarrow |\angle abc| - |\angle ebf| = |\angle acb| - |\angle dcf|$
i.e. $|\angle dbc| = |\angle ecb|$..Step 3

or

[Consider Δbcd and Δcbe]
$|\angle bcd| = |\angle cbe|$ [Δabc is isosceles]........................Step 1
$|\angle bdc| = |\angle bec|$ [given]..Step 2
$\therefore |\angle dbc| = |\angle ecb|$ [3rd pair of angles].....................Step 3

or

Δbce and Δbcd have the same circumcircle with diameter [*bc*].............Step 1
$\Rightarrow |\angle dbe| = |\angle dce|$ [angles standing on the same arc *de* are equal)...........Step 2
$|\angle cbe| = |\angle bcd|$ [Δabc is isosceles]
$\Rightarrow |\angle cbe| - |\angle dbe| = |\angle bcd| - |\angle dce|$
i.e. $|\angle dbc| = |\angle ecb|$...Step 3

* Some steps may be indicated on candidate's diagram
* Accept other valid proofs
* If prove Δbcd and Δcbe are congruent by A.S.A. full marks

Blunders (-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (2 marks)
A1 Diagram drawn with equal angles indicated
A2 $|\angle abc| = |\angle acb|$
A3 Mention of $[bc]$ as diameter of circumcircle of either Δbce or Δbcd
A4 Indication that the sum of the angles in a triangle = 180°

Worthless (0)
W1 Diagram from examination paper either partially or fully drawn

(c) (ii) **5 marks** **Att 2**

[Consider Δbef and Δcdf]
$|\angle efb| = |\angle dfc|$ [vertically opposite angles]
$|\angle bef| = |\angle cdf|$ [given]..Step 1
$\therefore |\angle ebf| = |\angle dcf|$ [third pair of angles]
$|bf| = |cf|$ [In Δbcf $|\angle fbc| = |\angle fcb|$, from part (i)]..................Step 2
$\therefore \Delta bef \equiv \Delta cdf$ (A.S.A.)
$\Rightarrow |ef| = |fd|$ [corresponding sides]..Step 3

* Some steps may be indicated on candidate's diagram
* Accept other valid proofs

Blunders (-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (2 marks)
A1 Diagram drawn with equal angles indicated
A2 Indication that the sum of the angles in a triangle = 180°
A3 Indication of a pair of sides or pair of angles relevant to proving congruency

Worthless (0)
W1 Diagram from examination paper either partially or fully drawn

QUESTION 4

Part (a)	**20(15,5) marks**	**Att (5,2)**
Part (b)	**20(15,5) marks**	**Att (5,2)**
Part (c)	**10(5,5) marks**	**Att (2,2)**

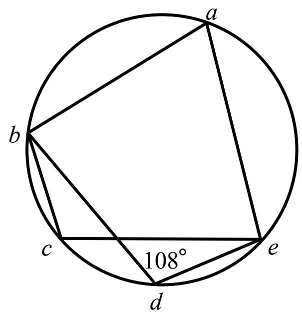
Part (a) **20(15,5) marks** **Att 5,2**

a, *b*, *c*, *d* and *e* are points on a circle and $|\angle bde| = 108°$.

Find **(i)** ✍ $|\angle bae|$,

(ii) ✍ $|\angle bce|$,

giving a reason for your answer in each case.



*Accept (i) and (ii) in any order

(a)(i) **15 marks** **Att 5**

$|\angle bae| = 180° - 108° = 72°$ [sum of opposite angles in cyclic quadrilateral = 180°]

* Some steps may be indicated on candidate's diagram

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 360° or 90° used instead of 180°

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (5 marks)

A1 Indication that the sum of the angles in a cyclic quadrilateral = 360°

A2 Indication that the sum of the opposite angles in a cyclic quadrilateral = 180°

Worthless (0)

W1 $|\angle bae| = 108°$

W2 Diagram from examination paper either partially or fully drawn

(a)(ii) **5 marks** **Att 2**

$|\angle bce| = 108°$ (angles standing on same arc are equal)
or
$|\angle bce| = 180° - 72° = 108°$ [sum of opposite angles in a cyclic quadrilateral = 180^{o}]

* Some steps may be indicated on candidate's diagram

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 360^{o} or 90^{o} used instead of 180^{o}

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)

A1 Indication that the sum of the angles in a cyclic quadrilateral = 360^{o}

A2 Indication that sum of the opposite angles in a cyclic quadrilateral = 180^{o}

A3 Indication that angles standing on the same arc are equal

Worthless (0)

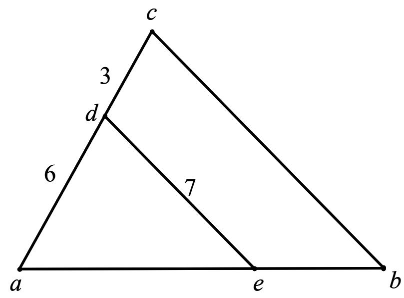
W1 $|\angle bce| = 72°$

W2 Diagram from examination paper either partially or fully drawn

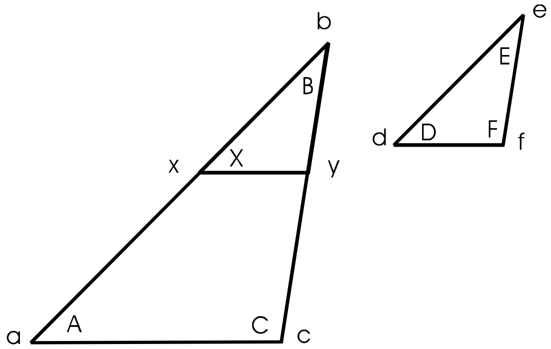
Part (b) **20 (15,5) marks** **Att 5,2**

(i) ✍ Prove that if two triangles are equiangular, the lengths of corresponding sides are in proportion.

(ii) In the triangle abc, de is parallel to cb.
$|ad| = 6$, $|dc| = 3$ and $|de| = 7$.
✍ Find $|cb|$.



(b) (i) **15 marks** **Att 5**



Given: Δabc and Δdef, with $|\angle A|=|\angle D|$, $|\angle B|=|\angle E|$ and $|\angle C|=|\angle F|$

To Prove: $\frac{|ab|}{|de|}=\frac{|bc|}{|ef|}=\frac{|ac|}{|df|}$ ....Step 1

Construction: Mark x on $[ab]$ such that $|xb|=|de|$
Mark y on $[cb]$ such that $|yb|=|ef|$..............Step 2
Mark in angle X

Proof: $|xb|=|de|$, $|yb|=|ef|$ [construction]
$|\angle B|=|\angle E|$ [given]
$\therefore \Delta xby \equiv \Delta def$ (S.A.S) ....Step 3
$\Rightarrow |\angle X|=|\angle D|$ (corresponding angles)
$\Rightarrow |\angle X|=|\angle A|$ ($|\angle A|=|\angle D|$)
$\Rightarrow xy \| ac$ (corresponding angles)
$\Rightarrow \frac{|ab|}{|xb|}=\frac{|bc|}{|yb|}$ (a line drawn parallel to one side of a triangle divides the other two sides in the same ratio)............Step 4

But $|xb|=|de|$ and $|yb|=|ef|$ [construction]

$\Rightarrow \frac{|ab|}{|de|}=\frac{|bc|}{|ef|}$

Similarly $\frac{|bc|}{|ef|}=\frac{|ac|}{|df|}$

$\therefore \frac{|ab|}{|de|}=\frac{|bc|}{|ef|}=\frac{|ac|}{|df|}$..Step 5

* Some steps may be indicated on candidate's diagram
* Accept other valid proofs
* Must have S.A.S. in Step 3 and one reason in Step 4

Blunders (-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (5 marks)
A1 Two separate triangles drawn with equal angles indicated
A2 The second diagram of the proof drawn

Worthless (0)
W1 Wrong theorem
W2 Two triangles drawn
W3 No diagram

(b) (ii) **5 marks** **Att 2**

$$\frac{|ac|}{|ad|} = \frac{|bc|}{|de|}$$

$$\Rightarrow \frac{9}{6} = \frac{|bc|}{7}$$

$$\Rightarrow |bc| = \frac{21}{2} \text{ or } 10 \cdot 5$$

* Some steps may be indicated on candidate's diagram

Blunders (-3)
B1 Correct answer without work shown (✍)
B2 $\frac{|ac|}{|ad|} = \frac{|de|}{|bc|}$ or similar incorrect ratio
B3 Error in cross multiplication

Slips (-1)
S1 Arithmetic errors to a max of *(-3)*

Attempts (2 marks)
A1 One correct relevant ratio
A2 9 written down without work
A3 Any use of 6 and 3 in a ratio

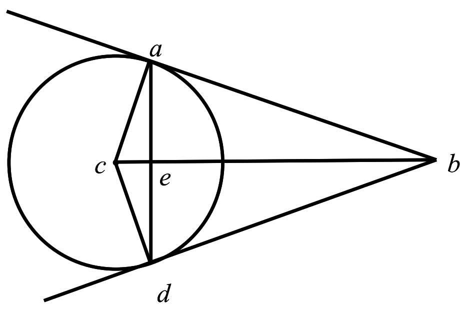
Worthless (0)
W1 Diagram from examination paper either partially or fully drawn

Part(c) **10 (5,5) marks** **Att 2,2**

ba and *bd* are tangents to the circle of centre *c*.
[*bc*] intersects the chord [*ad*] at the point *e*.

(i) ✍ Prove that $\Delta\, abc$ is congruent to $\Delta\, dbc$.

(ii) ✍ Hence, prove that [*bc*] bisects the chord [*ad*].



(c) (i) **5 marks** **Att 2**

[Consider Δabc and Δdbc]

$|ac| = |dc|$ [radii]......................................Step 1

$|\angle cab| = |\angle cdb|$ [right angles]

$|cb| = |cb|$ [common side]........................Step 2

$\therefore \Delta abc \equiv \Delta dbc$ (R.H.S.)..........................Step 3

* Some steps may be indicated on candidate's diagram

* Accept other valid proofs

Blunders (-3)

B1 Each step incorrect or omitted

B2 Each step incomplete

Attempts (2 marks)

A1 Indication of a pair of sides or pair of angles relevant to proving congruency

Worthless (0)

W1 Diagram from examination paper either partially or fully drawn

(c) (ii) **5 marks** **Att 2**

[Consider Δace and Δdce]

$|ac|=|dc|$[radii]

[$\Delta abc \equiv \Delta dbc$]

$\Rightarrow |\angle ace|=|\angle dce|$[corresponding angles].............Step 1

$|ce|=|ce|$[common side]

$\therefore \Delta ace \equiv \Delta dce$ (S.A.S.)...Step 2

$\Rightarrow |ae|=|de|$[corresponding sides]..........................Step 3

or

[Consider Δaeb and Δdeb]

$|ab|=|bd|$[$\Delta abc \equiv \Delta dbc \Rightarrow$ corresponding sides]

$|\angle abe|=|\angle dbe|$[$\Delta abc \equiv \Delta dbc \Rightarrow$ corresponding angles].............Step 1

$|be|=|be|$[common side]

$\therefore \Delta aeb \equiv \Delta deb$ (S.A.S.)..Step 2

$\Rightarrow |ae|=|de|$[corresponding sides]..Step 3

or

$|\angle cae|=|\angle cde|$[$\Delta acd$ is isosceles]

$\Rightarrow |\angle bac|-|\angle cae|=|\angle bdc|-|cde|$

i.e. $|\angle bae|=|\angle bde|$...Step 1

$\Rightarrow \Delta bad$ is isosceles

$|\angle abe|=|\angle dbe|$[corresponding angles since $\Delta abc \equiv \Delta dbc$]

$\Rightarrow bc \perp ad$...Step 2

$\Rightarrow [bc]$ bisects the chord $[ad]$[a diameter line perpendicular to a chord bisects the chord]...Step 3

* Some steps may be indicated on candidate's diagram

* Accept other valid proofs

Blunders (-3)

B1 Each step incorrect or omitted

B2 Each step incomplete

Attempts (2 marks)

A1 Indication of a pair of sides or pair of angles relevant to proving congruency

A2 Indication that a diameter perpendicular to a chord bisects the chord

Worthless (0)

W1 Diagram from examination paper either partially or fully drawn

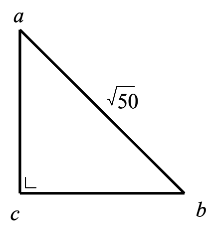
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(10,10) marks**	**Att (3,3)**
Part (c)	**20(10,10) marks**	**Att (3,3)**

Part (a) **10 marks** **Att 3**

abc is an isosceles triangle with $|ac| = |bc|$,
$|ab| = \sqrt{50}$ and $|\angle acb| = 90°$.

✍ Find $|bc|$.



(a) **10 marks** **Att 3**

$$|ab|^2 = |ac|^2 + |bc|^2$$
$$\left(\sqrt{50}\right)^2 = |bc|^2 + bc|^2$$
$$= 2|bc|^2$$
$$\Rightarrow |bc|^2 = 25$$
$$\Rightarrow |bc| = 5 \text{ or } \sqrt{25}$$

or

$$|\angle abc| = |\angle bac| = 45^\circ$$
$$\sin 45^\circ = \frac{|bc|}{|ab|} = \frac{|bc|}{\sqrt{50}}$$
$$\Rightarrow |bc| = \sqrt{50}\sin 45^\circ$$
$$\Rightarrow |bc| = 5 \text{ or } 4\cdot 9$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Pythagoras Theorem incorrect
B3 Incorrect squaring
B4 $|bc|^2 = 25$ and stops
B5 Incorrect ratio for sin function
B6 Error in cross multiplication
B7 Calculator in incorrect mode or page in tables used not relevant to sin/cos function
B8 Early rounding which affects the accuracy of the answer

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

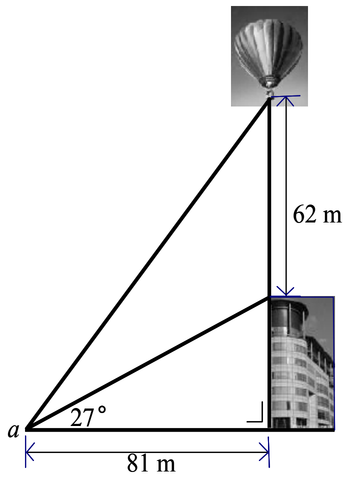
Attempts (3 marks)
A1 Indication of Pythagoras Theorem
A2 Indication of 45°
A3 Sine Rule with some substitution
A4 $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ or $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Part (b) **20 (10,10) marks** **Att 3,3**

The angle of elevation of the top of a building, as viewed from a point *a*, 81 m from the base of the building, is 27°.

(i) ✍ Find the height of the building correct to the nearest metre.

The bottom of a balloon is 62 m above the top of the building, as shown.



(ii) ✍ Find the angle of elevation of the bottom of the balloon as viewed from the point *a*. Give your answer correct to the nearest degree.

(b) (i) **10 marks** **Att 3**

Let h = height of the building.

$$\tan 27^\circ = \frac{h}{81} \Rightarrow h = 81\tan 27^\circ$$
$$= 41\cdot 27156141$$
$$= 41\text{m}$$

or

$$\frac{h}{\sin 27^\circ} = \frac{81}{\sin 63^\circ} \Rightarrow h = \frac{81\sin 27^o}{\sin 63^\circ}$$
$$= 41\cdot 27156141$$
$$= 41\text{m}$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect ratio for tan function
B3 Incorrect ratio for Sine Rule
B4 Error in cross multiplication
B5 Calculator in incorrect mode or page in tables used not relevant to tan/sin function
B6 Early rounding which affects the accuracy of the answer

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded off or incorrectly rounded

Attempts (3 marks)

A1 $\tan A = \frac{\text{opposite}}{\text{adjacent}}$
A2 $\tan 27^\circ$
A3 $\frac{h}{81}$
A4 Indication that the sum of the angles in a triangle = 180°
A5 Sine Rule with some substitution

Worthless (0)

W1 $\frac{h}{27} = \frac{81}{63}$

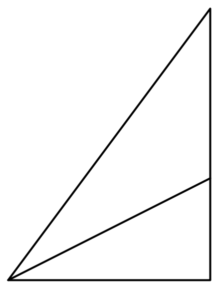
(b) (ii) **10 marks** **Att 3**

Let θ = angle of elevation.
The distance from the base of the building to the bottom of the balloon = 62+41=103m.

$$\tan\theta = \frac{103}{81} \Rightarrow \theta = \tan^{-1}\left(\frac{103}{81}\right)$$
$$= 51\cdot81821457^{\circ} = 52^{\circ}$$

or

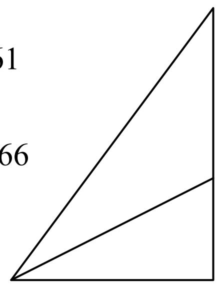
$$|ab|^2 = 103^2 + 81^2$$
$$= 10609 + 6561$$
$$= 17170$$
$$\Rightarrow |ab| = 131.0343466 = 131$$
$$\frac{\sin\angle bad}{62} = \frac{\sin 117^{\circ}}{131}$$
$$\Rightarrow \sin\angle bad = \frac{62\sin 117^{\circ}}{131}$$
$$= 0\cdot421697744$$
$$\Rightarrow |\angle bad| = \sin^{-1}(0\cdot421697744) = 24\cdot94^{\circ} = 25^{\circ}$$
$$\therefore |\angle bac| = 25^{\circ} + 27^{\circ} = 52^{\circ}$$



or

$$|ad|^2 = 41^2 + 81^2$$
$$= 1681 + 6561$$
$$= 8242$$
$$\Rightarrow |ad| = 90\cdot78546139$$
$$= 91$$

$$|ab|^2 = 103^2 + 81^2$$
$$= 10609 + 6561$$
$$= 17170$$
$$\Rightarrow |ab| = 131.0343466$$
$$= 131$$



$$\frac{\sin\angle abd}{91} = \frac{\sin 117^{\circ}}{131}$$
$$\Rightarrow \sin\angle abd = \frac{91\sin 117^{\circ}}{131} = 0\cdot618943463$$
$$\Rightarrow |\angle abd| = \sin^{-1}(0\cdot618943463) = 38\cdot24^{\circ} = 38^{\circ}$$
$$\Rightarrow |\angle bad| = 180^{\circ} - 117^{\circ} - 38^{\circ} = 25^{\circ}$$
$$\therefore |\angle bac| = 25^{\circ} + 27^{\circ} = 52^{\circ}$$

* Accept candidate's answer from part (i)
* Accept $25^{\circ} + 27^{\circ}$ for full marks

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect ratio for tan function
B3 Error in cross multiplication
B4 Calculator in incorrect mode or page in tables used not relevant to tan/sin function

B5 $\tan\theta = \frac{62}{81}$

B6 Early rounding which affects the accuracy of the answer

B7 Incorrect ratio for Sine Rule

B8 $|\angle abd|$ found as angle of elevation

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

S2 Answer not rounded off or incorrectly rounded

S3 Sum of angles not indicated

Attempts (3 marks)

A1 $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

A2 Indication of use of 81 in a ratio

A3 Attempt at finding the hypotenuse of either right angled triangle

A4 Sine Rule with some substitution

A5 Indication that the sum of the angles in a triangle = 180°

Worthless (0)

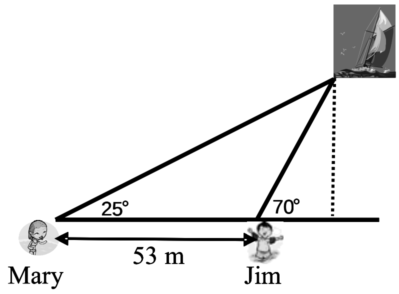
W1 $\frac{27}{81}$

Part (c) **20 (10,10) marks** **Att 3,3**

Mary and Jim are standing 53 m apart on a straight shoreline.
They observe a boat at sea making angles of 25° and 70° respectively with the shore, as shown.

(i) ✍ Find the distance Jim is from the boat correct to the nearest metre.

(ii) ✍ Calculate the shortest distance from the boat to the shoreline.



(c) (i) **10 marks** **Att 3**

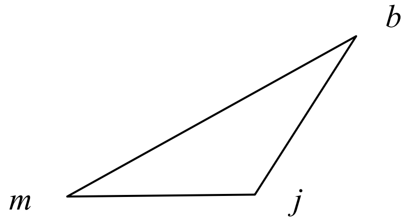
$$|\angle bjm| = 110^\circ \Rightarrow |\angle jbm| = 45^\circ$$

$$\frac{|bj|}{\sin 25^\circ} = \frac{53}{\sin 45^\circ}$$

$$\Rightarrow |bj| = \frac{53\sin 25^\circ}{\sin 45^\circ}$$

$$= 31\cdot 67664131$$

$$= 32\text{m}$$



Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect ratio for Sine Rule
B3 Error in cross multiplication
B4 Calculator in incorrect mode or page in tables used not relevant to sin function
B5 Early rounding which affects the accuracy of the answer
B6 Sum of the angles in a triangle $\neq 180^\circ$

Slips (-1)

S1 Arithmetic slips to a maximum of (-3)
S2 Answer not rounded off or incorrectly rounded

Misreadings (-1)

M1 The distance Mary is from the boat correctly found

Attempts (3 marks)

A1 Sine Rule with some substitution
A2 Indication that the sum of the angles in a triangle = 180°

Worthless (0)

W1 $\frac{|bj|}{25} = \frac{53}{45}$ or similar
W2 The triangle treated as right angled

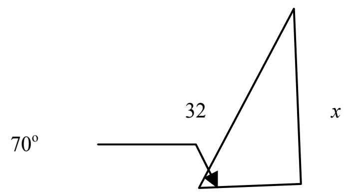
(c) (ii) **10 marks** **Att 3**

Let x = shortest distance.

$$\sin 70^{\circ} = \frac{x}{32} \Rightarrow x = 32\sin 70^{\circ}$$

$$= 30 \cdot 07016387$$

$$\text{or } 30 \text{ or } 30 \cdot 1 \text{ etc.}$$



* Accept candidate's answer from part (i)

* Accept answer not rounded off

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 Incorrect ratio for sin function

B3 Error in cross multiplication

B4 Calculator in incorrect mode or page in tables not relevant to sin/cos function used

B5 Early rounding which affects the accuracy of the answer

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)

A1 $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

A2 $\sin 70^{\circ}$

A3 Indication that the sum of the angles in a triangle = 180°

A4 Attempt at distance of Jim or Mary to the dotted line

A5 Attempt at distance of Mary to the boat

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(10,10) marks**	**Att (3,3)**
Part (c)	**20(5,5,5,5) marks**	**Att (2,2,2,2)**

Part (a) **10 marks** **Att 3**

8 is the mean of the five numbers 13, 6, 5, x and 7

✍ Find the value of x.

(a) **10 marks** **Att 3**

$$\frac{13+6+5+x+7}{5}=8$$

$$\Rightarrow \frac{x+31}{5}=8$$

$$\Rightarrow x+31=40$$

$$\Rightarrow x=9$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect denominator
B3 Transposition error
B4 $31x$ in numerator
B5 $24x$ in numerator

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Misreadings (-1)
M1 Mean of 13, 6, 5, x is 7 and continues

Attempts (3 marks)
A1 Some or all of the numbers added
A2 Indication of division by 5
A3 $\frac{13+6+5+x+7}{5}$

Part (b) **20 (10,10) marks** **Att (3,3)**

The weights, in kg, of 125 Junior Certificate students are given in the following frequency table.

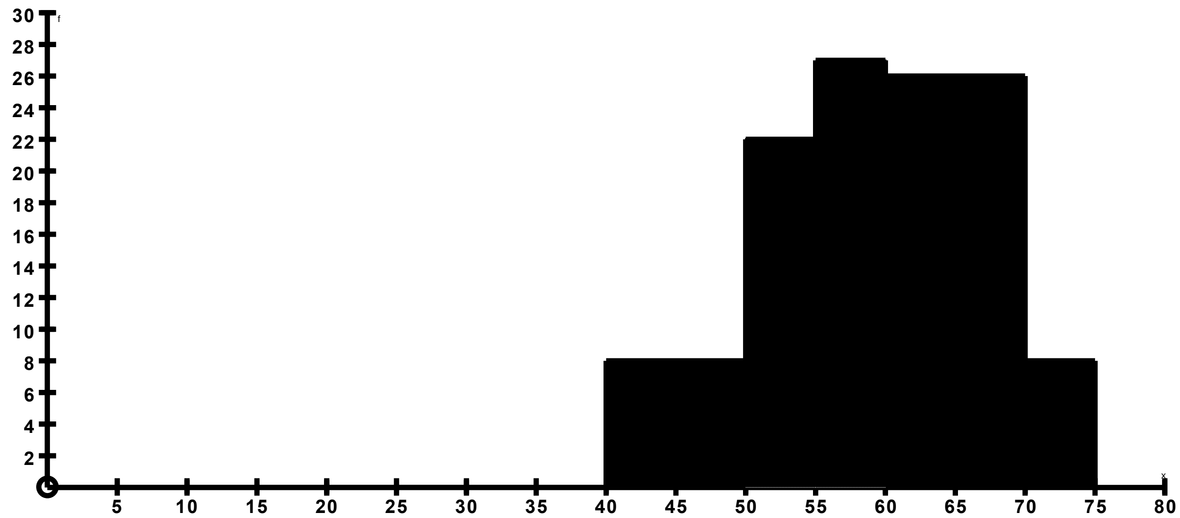
Weight in kg	40 – 50	50 – 55	55 – 60	60 – 70	70 – 75
Number of students	16	22	27	52	8

[Note: 40 – 50 means 40 or more but less than 50, etc.]

(i) Draw a histogram to illustrate the data in the frequency table.

(ii) ✍ Using mid interval values, calculate the mean weight of the Junior Certificate students.

(b) (i) **10 marks** **Att 3**



* Accept candidate's perpendicular axes

* The interval 0-40 need not be shown on "Weight" axis

Blunders (-3)

B1 Scale not uniform on each axis
B2 Incorrect width of rectangle
B3 Incorrect height of rectangle
B4 Rectangle omitted

Attempts (3 marks)
A1 Graph from frequency table drawn
A2 Scaled axes drawn and stops

Worthless (0)
W1 Table from examination paper copied

(b) (ii) **10 marks** **Att 3**

$$\text{Mean weight} = \frac{(16)(45)+(22)(52\cdot5)+(27)(57\cdot5)+(52)(65)+(8)(72\cdot5)}{16+22+27+52+8}$$

$$= \frac{720+1155+1552\cdot5+3380+580}{125}$$

$$= \frac{7387\cdot5}{125}$$

$$= 59\cdot1\text{kg}$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Consistent incorrect mid interval values
B3 Division by 5
B4 Division by sum of mid interval values
B5 Mid interval values added to frequencies instead of multiplied

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (3 marks)

A1 One correct multiplication in numerator
A2 Indication of division by 125
A3 One correct mid interval
A4 Sum of mid interval values divided by 125

Worthless (0)

W1 Sum of frequencies divided by 5

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

The salaries of the employees in a manufacturing firm were recorded. The following were the results.

Salary (in 1000's €)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of employees	7	12	20	29	7

[Note: 20 – 40 means 20 or more but less than 40, etc.]

(i) Construct the cumulative frequency table.

(ii) On graph paper construct the ogive.

(iii) ✍ Use your graph to estimate the median salary.

(iv) ✍ Estimate from your graph the percentage of employees whose salaries are between €70 000 and €90 000.

Give your answer correct to the nearest whole number.

(c) (i) **5 marks** **Att 2**

Salary (in 1000's €)	< 20	< 40	< 60	< 80	< 100
Number of employees	7	19	39	68	75

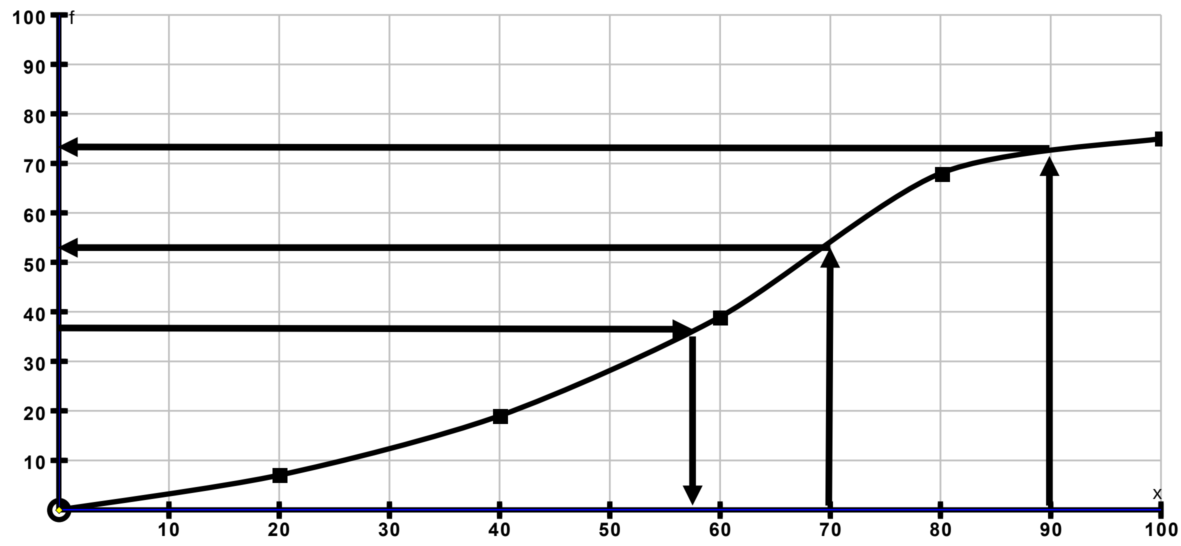
Blunders (-3)
B1 Number omitted (sum ≠ 75)

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)
A1 One value correctly filled into table
A2 Indication of addition of frequencies

Worthless (0)
W1 Table copied from examination paper

(c) (ii) **5 marks** **Att 2**



Salary (in 1000's €)

* Accept candidate's perpendicular axes

Blunders (-3)
B1 Scale not uniform
B2 Points plotted but not joined
B3 Cumulative cumulative ogive drawn

Slips (-1)
S1 Each point incorrectly plotted
S2 Each point omitted
S3 Points joined with straight lines
S4 Not on graph paper

Attempts (2 marks)
A1 Graph from frequency table drawn
A2 Scaled axes drawn

(c) (iii) **5 marks** **Att 2**

Median salary = €58 500 or 58·5

* Accept answer consistent with candidate's work
* Accept median in the range €55 000 - €60 000 or 55 - 60

Blunders (-3)
B1 Correct answer without work shown (✍)
B2 Line drawn from incorrect starting point of correct axis for median
B3 Median outside the range

Slips (-1)
S1 Work for median correct but not clearly marked
S2 Incorrect reading from graph with work shown

Attempts (2 marks)
A1 Horizontal line from 38 or 37·5 to the ogive drawn and stops
A2 Indication of use of 38 or 37·5
A3 Horizontal axis used for median

Worthless (0)
W1 Answer from trend graph

(c) (iv) **5 marks** **Att 2**

€70 000: 54 employees
€90 000: 73 employees
The number of people whose salaries are between €70 000 and €90 000
= 73 – 54 = 19

$$\text{Percentage} = \frac{19}{75} \times 100\%$$
$$= 25\tfrac{1}{3}\%$$
$$= 25\%$$

* Accept answer consistent with candidate's work

Blunders (-3)
B1 Correct answer without work shown (✍)
B2 Percentage not found

Slips (-1)
S1 Numerical slips to a maximum of *(-3)*
S2 Values added
S3 Answer not rounded off or incorrectly rounded
S4 Incorrect reading from graph with work shown

Attempts (2 marks)
A1 Graphical indication of use of €70 000 and /or €90 000
A2 73 or 54 indicated
A3 Use of 100

Worthless (0)
W1 Answer from trend graph

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded ***down***.
(e.g. 198 marks × 5% = 9.9 ⇒ bonus = 9 marks.)

If the mark awarded is above 225, the following table applies:

Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)	Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0