MARKING SCHEME
JUNIOR CERTIFICATE EXAMINATION 2007
MATHEMATICS - HIGHER LEVEL - PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips- numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 marks**	**Att 8**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 marks** **Att 3**

Express the speed 72 km/h in metres per second.

(a) **10 marks** **Att 3**

(a) 72 km=72000 m

1 hour =60×60 seconds =3600 seconds

$$\text{Speed} = \frac{72000}{60 \times 60} = \frac{72000}{3600} = 20 \text{ m/s}$$

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 Conversion error once only
B4 Error in using S/D/T formula
B5 Mathematical error

Slips (-1)

S1 Numerical errors to a max of 3
S2 Answer not in required or simplified form

Attempts (3 marks)

A1 Correct expression for S/D/T and stops
A2 Any correct conversion and stops

Worthless (0)

W1 Incorrect answer and no work shown

Part (b) **25 (15,10) marks** **Att (5,3)**

Part (b) (i) **15 marks** **Att 5**

✍ In 1981 the population of Peru was approximately $1\cdot8\times10^7$.
By 1988 the population had increased by 2·5 million.
What would be the approximate population of Peru in 1988?
Express your answer in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \le a < 10$.

b(i) **15 marks** **Att 5**

$1\cdot8\times10^7 = 18.0\times10^6$ (millions) or $1\cdot8\times10^7 = 18000000$

Increase = $2\cdot5 \times10^6$ $2\cdot5$ million =2500000

1988: $18.0\times10^6 + 2\cdot5 \times10^6$ 1988: $18000000 + 2500000$

1988: $2\cdot05\times10^7$ 1988: 20500000

1988: $2\cdot05\times10^7$

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 Incorrect operation e.g. subtracts instead of adding
B4 Only work shown 2·5 (millions) + $1\cdot8 \times 10^7 = 4\cdot3 \times 10^7$

Slips (-1)

S1 Numerical errors to a max of 3
S2 Answer not in required or simplified form

Misreadings (-1)

M1 1·8 taken as 2 and / or 2·5 taken as 3 and proceeds correctly to get $2\cdot3 \times 10^7$ with work shown

Attempts (5 marks)

A1 States 1 million $=10^6$

Worthless (0)

W1 Incorrect answer and no work shown

Part (b) (ii) **10 marks** **Att 3**

(ii) ✍ A snowman has a mass of 12 kg.
It melts at a rate of 0·2% of its mass per minute.
What will be the mass of the
snowman after 3 minutes?
Give your answer correct to 2 decimal places.

(b) (ii) **10 marks** **Att 3**

(ii) **Method I**

Start	= 12 ·000	= 12000
Loss min 1	= 0·024	= 24
After 1 min	= 11·976	= 11976
Loss min 2	= 0·023952	= 23·952
After 2 mins	= 11·952048	= 11952·048
Loss min 3	= 0·023904	= 23·904
After 3 mins	= 11·928144	= 11928·144
	= 11·93	= 11928·14

Method II

Start	= 12·000 or	= 12000
After 1 min	= 12×0·998	= 12000×0·998
	= 11·976	= 11976
After 2 mins	= 11·976×0·998	= 11976×0·998
	= 11·952048	= 11952·048
After 3 mins	= 11·952048×0·998	= 11952·048 ×0·998
	= 11·9281439	= 11928·1439
	= 11·93	= 11928·14

* Accept 11·93 kg or 11928·14 g
* Candidates may offer other correct versions- e.g. Compound Interest depreciation formula

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 Ignores cumulative loss of mass

Slips (-1)

S1 Numerical errors to max of 3
S2 Fails to round off or incorrect rounding or, early rounding off, which affects answer
S3 Incorrect operation e.g. Adds instead of subtracting
S4 Each minute omitted e.g. stops after 2 minutes. Note: Stops after 1 minutes merits 2 slips
S5 Consistent error in percentage e.g. uses incorrect % or 0.98 method II

Attempts (3 marks)

A1 Mentions 0.998 or 99.8% and stops

Worthless (0)

W1 Incorrect answer and no work shown

Part (c) **15 (10,5) marks** **Att (3,2)**

(c) (i) **10 marks** **Att 3**

(i) ✍ Simplify

$$\frac{2^5 \times 8^{\frac{2}{3}}}{64^{\frac{1}{2}} \times 4^2}.$$

Give your answer in the form 2^n, where $n \in \mathbf{N}$.

c(i) **10 marks** **Att3**

(i)

$$\frac{2^5 \times 8^{\frac{2}{3}}}{64^{\frac{1}{2}} \times 4^2} = \frac{2^5 \times (8^{\frac{1}{3}})^2}{8 \times 16} = \frac{2^5 \times (2^2)}{2^3 \times 2^4} = \frac{2^7}{2^7} = 2^0$$

or $$\frac{2^5 \times 8^{\frac{2}{3}}}{64^{\frac{1}{2}} \times 4^2} = \frac{32 \times 4}{8 \times 16} = \frac{128}{128} = 1 = 2^0$$

* With work $\frac{128}{128}$ or $\frac{2^5 \times 2^2}{2^3 \times 2^4}$ and stops merits 6 marks, 1 or $\frac{2^7}{2^7}$ and stops merits 9

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 Index error once only

Slips (-1)

S1 Numerical errors to max of 3
S2 Answer not in required or simplified form

Attempts (3 marks)

A1 Any correct calculation/re-write in index or decimal form shown and stops
A2 Correctly states a rule for indices

Worthless (0)

W1 Incorrect answer and no work shown

(c)(ii) **5 marks** **Att 2**

(ii) ✍ Simplify $(\sqrt{6}-2\sqrt{3})(5\sqrt{3}-3\sqrt{6})$,
without the use of a calculator.
Express your answer in the form $a\sqrt{2}+b$, where $a, b \in \mathbf{Z}$.

(c)(ii) **5 marks** **Att 2**

(ii)

$$(\sqrt{6}-2\sqrt{3})(5\sqrt{3}-3\sqrt{6}) = \sqrt{6}(5\sqrt{3}-3\sqrt{6})-2\sqrt{3}(5\sqrt{3}-3\sqrt{6})$$
$$= 5\sqrt{18}-3\sqrt{36}-10\sqrt{9}+6\sqrt{18}$$
$$= 5.3\sqrt{2}-18-30+6.3\sqrt{2}$$
$$= 15\sqrt{2}-48+18\sqrt{2}$$
$$= 33\sqrt{2}-48 \text{ or } -48+33\sqrt{2}$$

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Distribution error - once only
B3 Sign error
B4 Error in handling surds

Slips (-1)

S1 Numerical errors to max of 3
S2 Answer not in required or simplified form

Attempts (2 marks)

A1 Any attempt at handling surds

Worthless (0)

W1 $\sqrt{6}=2\cdot449$ and/or $\sqrt{3}=1\cdot7320$ without any relevant work
W2 -1·3309 without any relevant work

QUESTION 2

Part (a)	**15 marks**	**Att 5**
Part (b)	**15 marks**	**Att 5**
Part (c)	**20 marks**	**Att 6**

Part (a) **15 marks** **Att 5**

(a) ✍ An auctioneer sells a house for €830,000. The auctioneer's fee is 1·5% on the first €500,000 and 2·5% on the remainder.
Calculate the auctioneer's fee.

(a) **15 marks** **Att 5**

(a)

€830,000 - €500,000	=	€330,000
1·5% of €500,000	=	€7500
2·5% of €330,000	=	€8250
Total fee	=	€15,750

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 1·5 % of incorrect figure
B4 2·5 % of incorrect figure if not consistent with B3
B5 Mathematical error e.g. divides to find % or incorrect fraction for % each time if not consistent

Slips (-1)

S1 Numerical errors to a max of 3
S2 Does not total auctioneer's fee

Attempts (5 marks)

A1 Finds €330,000 and stops
A2 Finds 1·5% or 2·5% of some relevant figure and stops -with work

Worthless (0)

W1 Incorrect answer and no work shown

Part (b)	**(10, 5) marks**	**Att (3,2)**
Part (b) (i)	**10 marks**	**Att 3**

(b) (i) ✍ By putting the smallest number first, place the following numbers in order**:** $\frac{10}{7}$, $\sqrt{2}$, $\frac{7}{2\sqrt{6}}$, $(1\cdot19)^2$.

b(i)	**10 marks**	**Att 3**

(i)

$\frac{10}{7} = 1\cdot428571$ $\quad \sqrt{2} = 1\cdot41421$ $\quad \frac{7}{2\sqrt{6}} = \frac{7\sqrt{6}}{12} = 1\cdot42886$ $\quad (1\cdot19)^2 = 1\cdot4161$

$\sqrt{2}$,	$(1.19)^2$,	$\frac{10}{7}$,	$\frac{7}{2\sqrt{6}}$
b	b	b	b
$(1\cdot41421)$	$(1\cdot4161)$	$(1\cdot428571)$	$(1\cdot42886)$
A	**B**	**C**	**D**

* Accept decimal equivalents
* Accept candidates values when arranging

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point if affects answer
B3 Mathematical error, e.g. $\frac{7}{2\sqrt{6}}$ as $3\cdot5$ $(\sqrt{6})$, if affects answer
B4 No order or incorrect order, but see S2

Slips (-1)

S1 Numerical errors to a max of 3
S2 Reverses order

Attempts (3 marks)

A1 Finds decimal form of any of the given numbers and stops

b(ii) **5 marks** **Att 2**

(ii) ✍ What sum of money invested at 2% per annum compound interest would produce interest of €306·04 after three years?

b(ii) **5 marks** **Att 2**

Let P =100%	
P year 1 =100	P year 1 =x
I year 1 = 2	I year 1 = $0\cdot02x$
P year 2 = 102	P year 2 =$1\cdot02x$
I year 2 = $2\cdot04$	I year 2 = $0\cdot0204x$
P year 3 = $104\cdot04$	P year 3 = $1\cdot0404x$
I year 3 = $2\cdot0808$	I year 3 = $0\cdot020808x$
Amount = $106\cdot1208$	Amount = $1\cdot061208x$
I for 3 years = $6\cdot1208\%$	I for 3 years = $0\cdot061208x$
$6\cdot1208\%=€306\cdot04$	$0\cdot061208x=€306\cdot04$
$1\%=\frac{€306\cdot04}{6\cdot1208}=€50$	$x=\frac{€306\cdot04}{0\cdot061208}=€5000$
100% = €5000	

* Candidates may use other correct methods

* Candidates may use other starting figures e.g. €1000

Blunders (-3)

B1 Correct answer but no work shown (✍)

B2 Error in decimal point

B3 Each year omitted e.g. works on 2 years.

Slips (-1)

S1 Numerical errors to a max of 3

S2 Rounding off which affects answer once only

S3 Mathematical error when solving for x

Misreading (-1)

M1 Treats €306·04 as amount for 3 years (€288·39) or as interest for year 3 (€14 707·80) or P for year 1 (€324·77)

Attempts (2 marks)

A1 Let €306·04 = 6% and continues to find 100%

A2 Ignores cumulating /compounding interest

Worthless (0)

W1 Incorrect answer and no work shown

Part (c) **20 (10,5,5) marks** **Att (3,2,2)**

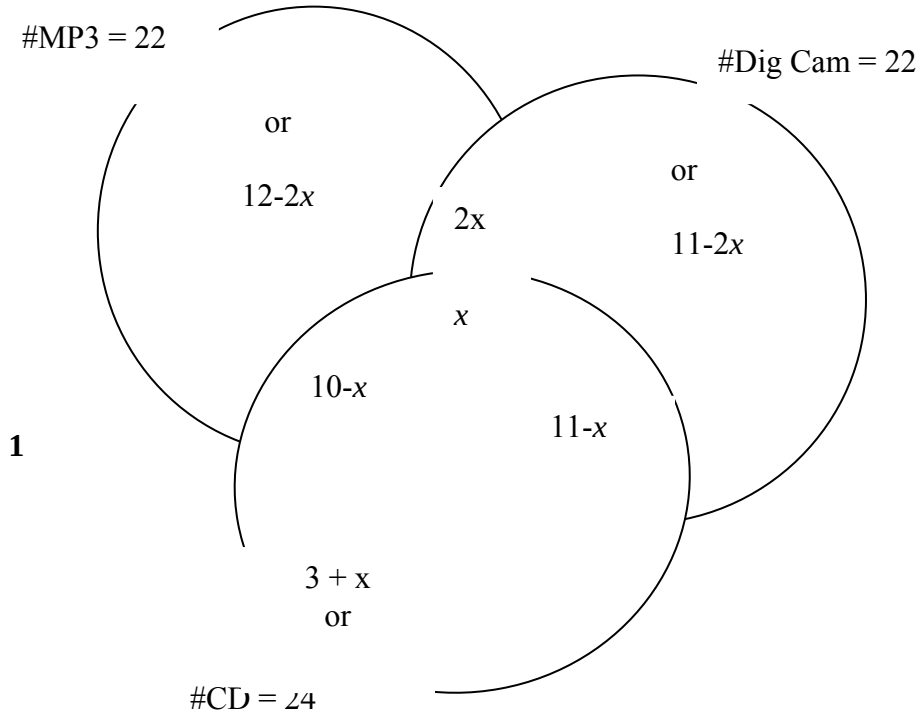
(c) A survey of 40 students was carried out to find how many owned an MP3 player, a digital camera or a CD player.
1 student does not own any of these.
x students own all three, while $2x$ own an MP3 player and a digital camera but not a CD player.
10 own an MP3 player and a CD player, while 11 own a digital camera and a CD player.
22 own an MP3 player, 22 own a digital camera and 24 own a CD player.

(i) ✍ Construct a Venn diagram and solve for x.

(ii) ✍ Hence, calculate the percentage of students who own one item only.

(c)(i) Venn Diagram **10 marks** **Att 3**

(i)U=40



1

* Follow candidates work Check #MP3, #Dig Cam and #CD

Slip (-1)

S1 Each missing or incorrect entry from the Venn diagram above; #U not required

Attempts (3 marks)

A1 Any single entry correct

A2 Draws 3 intersecting circles and stops

A3 No use of x in Venn diagram

(c)(i) **5 marks** **Att 2**

(i) ✍ Construct a Venn diagram and solve for x.

c(i) Finding x **5 marks** **Att 2**

Finding *x*

(i) $1 + 24 + 12 - 2x + 2x + 11 - 2x = 40$

$48 - 2x = 40$

$2x = 8$

$x = 4$

* Follow candidate's work

Blunders (-3)

B1 Correct answer but no work shown (✍)

Slips (-1)

S1 Numerical errors to a max of 3

S2 Each transposing error to a maximum of 3

S3 Each missing or incorrect entry from equation

Attempts (2 marks)

A1 Any correct entry in the equation e.g. has …..=40

Worthless (0)

W1 Incorrect answer and no work shown

c (ii)	**5 marks**	**Att 2**
(ii) ✍ Hence, calculate the percentage of students who own one item only		

c (ii)	**5 marks**	**Att 2**

(ii) $12-2x+11-2x+x+3=26-3x$ or $12-2x=12-8=4$

$x=4$ $11-2x=11-8=3$

$26-12=14$ $x+3=7$

$4+3+7=14$

% $\frac{14}{40}\times 100=35\%$

* Accept candidate's work from (i)

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Error in decimal point
B3 Mathematical error

Slips (-1)

S1 Numerical errors to a max of 3
S2 Fails to find percentage, stops at 14
S3 Finds % of incorrect figure or error in finding %

Misreading (-1)

M1 Finds % who own two items

Attempts (2 marks)

A1 Finds % who own three items

Worthless (0)

W1 Incorrect answer and no work shown

QUESTION 3

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 marks**	**Att 7**
Part (c)	**15 marks**	**Att 6**

Part (a) **15 marks** **Att 5**

(a) ✍ Solve $\frac{3-2m}{5} = 3$, where $m \in \mathbf{Z}$.

(a) **15 marks** **Att 5**

(a)

I

$$\frac{3-2m}{5} = 3$$
$$3-2m = 15$$
$$-2m = 15-3$$
$$-2m = 12$$
$$m = -6$$

or

II

$$\frac{3}{5} - \frac{2m}{5} = 3$$
$$-\frac{2m}{5} = 3 - \frac{3}{5}$$
$$-\frac{2m}{5} = 2\frac{2}{5} = \frac{12}{5}$$
$$m = \frac{12}{5} \times -\frac{5}{2}$$
$$m = -6$$

* Accept verification as work

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Mathematical error
B3 Sign error

Slips (-1)

S1 Numerical errors to a max of 3
S2 Answer not simplified or in required form
S3 Each transposing error to a maximum of 3

Attempts (5 marks)

A1 Trial and error but correct solution not found

Worthless (0)

W1 Incorrect answer and no work shown

(b) **20 (5,15) marks** **Att (2,5)**

(b) (i) **5 marks** **Att 2**

(i) ✍ Simplify

$$\frac{2x^2+4x-30}{x-3}.$$

(b) (i) **5 marks** **Att 2**

(i) I $\frac{2x^2+4x-30}{x-3}=\frac{2(x^2+2x-15)}{x-3}$ or $\frac{2(x+5)(x-3)}{x-3}$ or $2(x+5)$

II $\frac{2x^2+4x-30}{x-3}=\frac{(2x+10)(x-3)}{x-3}$ or $2x+10$

III
$$\begin{array}{r} 2x+10 \\ x-3\overline{)\,2x^2+4x-30} \\ \underline{2x^2-6x} \\ +10x-30 \\ \underline{10x-30} \\ 0 \quad 0 \end{array}$$

* Accept $2x+10$ or $2(x+5)$ with work

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Incorrect other factor of $2\boldsymbol{x}^2+4\boldsymbol{x}-30$

B3 Error when dividing.

Note: If uses formula for solving a quadratic equation, apply blunders (-3) as in part b(ii)

Misreadings (-1)

M1 $x^2 + 2x$ -15 and continues

Attempts (2 marks)

A1 Some effort at factorising

A2 Sets up division

A3 Multiplies numerator by denominator

A4 Any one entry correct

Worthless (0)

W1 Incorrect answer and no work shown

Part (b) (ii) **15 (5,10) marks** **Att (2,3)**

(ii) ✍ Solve $3x^2+9x+10=(2x+2)^2-1$ and give your answers correct to one decimal place.

Forming Quadratic **Solving Quadratic**

5 marks **Att 2** **10 marks** **Att 3**

(ii)

$3x^2+9x+10=(2x+2)^2-1$

$3x^2+9x+10=4x^2+8x+4-1$

$-x^2+x+7=0$ 5 marks

$x^2-x-7=0$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a=1 \quad b=-1 \quad c=-7$

$$\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-7)}}{2(1)}=\frac{1\pm\sqrt{1+28}}{2}$$

$$=\frac{1\pm\sqrt{29}}{2}=\frac{1\pm5\cdot385}{2}$$

$$x=\frac{1+5\cdot385}{2} \quad \text{or} \quad x=\frac{1-5\cdot385}{2}$$

$$x=\frac{6\cdot385}{2} \qquad x=\frac{-4\cdot385}{2}$$

$x=3\cdot19$ $\qquad$ $x=-2\cdot19$

$x=3\cdot2$ $\qquad$ $x=-2\cdot2$

Blunders (-3)

B1 Correct answer, but no work shown (✍)
B2 Error in squaring
B3 Error in substitution once only
B4 Error in quadratic formula once only
B5 Error when applying quadratic formula once only
B6 Correctly filled in formula and stops

Slips (-1)

S1 Numerical errors to a max of 3
S2 Fails to round off or rounds off incorrectly - once only
S3 Each transposing error to a maximum of 3

Misreadings (-1)

M1 Omits -1

Attempts (2 and/or 3 marks)

A1 Correct formula or identifies a, b or c correctly and stops
A2 Has simplified equation to linear and solves correctly for single value of x -max att 3

Worthless (0)

W1 Incorrect answer and no work shown

Part (c) **15(5, 5, 5) marks** **Att (2,2,2)**

(c) **(i)** ✍ Solve the equation $3a^2 + 5a = 2$.

(ii) ✍ Hence, or otherwise, find the two values of $t \in \mathbf{R}$ for which

$$3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$$

(iii) ✍ Verify your values for *t* from part **(ii)**, above.

c (i) **5 marks** **Att 2**

(i) ✍ Solve the equation $3a^2 + 5a = 2$.

(*i*)

$3a^2 + 5a = 2$

$3a^2 + 5a - 2 = 0$

$(3a-1)(a+2) = 0$

$3a - 1 = 0$ or $a + 2 = 0$

$a = \frac{1}{3}$ or $a = -2$

$3a^2 + 5a = 2$

$3a^2 + 5a - 2 = 0$

Factorising by Guide Number

$3a^2 + 5a - 2$ GN =-6

$3a^2 - a + 6a - 2$

$a(3a-1) + 2(3a-1)$

$(3a-1)(a+2)$

$(3a-1)(a+2) = 0$ as before

* If candidates uses formula apply slips and blunders as per b(ii)

Blunders (-3)

B1 Correct answer, but no work shown (✍)
B2 Mathematical errors
B3 Incorrect factors
B4 Incorrect roots from factors
B5 Only finds one solution

Slips (-1)

S1 Numerical errors to a max of 3
S2 Each transposing error to a maximum of 3

Attempts (2 marks)

A1 Correct formula or identifies a, b or c correctly and stops
A2 Linear equation and solves correctly for single value of *a* -max att 2
A3 Any attempt ay factorising

Worthless (0)

W1 Incorrect answer and no work shown

c(ii) **5 marks** **Att 2**

(ii) ✍ Hence, or otherwise, find the two values of $t \in \mathbf{R}$ for which

$$3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$$

c(ii) **5 marks** **Att 2**

(ii) **Method I**

$$3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$$

From c(i)

$$\frac{1}{t} \equiv a$$

$$\frac{1}{t} = \frac{1}{3} \quad \textbf{or} \quad \frac{1}{t} = -2$$

$$t = 3 \quad \textbf{or} \quad t = -\frac{1}{2}$$

Method II

$$3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$$

$$\frac{3}{t^2} + \frac{5}{t} - 2 = 0 \qquad \times t^2$$

$$3 + 5t - 2t^2 = 0$$

$$2t^2 - 5t - 3 = 0$$

$$(2t + 1)(t - 3) = 0$$

$$t = -\frac{1}{2} \quad \text{or} \quad t = 3$$

* Accept candidate's answers from part (i)

* If candidate uses Method II apply slips and blunders as per previous sections

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Mathematical /sign errors

B3 Only finds one solution. Note if part (i) linear possible to gain 2 marks

Slips (-1)

S1 Numerical errors to a max of 3

S2 Each transposing error to a maximum of 3

Attempts (2 marks)

A1 States $a = \frac{1}{t}$ and stops

Worthless (0)

W1 Incorrect answer and no work shown

c(iii) **5 marks** **Att 2**

(iii) ✍ Verify your values for t from part (ii), above. $3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$

c(iii) **5 marks** **Att 2**

(iii)

$$3\left(\frac{1}{t}\right)^2 + 5\left(\frac{1}{t}\right) = 2.$$

$t = 3$ $\qquad$ $t = -\frac{1}{2}$

$3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) = 2$ $\qquad$ $3\left(\frac{1}{-\frac{1}{2}}\right)^2 + 5\left(\frac{1}{-\frac{1}{2}}\right) = 2$

$2 = 2$ $\qquad$ $2 = 2$

* Candidates may use decimals
* Allow candidate's values from previous section, see B3

Blunders (-3)

B1 Correct answer, but no work shown (✍)
B2 Only tests one value
B3 Incorrect conclusion(s) for candidate's values

Attempts (2 marks)
A1 Substitutes for one and/or two values and stops

QUESTION 4

Part (a)	**15 marks**	**Att 5**
Part (b)	**25 marks**	**Att 9**
Part (c)	**10 marks**	**Att 4**

Part (a) **15 marks** **Att 5**

(a) ✍ When $x = \frac{1}{3}$, find the value of $\frac{3}{x+1} + \frac{4}{x+5}$.

(a) **15 marks** **Att 5**

I

$$\frac{3}{x+1} + \frac{4}{x+5}$$

$$= \frac{3}{\frac{1}{3}+1} + \frac{4}{\frac{1}{3}+5}$$

$$= \frac{3}{\frac{4}{3}} + \frac{4}{\frac{16}{3}}$$

$$= \frac{9}{4} + \frac{12}{16}$$

$$= \frac{48}{16}$$

$$= 3$$

II

$$\frac{3}{x+1} + \frac{4}{x+5}$$

$$\frac{1}{3} = 0\cdot333 = 0\cdot3$$

$$\frac{3}{0\cdot333+1} + \frac{4}{0\cdot333+5}$$

$$= \frac{3}{1\cdot333} + \frac{4}{5\cdot333}$$

$$= 2\cdot250 + 0\cdot7500$$

$$= 3\cdot00$$

III

$$\frac{3}{x+1} + \frac{4}{x+5}$$

$$= \frac{3(x+5)+4(x+1)}{(x+1)(x+5)}$$

$$= \frac{7x+19}{(x+1)(x+5)}$$

$$= \frac{7(\frac{1}{3})+19}{(\frac{1}{3}+1)(\frac{1}{3}+5)}$$

$$= \frac{\frac{64}{3}}{\frac{64}{9}}$$

$$= \frac{9}{3}$$

$$= 3$$

Blunders (-3)

B1 Correct answer but no work shown (✍)
B2 Each different error when working with fractions
B3 Decimal error each time - Method II
B4 Each different algebraic error - Method III
B5 Substitutes into expression and stops. Max loss after substitution is 3 marks

Slips (-1)

S1 Numerical errors to a max of 3

Attempts (3 marks)

A1 Finds Common Denominator and stops
A2 Equation $\frac{3}{x+1} + \frac{4}{x+5} = \frac{1}{3}$ plus some correct step

Worthless (0)

W1 Incorrect answer and no work shown

Part (b)	**(20,5) marks**	**Att (7,2)**
b(i)	**20 marks**	**Att 7**

(i) ✍ Factorise $6c+12bd-8d-9bc$.

b(i)	**20 marks**	**Att 7**

(b) (i)

$$6c+12bd-8d-9bc$$

$=6c-9bc-8d+12bd$ or $=6c-8d-9bc+12bd$

$=3c(2-3b)-4d(2-3b)$ $=2(3c-4d)-3b(3c-4d)$

$=(2-3b)(3c-4d)$ $=(3c-4d)(2-3b)$

* Candidates may offer other correct versions

Blunders (-3)

B1 Correct answer, but no work shown (✍).
B2 Error in sign when factorising and/or regrouping
B3 Stops at $3c(2-3b)-4d(2-3b)$ or similar
B4 Answer givens as *(2-3b)+(3c-4d)* but *(2-3b)* and *(3c-4d)*merits full marks
B5 Error in factors *3c(2-3b)+4d(3b-2)* given as *(2-3b)(3c+4d)* or similar

Attempts (7 marks)

A1 Any partial factorising and stops e.g. *6(c+2d)*-8d-9bc
A2 Any partial re-grouping and stops

Worthless (0)

W1 Incorrect answer and no work shown

b(ii) **5 marks** **Att 2**

(ii) ✍ Simplify
$(7x-2)(7x+2) - (5y-2)(5y+2)$
and fully factorise the simplified expression.

b(ii) **5 marks** **Att 2**

(ii)

$$(7x - 2)(7x + 2) - (5y - 2)(5y + 2)$$
$$= 49x^2 - 4 - [25y^2 - 4]$$
$$= 49x^2 - 4 - 25y^2 + 4$$
$$= 49x^2 - 25y^2$$
$$= (7x - 5y)(7x + 5y)$$

Blunders (-3)

B1 Correct answer, but no work shown (✍).
B2 Distribution error
B3 Sign error

Slips (-1)

S1 Numerical errors to a max of 3
S2 Does not factorise

Attempts (2 marks)

A1 Any effort at multiplying out the brackets

Worthless (0)

W1 Incorrect answer and no work shown

Part (c) **10 (5, 5) marks** **Att 4 (2,2)**

(c) The distance from town A to town B is half the distance from town B to town C. The total journey from town A to town C, through town B, is 60 km.
A car travels at x km/h from town A to town B. It increases its speed by 20 km/h on the journey from town B to town C.
The total time for the journey is 50 minutes.

✍ Find the value of x.

A ⟷ B ⟷ C

D 2D

3D=60

D=20

A to B =20 km B to C = 40 km I

Table

	A→B	B→C	Marks
D	20	40	5 **I**
S	x	x+20	***or*** 5 **II**
T	$\frac{20}{x}$	$\frac{40}{x+20}$	

Time from A to B = $\frac{20}{x}$

Time from B to C $\frac{40}{x+20}$

$$\frac{20}{x}+\frac{40}{x+20}=\frac{50}{60}=\frac{5}{6}$$

$$20(6)(x+20)+40(6)(x)=5(x)(x+20)$$

$$120x+2400+240x=5x^2+100x$$

$$5x^2-260x-2400=0$$

$$x^2-52x-480=0$$ III

$$(x-60)(x+8)=0$$ 5 Marks

$$x=60 \text{ km/h}$$

***** Note Correct Distances (20km and 40km) I **or** Speeds (x and x+20) II merits 5 marks
Final 5 marks for finishing

Part I

Blunders (-3)

B1 Mathematical error

Slips (-1)

S1 Reverses distances - answer is a decimal in III

Attempts (2 marks)

A1 Draws a diagram and stops

A2 States ratios and stops

Finishing **5 marks** **Att 2**

A ⟷ B ⟷ C

D 2D

3D=60

D=20

A to B =20 km B to C = 40 km I

Table

	A→B	B→C	Marks
D	20	40	5 **I**
S	x	x+20	***or*** 5 **II**
T	$\frac{20}{x}$	$\frac{40}{x+20}$	

Time from A to B = $\frac{20}{x}$

Time from B to C $\frac{40}{x+20}$

$$\frac{20}{x}+\frac{40}{x+20}=\frac{50}{60}=\frac{5}{6}$$

$$20(6)(x+20)+40(6)(x)=5(x)(x+20)$$

$$120x+2400+240x=5x^2+100x$$

$$5x^2-260x-2400=0$$

$$x^2-52x-480=0$$

III

$$(x-60)(x+8)=0$$

5 Marks

$x=60$ km/h

Blunders (-3)

B1 Correct answer, but no work shown (✍). (III)
B2 Incorrect S/D/T relationship once only
B3 Subtracts instead of adding 20km/h (II)
B4 Sign error in setting up equation
B5 Error when solving equation

Slips (-1)
S1 Numerical errors to a max of 3

Misreadings (-1)
M1 Expression not equal to 50/60 - conversion error

Attempts (*2* and/ or *2*)
A1 S/D/T relationship and stops -*2 marks* (II)
A2 x+20 and stops -*2 marks* (II) Note work at II can only merit A1 **or** A2
A3 Any effort at forming an equation (III)

QUESTION 5

Part (a)	**5 marks**	**Att 2**
Part (b)	**25 marks**	**Att 9**
Part (c)	**20 marks**	**Att 7**

Part (a) **5 marks** **Att 2**

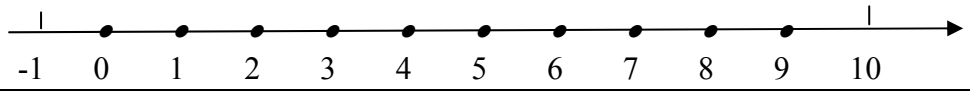
(a) ✍ Graph on the number line the solution set of

$$-98 \leq 10-12x,\ x \in \mathbf{N}.$$

(a) **5 marks** **Att 2**

$-98 \leq 10-12x$
$-98-10 \leq -12x$
$-108 \leq -12x$
$9 \geq x$

$-98 \leq 10-12x$
$12x \leq 10+98$
$x \leq 9$



* Only required to show 0 to 9 of number line

Blunders (-3)

B1 Correct answer but no work shown (✍)

Slips (-1)

S1 Numerical errors to a max of 3
S2 Each transposing error to a maximum of 3
S3 Solves and/or lists but does not graph on number line

Attempts (2 marks)

A1 Tests any value in the inequality and stops
A2 Draws a number line and stops

Worthless (0)

W1 Incorrect answer and no work shown

(b) **25 (20,5) marks** **Att (7,2)**

(b) **(i)** Let f be the function $f: x \to 2x^2 - 4x + 5$.

✍ Draw the graph of f for $-2 \le x \le 4,\ x \in \mathbf{R}$.

(ii) ✍ Use your graph to find the values of x for which $f(x) = 7$.

b(i) **20 marks** **Att 7**

f(x) =2x^2 -4x +5

f(-2) =2(-2)^2 -4(-2) +5 =8+8+5=21

f(-1) =2(-1)^2 -4(-1) +5 =2+4+5=11

f(0) =2(0)^2 -4(0) +5 = 0+0+5=5

f(1) =2(1)^2 -4(1) +5 = 2 - 4 + 5 = 3

f(2) =2(2)^2 -4(2) +5 = 8 - 8 + 5 = 5

f(3) =2(3)^2 -4(3) +5 = 18 - 12 +5 = 11

f(4) =2(4)^2 -4(4) +5 = 32 - 16 +5 = 21

x	*-2*	*-1*	*0*	*1*	*2*	*3*	*4*	
2x^2	*8*	*2*	*0*	*2*	*8*	*18*	*32*	
-4x	*8*	*4*	*0*	*-4*	*-8*	*-12*	*-16*	
+5	*+5*	*+5*	*+5*	*+5*	*+5*	*+5*	*+5*	
f(x)	*21*	*11*	*5*	*3*	*5*	*11*	*21*	

Values for quadratic graph

Blunders (-3)

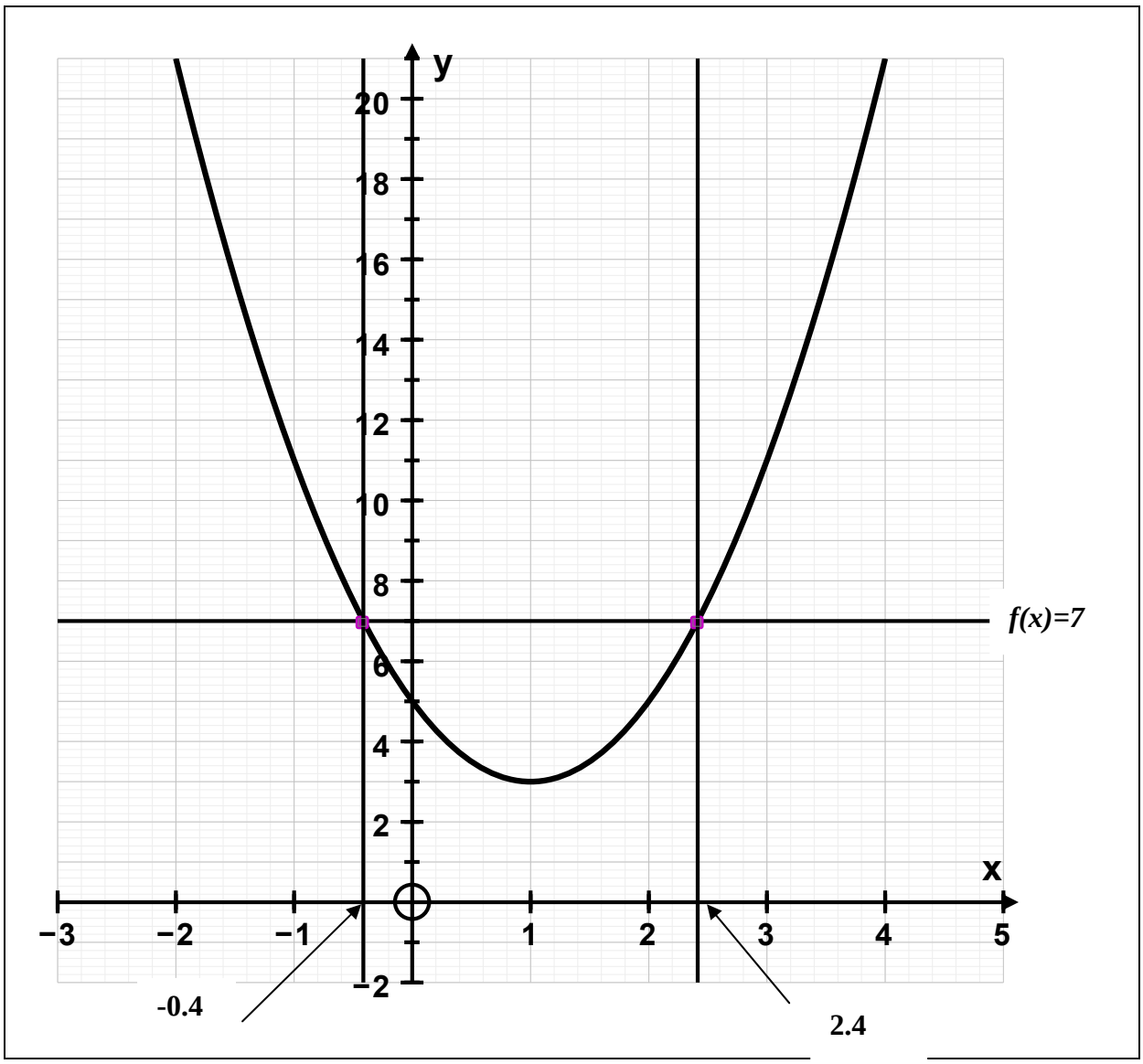
B1 Each incorrect $f(x)$ without work.
B2 x row added in, i.e. top row, or adds in extra row.
B3 Treating the domain as $-2 < x < 4$, can incur 2 Blunders if both omitted.
B4 Each different blunder which yields an incorrect row (full or part),
B5 Avoids square for some (not all) values. See Attempts below
B6 Mathematical errors in tots, - apply once only.

Slips (-1)

S1 Numerical errors to a max. of 3.

Attempts (7 marks)

A1 Omits $2x^2$ or does not treat as $2x^2$, - linear expression
A2 Correct or partly correct table / values, but no graph drawn.



Plotting the quadratic graph

* Accept candidate's values from his/her table.

Note If no values worked out, points on graph must be within tolerance, $\pm 0 \cdot 3$, of where the graph should be, otherwise B4 each time

Blunders (-3)

B1 Points not joined to form a reasonable graph or 'flat bottom'.

B2 (x, y) plotted as (y, x), but apply once only, or reverses axes.

B3 Scale not reasonably uniform once only

B4 Blunder in plotting points from candidate's table / values.

B5 Each point omitted if graph does not go reasonably close to where point should be

B6 Points joined with straight lines.

Attempts (7 marks)

A1 Scaled axes drawn graded but not labelled

(b) (ii) **5 marks** **Att 2**

✍ Use your graph to find the values of x for which $f(x) = 7$.

b(ii) **5 marks** **Att 2**

(ii) See graph

$x = -0{\cdot}4$ and $x = 2{\cdot}4$

*Accept answer consistent with candidates curve tolerance $\pm 0{\cdot}3$

Blunders (-3)

B1 No indication on graph, each value

B2 Indication on graph but no value given or value outside tolerance, each time

B3 Only finds one value

Attempts (2 marks)

A1 Correctly solves *f(x)* =7 by formula - graph not used

A2 *f(7)* found

Worthless (0)

W1 Incorrect answer and no work shown

Part (c) **20 (10, 5 , 5) marks** **Att (3,2,2)**

(c) **(i)** Let f be the function $f : x \to 2x - 1$ and g be the function $g : x \to 4x - 4$.

✍ Using the same axes and scales, draw the graph of f and the graph of g, for $0 \le x \le 2$, $x \in \mathbf{R}$.

(ii) From your graphs, write down the co-ordinates of the point of intersection of the two lines.

(iii) ✍ Check your answer to part **(ii)** by solving the simultaneous equations

$y = 2x - 1$

$y = 4x - 4$.

(c) (i) **10 marks** **Att 3**

$f : x \to 2x - 1$ **$g : x \to 4x - 4$.**

f(0)= 2(0)-1=-1 (0,-1) *g(0)=4(0)-4=-4 (0.-4)*

f(1)= 2(1)-1=1 (1,1) *g(1)=4(1)-4=0 (1,0)*

f(2)= 2(2)-1=3 (2,3) *g(2)=4(2)-4=4 (2,4)*

x	*0*	*1*	*2*		*x*	*0*	*1*	*2*
2x	*0*	*2*	*4*		*4x*	*0*	*4*	*8*
-1	*-1*	*-1*	*-1*		*-4*	*-4*	*-4*	*-4*
f(x)	*-1*	*1*	*3*		*g(x)*	*-4*	*0*	*4*
Point	*(0,-1)*	*(1,1)*	*(2,3)*			*(0,-4)*	*(1,0)*	*(2,4)*

* Only two points needed for each function but must have lines within domain

Values and plotting for linear graphs

Blunders (-3)

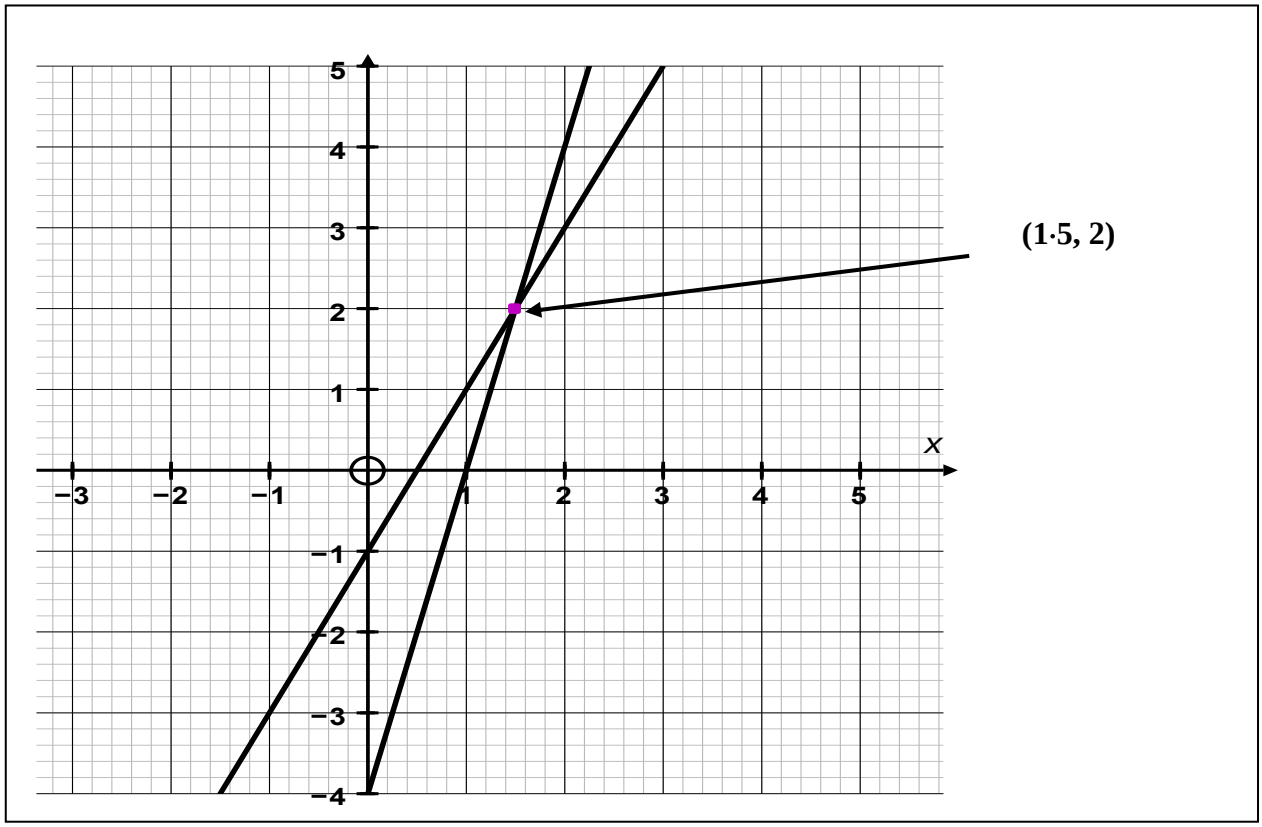
B1 Each incorrect value without work -once per line
B2 *x* row added in, or adds in extra row - once if consistent
B3 Mathematical error in calculation - once if consistent
B4 Points not joined to form lines - once only
B5 (x, y) plotted as (y, x), but apply once only, or reverses axes.
B6 Scale not reasonably uniform once only
B7 Each different blunder in plotting points from candidate's table / values.
B8 Only considers one function
B9 Lines not extended to include full domain, once only

Misreading (-1)

M1 Uses separate graphs for *f(x)* and *g(x)*

Attempts (3 marks)

A1 Correct or partially correct tables/values but no graph drawn
A2 Scaled axis drawn (for this part)



(c) (ii) **5 marks** **Att 2**

(ii) From your graphs, write down the co-ordinates of the point of intersection of the two lines.

(ii)	Point of intersection = **(**1·5, 2**)** tolerance ±0·3

* Follow candidates work from (i)

Blunders (-3)

B1 Reverses order of co-ordinates

B2 Indicates on graph only but does not name

(c)(iii) **5 marks** **Att 2**

✍ Check your answer to part **(ii)** by solving the simultaneous equations

$y = 2x - 1$

$y = 4x - 4.$

(c) (iii) 5 marks Att 2

c(iii)

$y = 2x - 1$

$y = 4x - 4$

I

$y = 2x - 1$

$y = 4x - 4$

$4x - 4 = 2x - 1$

$2x = 3$

$x = 1 \cdot 5$

$y = 2(1 \cdot 5) - 1$

$y = 3 - 1$

$y = 2$

II

$y = 2x - 1$ x -1

$y = 4x - 4$

$-y = -2x + 1$

$y = 4x - 4$

$0 = 2x - 3$

$x = 1 \cdot 5$

$y = 2(1 \cdot 5) - 1$

$y = 3 - 1$

$y = 2$

* Candidates may use other correct algebraic versions

Blunders (-3)

B1 Sign error

B2 Mathematical error

B3 Only finds one value

Slips (-1)

S1 Numerical errors to a max. of 3.

S2 Each transposing error to a maximum of 3

Attempts (2 marks)

A1 Sets up equations and stops e.g. multiplies an equation by -1 or similar

A2 Correctly substitutes their values into both equations and stops

QUESTION 6

Part (a)	**20 marks**	**Att 7**
Part (b)	**10 marks**	**Att 4**
Part (c)	**20 marks**	**Att 7**

Part (a) **20 marks** **Att 7**

(a) ✍ Given that $f: x \to 3x + 1$ and $g: x \to 1 + x^2$,
solve for x: $f(x) = g(x)$, $x \in \mathbf{N}$.

(a) **20 marks** **Att 7**

(a)

$f(x) = g(x)$

$3x + 1 = 1 + x^2$

$3x = x^2 \quad \Rightarrow \quad x = 0 \text{ or } x = 3$

or

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0 \quad x = 3$

Formula

$f(x) = g(x)$

$3x + 1 = 1 + x^2$

$x^2 - 3x = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1 \quad b = -3 \quad c = 0$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(0)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9 - 0}}{2}$

$x = \frac{3+3}{2}$ or $x = \frac{3-3}{2}$

$x = 3$ or $x = 0$

Blunders (-3)

B1 Correct answer, but no work shown (✍)
B2 Mathematical errors
B3 Incorrect factors
B4 Incorrect roots from factors
B5 Only finds one solution
B6 Error in quadratic formula
B7 Error when applying quadratic formula
B8 Correct equation formed but fails to find roots

Slips (-1)

S1 Numerical errors to a max of 3
S2 Each transposing error to a maximum of 3

Attempts (7 marks)

A1 Solves *f(x)* and/ or *g(x)* =0
A2 Correct formula or identifies a, b or c correctly and stops
A3 Graphical unless both solutions tested in both equations -merits full marks

Worthless (0)

W1 Incorrect answer and no work shown

Part (b) **10 (5,5) marks** **Att (2,2)**

(b) **(i)** ✍ Given that $x = 2a + 1$ and $y = 2ax - 4a^2$, express y in terms of a.

(ii) ✍ Hence, or otherwise, find the value of x for which $y = 4$.

Part (b) (i) **5 marks** **Att 2**

(i) $x = 2a + 1$ $y = 2ax - 4a^2$

$y = 2a(2a + 1) - 4a^2$

Blunder (-3)

B1 Correct answer, but no work shown (✍)
B2 Each incorrect substitution but see M1

Misreadings (-1)

M1 Expresses y in terms of x Answer $y = x$-1 (Allow in part (ii))

Attempts (2 marks)

A1 Substitutes a numerical value for x e.g. 0

Worthless (0)

W1 Incorrect answer and no work shown

Part (b) (ii) **5 marks** **Att 2**

(b)(ii) ✍ Hence, or otherwise, find the value of x for which $y = 4$.

b (ii) **5 marks** **Att 2**

$x = 2a + 1$ **I** $y = 2ax - 4a^2$ **II**

$y = 2a(2a + 1) - 4a^2$ (i)

$y = 4a^2 + 2a - 4a^2$

$y = 2a$

$y = 2a$

$y = 4$

$x = 4+1 = 5$

or

$y = 2a$

$4 = 2a$

$2 = a$

$x = 2(2) + 1 = 5$

* Errors in simplification of (i) are applied here

Blunder (-3)

B1 Correct answer, but no work shown (✍)
B2 Mathematical errors e.g distribution
B3 Sign error
B4 Finds value of a and stops

Attempts (2 marks)

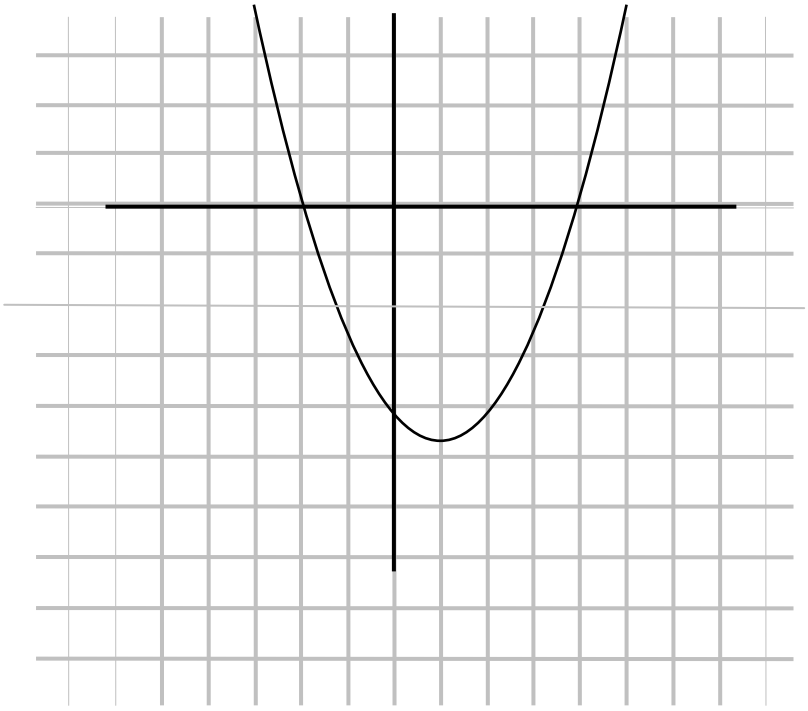
A1 Subs $y = 4$ in II and stops

Worthless (0)

W1 Incorrect answer and no work shown

(c) The diagram shows part of the graph of the function

$$f : x \to x^2 + bx + c, \text{ where } x \in \mathbf{R} \text{ and } b, c \in \mathbf{Z}.$$



The graph intersects the x-axis at (-1, 0) and (2, 0).

(i) ✍ Calculate the value of b and the value of c.

(ii) ✍ $(k, -k+14)$ is a point on the graph, where $k \in \mathbf{Z}$.
Find the values of k.

(c)(i) **15 marks** **Att 5**

The graph intersects the *x*-axis at (-1, 0) and (2, 0).

(i) ✍ Calculate the value of *b* and the value of *c*.

(c) (i) **15 marks** **Att 5**

(c) (i)

$$f(x) = x^2 + bx + c$$

$$f(-1) = (-1)^2 + b(-1) + c = 0 \quad \text{Eq 1}$$

$$1 - b + c = 0$$

$$-b + c = -1$$

$$f(2) = (2)^2 + b(2) + c = 0 \quad \text{Eq 2}$$

$$4 + 2b + c = 0$$

$$2b + c = -4$$

$$-b + c = -1 \qquad \times -1$$

$$b - c = 1$$

$$\underline{2b + c = -4}$$

$$3b = -3$$

$$b = -1$$

$$2(-1) + c = -4$$

$$-2 + c = -4$$

$$c = -2$$

* May read *c = -2* off graph and then finds *b,* accept, but see 'Note' in S2

Blunder (-3)

B1 Correct answer, but no work shown (✍)
B2 Mathematical error.
B3 Forms correct simultaneous equations and stops - max loss in solving 3 marks
B4 Incorrect factors from roots
B5 Incorrect rule for 'Sum ' and 'Product' once if consistent
B6 Takes an incorrect value of *c* and continues to find a value of *b*

Slips (-1)

S1 Numerical errors to a max of 3
S2 Only finds one value (Note reads *c* off graph as -2 and stops, apply B1 also)
S3 Finds correct quadratic but does not identify *b* and/or *c*

Attempts (5 marks)

A1 Substitutes one value into *f(x)* and stops $\neq 0$
A2 States 'Sum'/ and or 'Product' rule and stops
A3 Correctly marks in both points on a graph

Worthless (0)

W1 Incorrect answer and no work shown

(c) (ii) **5 marks** **Att 2**

(ii) ✍ $(k, -k+14)$ is a point on the graph, where $k \in Z$.
Find the values of k.

c(ii) **5 marks** **Att 2**

(ii) $f(x)=x^2 - x - 2$

$f(k)=k^2 - k - 2 = -k+14$

$k^2 - k - 2 = -k+14$

$k^2 = 16$ *or* $k^2 - 16 = 0$ or Formula

$k = 4$ *or* $k = -4$ $(k-4)(k+4) = 0$

$k = 4$ *or* $k = -4$

* Accept candidate's answer from (ii)

Blunder (-3)

B1 Correct answer, but no work shown (✍)
B2 Mathematical error
B3 Incorrect roots from factors
B4 Incorrect substitution

Slips (-1)

S1 Numerical errors to a max of 3
S2 Only finds one value for k

Attempts (2 marks)

A1 Substitutes k into $f(x)$ and stops $f(x) = k^2 + b(k) + c \neq -k+14$
A2 Carries down from (i) and stops

Worthless (0)

W1 Incorrect answer and no work shown