# Coimisiún na Scrúduithe Stáit State Examinations Commission 

JUNIOR CERTIFICATE EXAMINATION 2006<br>MATHEMATICS - HIGHER LEVEL - PAPER 2<br>MARKING SCHEME

## GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions
- Slips- numerical errors
- Misreadings (provided task is not oversimplified)

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.
2. When awarding attempt marks, e.g. Att(3), note that

- any correct, relevant step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase "hit or miss" means that partial marks are not awarded - the candidate receives all of the relevant marks or none.
5. The phrase "and stops" means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists - there may be other correct solutions.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts even when attempts have been cancelled.
9. The same error in the same section of a question is penalised once only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. $€ 5.50$ may be written as $€ 5,50$.

## QUESTION 1

| Part (a) | 10 marks | Att 3 |
| :--- | :---: | ---: |
| Part (b) | $20(5,5,10)$ marks | Att $2,2,3$ |
| Part (c) | $20(10,10)$ marks | Att 3,3 |

Part (a)
10 marks
Att 3
The height and the diameter of a solid cylinder are both 9 cm in length.
Find the volume of the cylinder correct to one decimal place.
(a)

10 marks
Att 3
(a) Volume of cylinder $=\pi \mathrm{r}^{2} \mathrm{~h}=\pi(4.5)^{2} .9=182.25 \pi \approx 572 \cdot 6 \mathrm{~cm}^{3}$

Blunders (-3)
B1 Correct answer with no work shown (hand)
B2 Incorrect substitution into correct formula
B3 Incorrect squaring
B4 Incorrect relevant volume formula or inappropriate value of $\pi$
B5 Answer in terms of $\pi$
Slips (-1)
S1 Arithmetic slips
S2 Answer not in required form
Attempts (3)
A1 Correct formula with some substitution
A2 Correct radius
Worthless (0)
W1 Surface area(s)
(i) The perimeter of a square lawn is 96 m .

R Find the area of the lawn in $\mathrm{m}^{2}$.
(ii) A garden roller, in the shape of a cylinder, has a diameter of 75 cm and is 1 m wide as shown in the diagram.

Calculate the curved surface area of the roller in $\mathrm{m}^{2}$ correct to one decimal place.

(iii) What percentage of the lawn will be rolled when the roller has completed 9 revolutions?
(b) (i)

5 marks
Att 2
(i) Length of side: $\frac{96}{4}=24 \mathrm{~m}$

$$
\text { Area of lawn } \quad=\quad 24^{2}=576 \mathrm{~m}^{2} .
$$

## Blunders (-3)

B1 Correct answer with no work shown (hand)
B2 Incorrect relevant area formula
B3 Perimeter divided by 4 neither indicated nor implied
B4 Incorrect squaring
Slips (-1)
S1 Arithmetic slips
Attempts (2)
A1 Length of one side of square only
A2 Correct area formula but no substitution
(ii) Curved Surface Area $=2 \pi \times(0 \cdot 375) \times(1)=2 \cdot 356 \approx 2 \cdot 4 \mathrm{~m}^{2}$.

## Blunders (-3)

B1 Correct answer with no work shown (hand)
B2 Incorrect substitution into correct formula
B3 Incorrect relevant area formula
B4 Measurements not converted to same units
B5 Answer in terms of $\pi$ or inappropriate value of $\pi$
Slips (-1)
S1 Arithmetic errors
S2 Answer not in required form
S3 Incorrect round off/ failure to round off
Attempts (2)
A1 Correct radius indicated
A2 $1 \mathrm{~m}=100 \mathrm{~cm}$ and/or $75 \mathrm{~cm}=.75 \mathrm{~m}$
Worthless (0)
W1 Volume of cylinder

## (b) (iii)

10 marks
Att 3
(iii) $\%$ completed $=\frac{9 \times 2 \cdot 4}{576} \times 100=3 \cdot 75 \%$

* $\quad$ Accept candidates answers from (i) and (ii) for (iii)

Blunders (-3)
B1 Correct answer with no work shown (hand)
B2 Calculations for one revolution only
B3 Calculations inverted
B4 Fraction calculated as a decimal, but percentage not found
Slips (-1)
S1 Arithmetic errors
Attempts (3)
A1 CSA x 9
(c) An egg-timer consists of two identical cones of height 6 cm and base radius 4 cm . Sand occupies half the volume of one cone and flows from one to the other at a rate of $\frac{4 \pi}{45} \mathrm{~cm}^{3}$ per second.
(i) Calculate the volume of each cone in terms of $\pi$.
(ii) Calculate the length of time it takes for the sand to flow from one cone into the other.

(c) (i)
(i) Volume each cone $=\frac{1}{3} \pi \mathrm{r}^{2} \mathrm{~h}=\frac{1}{3} \pi 4^{2} 6=32 \pi \mathrm{~cm}^{3}$

## Blunders (-3)

B1 Correct answer with no work shown (hand)
B2 Incorrect substitution into correct formula
B3 Incorrect relevant formula
B4 Incorrect squaring

## Misreading(-1)

M1 $\mathrm{h}=3$
Slips (-1)
S1 Arithmetic slips
Attempts (3)
A1 Correct formula with some substitution
A2 Diagram with r and/ or h shown correctly
(c) (ii)
(ii) $\quad$ Time $=\frac{16 \pi}{4 \pi}=180 \mathrm{~s}$

Blunders (-3)
B1 Correct answer with no work shown (hand)
B2: Ratio not simplified
B3 Ratio inverted
B4 Use of $32 \pi$ or answer not halved from ( c)(i)
Slips (-1)
S1 Arithmetic slips
Attempts (3)
A1 Identifies half the volume of one cone

## QUESTION 2

| Part (a) | 10 marks | Att 3 |
| :---: | :---: | :---: |
| Part (b) | 25 (5,5,5,10) marks | Att( 2,2,2,3) |
| Part (c) | 15 marks | Att 5 |
| Part (a) | 10 marks | Att 3 |
| (a) $\quad a(4,-6)$ and $b(6,-2)$ are two points. <br> Write $\|a b\|$ in surd form. |  |  |

(a) 10 marks

Att 3

$$
\begin{aligned}
& |a b|=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(6-4)^{2}+(-2--6)^{2}} \\
& \sqrt{2^{2}+4^{2}} \\
& =\sqrt{4+16} \sqrt{20}
\end{aligned}
$$

Blunders (-3)
B1 Correct answer with no work shown (hand)
B2 Incorrect relevant formula
B3 Mixes both x and y in substitution
B4 Squares incorrectly
Slips (-1)
S1 Arithmetic slips
S2 One incorrect substitution into x or y
Attempts (3)
A1 Correct formula and stops
A2 Some attempt at difference of $y$ 's and /or difference of $x$ 's
$p(-1,2)$ and $r(3,4)$ are two points.
(i) Find $m$, the midpoint of [ $p r$ ].
(ii) Find the slope of $p r$.
(iii) Find the equation of the line $L$, the perpendicular bisector of [pr].
(iv) The equation of the line $K$ is $x-2 y=0$.

Find $n$, the point of intersection of $L$ and $K$.
(b) (i) 5 marks

Att 2
(i) $m=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)=\left(\frac{-1+3}{2}, \frac{2+4}{2}\right)=\left(\frac{2}{2}, \frac{6}{2}\right)=(1,3)$

## Blunders (-3)

B1 Correct answer with no work shown (2)
B2 Incorrect midpoint formula and continues
B3 Mixes both $x$ and $y$ in substitution
B4 Substitutes correctly but midpoint not found
Slips (-1)
S1 One incorrect sign after substitution
S2 One incorrect substitution
S3 Arithmetic errors
Attempts (2)
A1 Writes midpoint formula with or without some substitution
A2 Correct graphical solution
(b) (ii)

5 marks
Att 2
(ii) slope of $p r=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{4-2}{3--1}=\frac{2}{4}$ or $\frac{1}{2}$

## Blunders (-3)

B1 Correct answer with no work shown (ك)
B2 Incorrect slope formula and continues
B3 Mixes both $x$ and $y$ in substitution
B4 Substitutes correctly but slope not found
Slips (-1)
S1 One incorrect sign after substitution
S2 One incorrect substitution
S3 Arithmetic errors
Attempts (2)
A1 Writes slope formula with or without some substitution
A2 Some attempt at difference of $y$ 's and /or difference of $x$ 's
(b) (iii)
(iii) Equation of $L \quad y-3=-2(x-1)$

* Accept candidates answers from b(i) and b(ii)


## Blunders (-3)

B1 Correct answer with no work shown (2)
B2 Incorrect relevant formula and continues
B3 Switches both x and y in substitution
B4 Substitutes correctly for x and y but no slope
B5 Use of incorrect point and/or incorrect slope
Slips (-1)
S1 Arithmetic errors
S2 One incorrect substitution of $x$ or $y$
Attempts (2)
A1 Correct line formula
(b) (iv)
(iv) $y-3=-2 x+2 \Rightarrow 2 x+y=5$
$2 x+y=5 \Rightarrow 4 x+2 y=10$
$x-2 y=0$
$5 x \quad=10 \Rightarrow x=2$
$2-2 y=0 \Rightarrow y=1$

* $(2,1)$ without work Att 3 subject to below
* $\quad$ Accept $(2,1) \in \mathrm{L}$ and $(2,1) \in \mathrm{K}$ shown in each case.


## Blunders (-3)

B1 Error in manipulation of equations
B2 Transposition error
B3 No substitution for second value
Slips (-1)
S1 Arithmetic slips

Attempts (3)
A1 Any correct step and stops
A2 Graphical solution correct
(c)

15 marks
Att 5


Given: $\square$ $a b c d$
To prove: (i) $|<b a d|=|<b c d|$ and $|<a d c|=|<a b c|$
(ii) $|a b|=|d c|$ and $|a d|=|b c|$

Construction: Join ac
Step 1
Step2
Proof: Taking triangles $a d c$ and $a b c$
$|<d a c|=|<a c b|$ (alternate angles since $a d$ parallel to $b c$ )
$|<d c a|=|<b a c|$ (alternate angles since $a b$ parallel to $d c$ ) ac common to both triangles
$\Rightarrow$ ASA $\Rightarrow$ Both triangles are congruent
Step 4
$\Rightarrow$ (i) $|a b|=|d c|$ and $|a d|=|b c| \quad$ corresponding sides
(ii) $|<d a c|+|<b a c|=|<a c b|+|<d c a|$
$\Rightarrow|<b a d|=|<b c d|$. Similarly $|\angle a d c|=|\angle a b c|$
Step 5
or $\quad|\angle a d c|=|\angle a b c|$. Similarly $|<b a d|=|<b c d|$
Some steps may be partially indicated on diagram

## Blunders (-3)

B1 Each step incorrect or omitted
B2 Each step incomplete
Attempts (5)
A1 Diagram with parallelogram drawn, and diagonal indicated
Worthless (0)
W1 Wrong Theorem
W2 Parallelogram and nothing else
Part (a) $10(5,5)$ marks
(a) The triangle $a b c$ has $|a b|=|a c|$.

The line $m n$ is parallel to $b c$ and $|\angle n m b|=115^{\circ}$
\& Find $|\angle a b c|$ and $|\angle b a c|$.

(a)
(a) $|\angle n m a|=180^{\circ}-115^{\circ}=65^{\circ}$

$$
|\angle a b c|=|\angle n m a|=65^{\circ} \text { or }|\angle a b c|+|\angle n m b|=180^{\circ} \Rightarrow|\angle a b c|=65^{\circ}
$$

## Blunders (-3)

B1 Correct answer with no work shown (S)
B2 Use of $360^{\circ}$ instead of $180^{\circ}$
Slips (-1)
S1 Arithmetic slip
Attempts (2)
A1 Indicates $|\angle n m b|+|\angle n m a|=180^{\circ}$
A2 $115^{\circ}$ correctly marked in diagram
Worthless (0)
W1 Use of $90^{\circ}$ instead of $180^{\circ}$
(a) (ii)

$$
\begin{aligned}
& |a b|=|a c| \Rightarrow|\angle a b c|=|\angle a c b| \\
& |\angle b a c|=180^{\circ}-\left(65^{\circ}+65^{\circ}\right)=50^{\circ}
\end{aligned}
$$

* Accept answer from above for this section

Blunders (-3)
B1 Correct answer with no work shown (S)
B2 Sum of angles in triangle $\neq 180^{\circ}$
Slips (-1)
S1 Arithmetic slip
Worthless(0)
W1 Assuming $\Delta a b c$ right angled triangle
(i) Prove that an exterior angle of a triangle equals the sum of the two interior opposite angles in measure.
(ii) Calculate the value of $x$ and the value of $y$ in the diagram.

(b)(i)

10 marks
Att 3


Given: Triangle with angles A, B, C. and one side extended with exterior angle D
To prove: $\quad|\angle A|+|\angle C|=|\angle D|$ Step 1
Proof: $\quad|<\mathrm{A}|+|\angle \mathrm{B}|+|\angle \mathrm{C}|=180^{\circ}$ (Angles in triangle sum to $180^{\circ}$ )

$$
\begin{array}{ll}
|\angle \mathrm{B}|+|\angle \mathrm{D}|=180^{\circ} & \text { (Angles on straight line) } \\
& |\angle \mathrm{A}|+|\angle \mathrm{B}|+|\angle \mathrm{C}|=|\angle \mathrm{B}|+|\angle \mathrm{D}| \\
& \Rightarrow|\angle A|+|\angle C|=|\angle \mathrm{D}|
\end{array}
$$

## Blunders (-3)

B1 Each step incorrect or omitted
B2 Each step incomplete
Attempts (3)
A1 Triangle with exterior angle drawn
A2 Indicates sum of angles in a triangle equals $180^{\circ}$
(b)(ii)

$$
\begin{array}{lc}
\text { (ii) } x^{\circ}=40^{\circ}+72^{\circ} \\
x^{\circ}=112^{\circ} & \Rightarrow 2 y^{\circ}+x^{\circ}=2 y^{\circ}+x^{\circ}=182^{\circ}=180^{\circ} \Rightarrow 2 y^{\circ}=68^{\circ} \Rightarrow y=34^{\circ} \\
\hline
\end{array}
$$

Blunders (-3)
B1 Correct answer with no work shown (es)
B2 $x=68^{\circ}$
B3 Sum of angles in triangle $\neq 180^{\circ}$
B4 Incorrectly indicates equal angles in isosceles triangle
Slips (-1)
S1 Arithmetic slips
Attempts (2)
A1 Indicates sum of angles in triangle equals $180^{\circ}$ once only
A2 Recognition of equal angles in isosceles triangle
(i) Construct a triangle of sides $8 \mathrm{~cm}, 7 \mathrm{~cm}$ and 6 cm .
(ii) Construct the incircle of the triangle.

All construction lines must be clearly shown in each case.
(c) (i) Triangle 10 Marks

Att 3


* Accept constructions with tolerance of 2 mm


## Blunders (-3)

B1 Each incorrect side
B2 Inserting right angles between two sides
B3 Correct triangle but no construction lines
Attempts(3)
A1 No triangle but one correct length drawn
Worthless (0)
W1 Triangle drawn with no correct length
(c) (ii) Incircle 10 marks

Att 3
(ii)

Incircle

* Accept constructions with tolerance of 2mm
* If candidate draws a separate correct triangle for (c )(ii), then accept this for construction of incircle.
* 4 marks: One angle bisected correctly
* 7 marks: Two angles correctly bisected


## Blunders (-3)

B1 Incentre indicated but incircle not drawn
Attempts (3)
A1 Effort at bisecting any angle
A2 Triangle and incircle drawn with no construction shown
A3 Circumcircle drawn with construction lines shown

## QUESTION 4

| Part (a) | 10 marks | Att 3 |
| :--- | :---: | ---: |
| Part (b) | 20 marks | Att 7 |
| Part (c) | $20(5,5,10)$ marks | Att(2,2,3) |

Part (a) 10 marks Att 3
$a b c d$ is a cyclic quadrilateral.
2s Given that $|\angle d a b|=73^{\circ}$ and

$$
|\angle a b c|=84^{\circ},
$$

find $|\angle a d c|$ and $|\angle b c d|$.


## (a) <br> 10 marks <br> Att 3

| (a) | $\|\angle a d c\|+84^{\circ}$ | $=180^{\circ}$ |
| :--- | :--- | :--- |
|  | $\|\angle a d c\|$ | $=96^{\circ} \quad\|\angle b c d\|$ |

## Blunders (-3)

B1 Correct answer with no work shown (S)
B2 Uses $360^{\circ}$ instead of $180^{\circ}$
B3 Sum of opposite angles $=90^{\circ}$
B4 One angle found only

## Slips (-1) <br> S1 Arithmetic error

## Attempts (3)

A1 Indicates sum of angles in cyclic quadrilateral $=360^{\circ}$
A2 Indicates sum of opposite angles in cyclic quadrilateral $=180^{\circ}$ in measure
Worthless (0)
W1 $|\angle a d c|=84^{\circ}$ or $73^{\circ}$
2. Prove that in a right-angled triangle, the square of the length of the side opposite to the right angle is equal to the sum of the squares of the lengths of the other two sides.


Given: $\quad \Delta a b c$, with $|\angle b a c|=90^{\circ}$
To Prove: $\quad|b c|^{2}=|a b|^{2}+|a c|^{2} \quad$ Step 1
Construction: Draw $a d \perp b c \quad$ Step 2
Proof: $\quad$ Taking $\Delta a b c$ and $\Delta a b d$
$|\angle b a c|=|\angle b d a|$ (both right angles)
$|\angle a b c|=|\angle a b d|$ (same angle)
$|\angle b a d|=|\angle b c a|\left(\right.$ Sum of angles in a triangle $\left.=180^{\circ}\right)$
$\Rightarrow$ triangles equiangular or triangles similar

$$
\begin{aligned}
& \Rightarrow \frac{|a b|}{|b c|}=\frac{|b d|}{|a b|} \\
& \Rightarrow|a b|^{2}=|b c| .|b d|
\end{aligned}
$$

Similarly taking $\Delta a b c$ and $\Delta a d c$
$|\quad a c|^{2}=|b c||d c|$
Therefore $|a b|^{2}+|a c|^{2}=|b c| .|b d|+|b c| .|d c| \quad$ Step 5

$$
\begin{aligned}
& =|b c|(|b d| \quad+|d c|) \\
& =|b c| .|b c|
\end{aligned}
$$

* Some steps may be partially indicated on diagram


## Blunders (-3)

B1 Each step incorrect or omitted
B2 Each step incomplete
Attempts (7)
A1 Diagram with perpendicular indicated

Alternative Proof


Given: Right-angled triangle with length of sides $a, b, c$, where $c$ is the hypotenuse.
RTP: $\quad a^{2}+b^{2}=c^{2}$.
Step 1


Step 2
Proof:
Each of the four inscribed triangles is congruent to the original triangle
Step 3
$\therefore \quad$ Each side of the inner quadrilateral has length $c$.
$|\angle 1|+|\angle 2|=90^{\circ} \quad$...Angle sum of triangle
$|\angle 1|=|\angle 3| \quad$...Corresponding parts in congruent triangles
$\therefore \quad|\angle 2|+|\angle 3|=90^{\circ}$
$\therefore \quad|\angle 4|=90^{\circ} \ldots$..Straight angle
Step 4
$\therefore \quad$ the inscribed quadrilateral is a square
Area of large square $=(a+b)^{2}=4($ area of one triangle $)+c^{2}$
$(a+b)^{2}=4(1 / 2 a b)+c^{2}$
Step 5
$a^{2}+2 a b+b^{2}=2 a b+c^{2}$
$\therefore a^{2}+b^{2}=c^{2}$

* $\quad$ Some steps may be indicated partially on diagram


## Blunders(-3)

B1 Each step incorrect or omitted
B2 Each step incomplete
Attempts(7)
A1 Diagram only

A circle, centre $o$, has a radius of length 17.
[ $1 k]$ is a chord of length 30 .
$m$ is a point on [lk] and $l k$ is perpendicular to $m o$.
(i) Write down the length of $[k m]$, giving a reason for your answer.

(ii) Calculate |om|.
(iii) Find the area of the triangle klo.
(c) (i)
(i) $|k m|=\frac{1}{2}|k l|=15$

Diameter/radius/line through centre/om perpendicular to chord bisects the chord

## Blunders (-3)

B1 Correct answer with no work shown (\&)
B2 Reason not given
Slips (-1)
S1 Arithmetic slip
Attempts (2)
A1 Some use of $\frac{1}{2}$
(c) (ii)

5 marks
Att 2
(ii) $\quad 17^{2}=15^{2}+|o m|^{2} \quad \Rightarrow|o m|^{2}=64 \quad \Rightarrow|o m|=8$

Blunders (-3)
B1 Correct answer with no work shown (S)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $|\mathrm{om}|^{2}=64$ and stops
B5 Use of 30 rather than 15
Slips (-1)
S1 Arithmetic slip
Attempts (2)
A1 Pythagoras indicated

## Blunders (-3)

B1 Correct answer with no work shown (\&)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
Slips (-1)
S1 Arithmetic slips
Attempts (3)
A1 Correct formula with some substitution
Worthless (0)
W1 Assuming angle at centre $=90^{\circ}$

## QUESTION 5

| Part (a) | 10 marks | Att 3 |
| :--- | :---: | ---: |
| Part (b) | $20(5,15)$ marks | Att (2,5) |
| Part (c) | $20(10,10)$ marks | Att (3,3) |

Part (a)
10 marks
Att 3
2. Without using a calculator or the tables, construct the angle $A$ such that

$$
\tan A=\frac{3}{4}
$$

(a)

10 marks
Att 3


* Tolerance of 2 mm
* Measure each side to check ratio
* Measure for right angle (need not be specifically marked)
* Check that relevant angle marked A


## Blunders (-3)

B1 Angle A not indicated
B2 Incorrect use of ratio
Attempts (3)
A1 Draws one side of length 3 or 4
A2 Indicates 3 as 'opposite' and /or 4 as 'adjacent'
A3 Indicates hypotenuse = 5
A4 States any correct trig. Ratio
A5 Pilot diagram
Worthless (0)
W1 Triangle with no length indicated
$a b c$ is an isosceles triangle with $|a b|=|a c|=9$.


25 Given that $|\angle a b c|=21 \cdot 7^{\circ}$, calculate the area of the triangle $a b c$, giving your answer correct to two decimal places.
(b)

First 5 marks
$|\angle b a c|=180^{\circ}-\left(21 \cdot 7^{\circ}+21 \cdot 7^{\circ}\right)=136 \cdot 6^{\circ}$

## Second 15 marks

Area of triangle $=\frac{1}{2} \times 9 \times \sin 136 \cdot 6^{\circ}$

$$
=\quad 27.827
$$

$$
\approx \quad 27 \cdot 83
$$

OR


First 5 marks
$a d \perp b c$
$\operatorname{Sin} 21 \cdot 7^{\circ}=\frac{|a d|}{9} \Rightarrow|a d|=3 \cdot 328$
Second 15 marks
$|b d|^{2}=81-11.076$
$|b d|=8 \cdot 362$
Area $=8 \cdot 362 \times 3 \cdot 328=27 \cdot 828 \approx 27 \cdot 83$

## Blunders (-3)

B1 Correct answer with no work shown (2)
B2 $|\angle b a c|$ not obtuse
B3 Incorrect substitution into correct formula
B4 Incorrect relevant formula
B5 Reads tables incorrectly or uses calculator in incorrect mode
B6 Early rounding off which affects answer
B7 Incorrect ratio for Sine function
Slips (-1)
S1 Arithmetic slips
S2 Slip reading tables (e.g. wrong column)
S3 Fails to distinguish between degrees and minutes and degrees in decimal format
S4 Not rounded off

## Attempts $(2,5)$

A1 Sum of angles in triangle equals $180^{\circ}$
A2 Correct formula with some substitution

## Part (c)

$20(10,10)$ marks
Att 3,3
$d$ and $e$ are points on a river bank 80 m apart and $f$ is a point on the opposite bank as shown in the diagram.
$|\angle f d e|=38^{\circ}$ and $|\angle f e d|=65^{\circ}$.

(i) Find $|e f|$, correct to the nearest metre.
(ii) Find the width of the river, as measured from $f$, correct to the nearest metre.
(c) (i)
(i)

$$
\begin{aligned}
& \frac{\operatorname{Sin} 77^{0}}{80}=\frac{\operatorname{Sin} 38^{0}}{|e f|} \Rightarrow|e f|=\frac{80 \operatorname{Sin} 38^{0}}{\operatorname{Sin} 77}=\frac{80(0.61566)}{0.97437} \\
& |e f| \quad=\quad 50 \cdot 5 \approx 51 \mathrm{~m}
\end{aligned}
$$

## Blunders (-3)

B1 Correct answer with no work shown (S)
B2 Incorrect ratio in use of Sine Rule
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode
B5 Early rounding off which affects answer
Misreading(-1)
M1 $|d f|$ found

## Slips (-1)

S1 Arithmetic slips
S2 Slip reading tables (e.g. wrong column)
Attempts (3)
A1 Sine Rule substituted
A2 $|\angle d f e|=77^{\circ}$
A3 Indicates sum of angles of triangle $=180^{\circ}$
Worthless (0)
W1 Treats triangle as right angled
(ii)

Let $h$ be the width

$$
\begin{aligned}
& \operatorname{Sin} 65=\frac{h}{51} \Rightarrow h=51 \operatorname{Sin} 65=51(0.9063) \\
& h=46 \cdot 2 \approx 46 \mathrm{~m}
\end{aligned}
$$

* Accept candidates answer from (c)(i)


## Blunders (-3)

B1 Correct answer with no work shown (2)
B2 Incorrect ratio for Sin function
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode
B5 Incorrect ratio for Sine Rule
Slips (-1)
S1 Arithmetic slips
S2 Slip reading tables (e.g. wrong column)
Attempts (3)
A1 Indicates use of $h$ in a ratio
A2 Indicates use of (c)(i) answer in a ratio
A3 Indicates use of $h$ in a right angled triangle
A4 Finds value of another acute angle and stops
A5 Any correct trig. Ratio or states Pythagoras

## QUESTION 6

| Part (a) | 10 marks | Att 3 |
| :--- | :---: | ---: |
| Part (b) | $20(10,10)$ marks | Att $(3,3)$ |
| Part (c) | $20(5,10,5)$ marks | Att $(2,3,2)$ |

Part (a)
10 marks
Att 3
\& Draw a pie chart to indicate how a lottery prize could be divided in the ratio of $3: 2: 1$.


* Allow a tolerance of $5^{\circ}$ in chart

Blunders (-3)
B1 Correct answer with no work shown (
B2 Sum of angles $\neq 360^{\circ}$
B3 Divisor other than 6
B4 Each incorrect plot
Slips (-1)
S1 Arithmetic slips
Attempts (3)
A1 Use of 360 indicated or implied
A2 Circle drawn
Worthless (0)
W1 Bar chart

The marks obtained by 25 candidates in an exam are as follows:

| 25 | 85 | 55 | 74 | 60 |
| :--- | :--- | :--- | :--- | :--- |
| 54 | 48 | 41 | 79 | 81 |
| 88 | 74 | 38 | 57 | 65 |
| 76 | 98 | 42 | 50 | 59 |
| 68 | 79 | 20 | 64 | 45 |

(i) Complete the following frequency table.

| Marks | $0-40$ | 4060 | $60-80$ | $80-100$ |
| :---: | :---: | :---: | :---: | :---: |
| Number of Students |  |  |  |  |

[Note: $40-60$ means 40 or more but less than 60 , etc.]
(ii) Taking mid-interval values, calculate the student mean mark

## (b) (i)

10 marks
Att 3
(i)

| Marks | $0-40$ | $40-60$ | $60-80$ | $80-100$ |
| :---: | :---: | :---: | :---: | :---: |
| Number of Students | $\mathbf{3}$ | $\mathbf{9}$ | $\mathbf{9}$ | $\mathbf{4}$ |

Blunders (-3)
B1 Omits any number
B2 Cumulative frequencies
Attempts (3)
A1 One entry correct
A2 No entry correct but sum of entries $=25$
Worthless(0)
W1 No correct entry and sum of entries $\neq 25$
(b) (ii)
(ii) Mean $=\frac{20 \times 3+50 \times 9+70 \times 9+90 \times 4}{25}=60$

* Accept candidates work from (b) (i)

Blunders (-3)
B1 Correct answer with no work shown (2)
B2 Division by 4
B3 Division by sum of mid interval
B4 Use of value other than mid interval values
B5 Consistently adds mid interval value to frequency instead of multiplying
Slips (-1)
S1 Arithmetic slips to max of -3
Attempts (3)
A1 Some or all mid intervals identified
A2 One correct multiplication in numerator
A3 Indicates division by 25
A4 Sum of frequencies divided by 4 or sum of mid intervals divided by 4

The cumulative frequency table below shows the times in minutes that 100 Olympic athletes completed the marathon after the winner crossed the line.

| Time in Minutes | $<2$ | $<5$ | $<7$ | $<9$ | $<11$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Athletes | 10 | 25 | 55 | 70 | 100 |

(i) Complete the following frequency table.

| Time in Minutes | $0-2$ | $2-5$ | $5-7$ | $7-9$ | $9-12$ |
| ---: | :--- | :--- | :--- | :--- | :--- |
| Number of Athletes | 1 |  |  |  |  |

[Note: $2-5$ means 2 or more but less than 5, etc.]
(ii) Draw a histogram to illustrate the data in the frequency table.
(iii) In which class interval does the $63^{\text {rd }}$ athlete to finish lie?
(c) (i)

5 marks
Att 2
(i)

| Time in Minutes | $0-2$ | $2-5$ | $5-7$ | $7-9$ | $9-12$ |
| ---: | :---: | :---: | :---: | :---: | :---: |
| Number of Athletes | $\mathbf{1 0}$ | $\mathbf{1 5}$ | $\mathbf{3 0}$ | $\mathbf{1 5}$ | $\mathbf{3 0}$ |

## Attempts (2)

A1 Any one value filled correctly into table
A2 Any indication of subtraction of frequencies
A3 Cumulative 'cumulative ' table
Worthless (0)
W1 Copies table and stops


* Accept candidate's work from (c )(i)
* No penalty for vertical scale instead of area scale Blunders(-3)
B1 Incorrect base scale
B2 Incorrect transfer of frequency to histogram
B3 Draws a trend graph from (c) (i)
B4 Each rectangle omitted
(c) (iii)

5 marks
Att 2
(iii) $10+15+30=55$ Thus $55^{\text {th }}$ athlete in the 5-7 interval
$10+15+30+15=70$ Thus $70^{\text {th }}$ athlete in 7-9 interval
$\Rightarrow \quad 63^{\text {rd }}$ athlete in the $\mathbf{7 - 9}$ class interval.

* Accept answer consistent with candidate's work
* Accept answer clearly identified on graph

Blunders (-3)
B1 Correct answer with no work shown (2)
B2 Chooses incorrect interval with work
Attempts (2)
A1 Adds a number of frequencies

