Coimisiún na Scrúduithe Stáit
State Examinations Commission

MARKING SCHEME 2003

JUNIOR CERTIFICATE EXAMINATION

MATHEMATICS

HIGHER LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work, as follows:

- Blunders - mathematical errors / sign errors / omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1)

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3, ….. S1, S2, S3, ….. M1, M2, …. etc.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that:

- any correct relevant step in a part of a question merits, at least, the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is not awarded.

3. Worthless work must be awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2, …. etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for the attempt mark only.

7. The phrase "and stops" means that no more work is shown by the candidate.

8. ✍ Failure to show supporting work, will, in general, result in a Blunder (-3).

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

✍ Express 45 centimetres as a fraction of 15 metres and write your answer in its simplest form.

(a) **10 marks** **Att 3**

$\frac{45}{1500} = \frac{3}{100}$ or $0 \cdot 03$

* Accept a decimal answer.

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Each error in conversion, e.g. 15 metres = 150 centimetres or similar.

B3 Ratio inverted, i.e. $\frac{1500}{45}$, and continues, but **Note:** May incur other errors also.

B4 Stops at $\frac{45}{1500}$, but **Note:** Stops at $\frac{1500}{45}$ incurs 2 blunders $\rightarrow$ 4 marks.

B5 No conversion, but Note stops at $\frac{45}{15}$ incurs 2 blunders $\rightarrow$ 4 marks.

B6 Error in division / cancelling.

Slips (-1)

S1 Numerical errors to a max of 3

S2 Not fully simplified, e.g. stops at $\frac{9}{300}$ or $\frac{15}{500}$ or similar.

Misreadings (-1)

M1 Reads as 54 centimetres or similar

Attempts (3 marks)

A1 Some correct conversion, e.g. 15 metres = 1500 centimetres and stops.

A2 Gets $\frac{15}{45}$ and continues with some cancellation or gives a decimal answer.

Worthless (0)

W1 Stops at $\frac{15}{45}$.

W2 Incorrect answer and no work.

Part (b) **20 (10 + 10) marks** **Att 3, 3**

€6000 was invested at compound interest. The rate for the first year was 4% per annum.

(i) ✍ Calculate the amount of the investment at the end of the first year.

(ii) ✍ At the end of the second year the investment amounted to €6520·80.
Calculate the rate per annum for the second year.

(b) (i) **10 marks** **Att 3**

(i) $6000 \times 1{\cdot}04 = 6240$

or

$P_1 = 6000$

$I_1 = 240$

$A_1 = 6240$

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each error in decimal point.

B3 Failure to get the amount, i.e. stops at 240.

B4 Mathematical error, e.g. 4% taken as some incorrect fraction.

B5 Subtracts 240 to get €5760.

B6 Uses 0·96 instead of 1·04

B7 Each incorrect substitution into correct formula.

Slips (-1)

S1 Numerical errors to max of 3

Attempts (3 marks)

A1 $4\% = \frac{4}{100}$ and stops.

A2 Writes down correct formula and stops.

Worthless (0)

W1 Incorrect formula, with or without work.

(b) (ii) **10 marks** **Att 3**

(ii) $\dfrac{6520 \cdot 80}{6240} = 1 \cdot 045 \;\Rightarrow\; 4 \cdot 5\%$

or

$P_2 = 6240$

$A_2 = 6520 \cdot 80$

$I_2 = 280 \cdot 80$

$\text{Rate} = \dfrac{280 \cdot 80}{6240} \times 100 = 4 \cdot 5\ \%$

* Accept candidate's answer from **(b) (i)** above.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each error in decimal point.

B3 Stops at $1 \cdot 045$, i.e. 4·5 % not stated.

B4 Stops at $\dfrac{6520 \cdot 80}{6240}$, but **Note:** will also incur B3 above $\rightarrow$ 4 marks.

B5 Fails to multiply by 100, i.e. stops at $0 \cdot 045$.

B6 Stops at $\dfrac{280 \cdot 80}{6240}$, but **Note:** will also incur B5 above $\rightarrow$ 4 marks.

B7 Inverts ratio, but **Note:** may also incur other blunders.

B8 Uses incorrect principal for year 2.

B9 Transposition error when using correct formula.

Slips (-1)

S1 Numerical errors to max of 3

Attempts (3 marks)

A1 Calculates interest and stops.

A2 Writes down correct formula for rate and stops.

Part (c) **20 (15 + 5)marks** **Att 5, 2**

The standard rate of income tax is 20% and the higher rate is 42%.
Fiona has tax credits of €1493 for the year and a standard rate cut-off point of €30 000.
She has a gross income of €31 650 for the year.

(i) ✍ After tax is paid, what is Fiona's income for the year?

(ii) ✍ What would Fiona's gross income for the year need to be in order for her to have an after tax income of €29 379?

(c) **15 marks** **Att 5**

(i)

Tax at 20% on 30 000	=	6000
Tax at 42% on 1650	=	693
Total tax	=	6693
Less tax credits of		1493
Net tax due	=	5200

Income after tax is 31 650 – 5200 = 26 450

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each error in decimal point.

B3 Mathematical error, e.g. 20% or 42% taken as some incorrect fraction – can incur 2 blunders.

B4 20% of some incorrect figure or 42% of some incorrect figure, but **Note:** may incur 2 Blunders.

B5 Incorrect total tax based on candidate's figure or no total tax.

B6 Mishandles the tax credit.

B7 Fails to calculate net income for the year.

B8 Stops at 6693, but **Note:** also incurs B7 → 9 marks.

Slips (-1)

S1 Numerical errors to max of 3

Attempts (5 marks)

A1 Correct calculation of 20% or 42% and stops.

(c) **5 marks** **Att 2**

(ii) **Method 1**	**Method 2**
Increase in after tax income	29379 – 24000 = 5379
29 379 – 26 450 = 2929.	5379 – 1493 = 3886
Equivalent to 0·58 of gross income.	58% = 3886 → 100% = 6700
$\frac{2929}{0.58} = 5050$ additional gross income.	Needs to earn 30 000 + 6700 = **36 700**
Needs to earn 31 650 + 5050 = **36 700**	

* Accept candidates' answer from **(c) (i)** above.

* Candidates may present other correct methods.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each error in decimal point.

B2 Uses 20% instead of 42%.

B3 Inverts ratio, but **Note:** may also incur other blunders.

B4 Multiplies / divides by 42%.

Slips (-1)

S1 Numerical errors to max of 3

S2 Stops at 5050, or some incorrect work with 5050.

Attempts (2 marks)

A1 Calculates increase i.e. 2929 and stops.

A2 Arrives at 5379 when using method 2.

A3 Any correct relevant step, e.g. some correct calculation on candidate's figures.

QUESTION 2

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 (5 + 5) marks** **Att 2, 2**

(i) List the first six multiples of 3 and the first six multiples of 5.

(ii) Hence, write down the lowest common multiple of 3 and 5.

(a) **5 marks** **Att 2**

(i) Multiples of 3: 3, 6, 9, 12, 15, 18
Multiples of 5: 5, 10, 15, 20, 25, 30

* Accept 0 given a multiple of 3 or 5 in this part

Slips (-1)

S1 Each multiple missing to a max of 3 – assuming some are correct!

(a) **5 marks** **Att 2**

(ii) LCM 15

* Accept candidate's answer from **(a) (i)** above.

* Accept correct answer and no work.

* No marks if 0 given as LCM in this part.

Part (b) **10 (5 + 5)marks** **Att 2, 2**

(i) ✍ By rounding correct to the nearest whole number, estimate the value of

$$\frac{1}{3\cdot 67} + (7\cdot 9)^2 \times \sqrt{16\cdot 32}\,.$$

Then, evaluate $\frac{1}{3\cdot 67} + (7\cdot 9)^2 \times \sqrt{16\cdot 32}$, correct to two decimal places.

(b) (i) Estimate **5 marks** **Att 2**

$$\frac{1}{4} + (8)^2 \times \sqrt{16} = 0\cdot 25 + 64\times 4 = 256\cdot 25$$

* Accept rounding down / up of final answer.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each error in decimal point.

B3 Error in precedent.

B4 Rounds $\frac{1}{3\cdot 67}$ to 1, but accept $\frac{1}{3\cdot 67}$ rounded to 0 and continues.

B5 Misreads + and × signs, i.e. reads as $\frac{1}{3\cdot 67}\times(7\cdot 9)^2 + \sqrt{16\cdot 32}$

or $\frac{1}{3\cdot 67}\times(7\cdot 9)^2 \times \sqrt{16\cdot 32}$ or similar.

Slips (-1)

S1 Numerical errors to max of 3

S2 Error in rounding off, e.g. $\frac{1}{3\cdot 67} = \frac{1}{3}$

Attempts (2 marks)

A1 Some correct rounding to nearest whole number.

(b) (i) Evaluate **5 marks** **Att 2**

$\frac{1}{3\cdot 67} + (7\cdot 9)^2 \times \sqrt{16\cdot 32}$ $= 0\cdot 272 + 62\cdot 41\times 4\cdot 039$ $= 0\cdot 272 + 252\cdot 073$ $= 252\cdot 345$ $= 252\cdot 35$	Using calculator from beginning: $\rightarrow \quad 252\cdot 396 = 252\cdot 40$

* Accept correct answer and no work.

* Do not penalise same error if already penalise in **Estimate** above.

Blunders (-3)

B1 Each error in decimal point.

B2 Error in precedent.

Slips (-1)

S1 Numerical errors to max of 3.

S2 Failure to round off or incorrect rounding off.

Attempts (2 marks)

A1 Some correct calculation done.

Worthless (0)

W1 Incorrect answer and no work, but **Note** S2 above.

Part (b) **10 marks** **Att 3**

(ii) ✍ Simplify $\dfrac{\sqrt[3]{27} \times 3}{9^{\frac{1}{2}} \times 3^4}$ into the form 3^n where $n \in \mathrm{Z}$.

(b) (ii) **10 marks** **Att 3**

$$\frac{\sqrt[3]{27} \times 3}{9^{\frac{1}{2}} \times 3^4} = \frac{3 \times 3}{3 \times 3^4} = 3^{-3}$$

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each different error in laws of indices.

B3 Error in sign – of index.

B4 Answer not given in form 3^n where $n \in \mathrm{Z}$, but **Note:** may incur other blunders also.

Slips (-1)

S1 Numerical errors to max of 3

Attempts (3 marks)

A1 Some correct work with the indices.

Worthless (0)

W1 Uses calculator to get an answer of $0 \cdot 037$ or similar, with no work.

Part (c) **10 (5 + 5) marks** **Att 2, 2**

(i) $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5\}$ and $C = \{1, 3, 4, 5, 6\}$ [Irish Version $C = \{3, 4, 5, 6\}$]

✍ List the elements of $(A \setminus B) \cup (C \cap B)$ and the elements of $(A \cup B) \cap (C \setminus B)$.

Part (c) (i) "$\cup$" **5 marks** **Att 2**

$(A \setminus B) = \{1, 4\}$ $(C \cap B) = \{3, 5\}$

$(A \setminus B) \cup (C \cap B) = \{1, 4\} \cup \{3, 5\} = \{1, 3, 4, 5\}$

* Accept correct indication on Venn diagram.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each element incorrect or omitted at any step.

B3 Interprets $\cup$ as $\cap$ or similar.

Attempts (2 marks)

A1 One correct set but no other work.

A2 Some correct work with given set operations.

A3 Correct Venn diagram, and stops, but **Note:** award once only.

Worthless (0)

W1 Incorrect answer and no work.

Part (c) (i) "$\cap$" **5 marks** **Att 2**

$(A \cup B)$ = {1, 2, 3, 4, 5}

$(C \setminus B)$ = {1, 4, 6} [Irish Version $(C \setminus B) = \{4, 6\}$]

$(A \cup B) \cap (C \setminus B)$ = {1, 2, 3, 4, 5} $\cap$ {1, 4, 6} = {1, 4}

[Irish Version $(A \cup B) \cap (C \setminus B)$ = {1, 2, 3, 4, 5} $\cap$ {4, 6} = {4}]

* Allow either answer for Irish scripts.

* Accept correct indication on Venn diagram.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each element incorrect or omitted at any step.

B3 Interprets $\cup$ as $\cap$ or similar.

Attempts (2 marks)

A1 One correct set but no other work.

Worthless (0)

W1 Incorrect answer and no work.

Part (c) **10 (5 + 5) marks** **Att 2, 2**

(ii) U is the universal set and P and Q are two subsets of U.

$\#U = 20$

$\#(P \cap Q) = x$

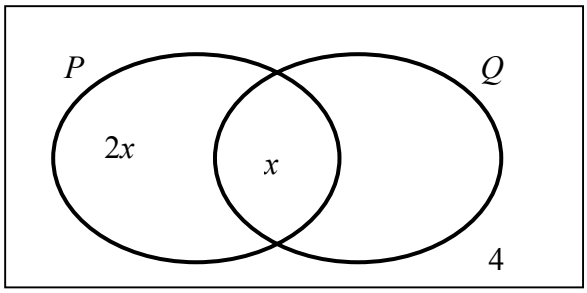
$\#(P \setminus Q) = 2x$

$\#((P \cup Q)') = 4$

$\#Q = 2(\#P)$.

✍ Represent the above information on a Venn diagram and hence find $\#Q$.

(c) (ii) Represent **5 marks** **Att 2**



* Accept correct indication on Venn diagram.

Blunders (-3)

B1 Each element incorrectly placed or omitted, but **Note:** may only merit attempt.

Attempts (2 marks)

A1 Any one correct element filled in on diagram.

(c) (ii) Find #Q **5 marks** **Att 2**

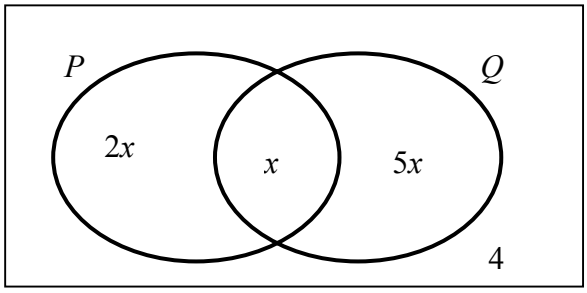
$\#Q = 2(\#P) \Rightarrow 6x = 2(3x)$

$2x + x + 5x + 4 = 20$

$8x = 16$

$x = 2$

$\#(Q) = 12$



Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each element incorrect or omitted at any step.

B2 Algebraic errors – transposition etc.

B4 Correct value of x, but incorrect #Q.

Slips (-1)

S1 States #Q = 6x, finds $x = 2$, but fails to evaluate.

Attempts (2 marks)

A1 Any correct relevant step.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

✍ Given that $p = \dfrac{x + 2y}{3}$, express y in terms of x and p.

(a) **10 marks** **Att 3**

$$p = \frac{x + 2y}{3} \quad \Rightarrow \quad 3p = x + 2y \quad \Rightarrow \quad 2y = 3p - x \quad \Rightarrow \quad y = \frac{3p - x}{2}$$

* Accept $y = \dfrac{x - 3p}{-2}$

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each different transposition error or error with sign.

B3 Mishandles cross multiplication e.g. $p - 3 = x + 2y$ and continues.

B4 Stops at $2y = 3p - x$, but **Note:** if stops at $3p = x + 2y$ also incurs B2 $\rightarrow$ 4 marks.

B5 Stops at $-y = \dfrac{3p - x}{-2}$ or $-y = \dfrac{x - 3p}{2}$.

B6 Solves correctly for x to get $x = 3p - 2y$, i.e a misreading that simplifies the question.

Attempts (3 marks)

A1 Some correct relevant work, e.g. one correct transposition or cross multiplication, but **Note:** B4.

Worthless (0)

W1 Incorrect answer and no work.

W2 Interchanges p and y to get $y = \dfrac{x + 2p}{3}$.

Part (b) **20 (10 + 10) marks** **Att 3, 3**

(i) ✍ Multiply out: $(3x - 1)(2x^2 + x - 4)$.

(ii) ✍ Evaluate your answer to part (i) when $x = -2$.

(b) **10 marks** **Att 3**

(i) $(3x - 1)(2x^2 + x - 4) = 6x^3 + 3x^2 - 12x - 2x^2 - x + 4 = \quad [6x^3 + x^2 - 13x + 4]$

* Accept cubic solution without tidying up, i.e. stops at $6x^3 + 3x^2 - 12x - 2x^2 - x + 4$.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each different error in index and sign.

B3 Each term omitted or incorrect

Slips (-1)

S1 Numerical slips to a max of 3.

Attempts (3 marks)

A1 Some correct relevant work, e.g. one correct multiplication.

Worthless (0)

W1 Incorrect answer and no work, but **Note:** A1.

(b) **10 marks** **Att 3**

(ii) When x = -2: $6x^3 + x^2 - 13x + 4 = 6(-2)^3 + (-2)^2 - 13(-2) + 4$ = -48 + 4 + 26 + 4 = -14

* Accept candidate's answer from **(b) (i)** above, if not oversimplified.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each different error in index and sign.

B3 Mathematical errors, e.g. $6(-2)^3 = (-12)^3$ etc.

B4 Evaluates correctly for $x = 2$.

Slips (-1)

S1 Numerical slips to a max of 3.

Attempts (3 marks)

A1 Some correct relevant work, e.g. one correct substitution.

A2 Any term evaluated correctly.

Worthless (0)

W1 Incorrect answer and no work.

Part (c) **20 (10 + 10)marks** **Att 3, 3**

(i) ✍ Solve $x^2 - 13x + 36 = 0$.

(ii) ✍ Hence, find the two values of $t \in \mathrm{R}$ for which

$$\left(\frac{1}{t} + 2\right)^2 - 13\left(\frac{1}{t} + 2\right) + 36 = 0.$$

(c) **10 marks** **Att 3**

(i) $x^2 - 13x + 36 = 0 \Rightarrow (x-4)(x-9) = 0 \Rightarrow x - 4 = 0$ or $x - 9 = 0 \Rightarrow x = 4$ or $x = 9$.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Incorrect factors each time and continues, but **Note:** incorrect factors containing 4 & 9 gives 1 Blunder .

B3 Correct factors of quadratic and stops, i.e. $(x-4)(x-9) = 0$ and stops.

B4 Each error in using formula.

B5 Transposition error.

Slips (-1)

S1 Numerical errors to a max of 3.

Attempts (3 marks)

A1 Some quadratic curve drawn.

A2 Incorrect factors and stops.

A3 Correct test on one or more correct values.

(c) **10 marks** **Att 3**

(ii) $\frac{1}{t}+2=4\Rightarrow\frac{1}{t}=2\Rightarrow t=\frac{1}{2}$

$\frac{1}{t}+2=9\Rightarrow\frac{1}{t}=7\Rightarrow t=\frac{1}{7}$

* Accept candidates' values for x from **(c) (i)** above.

* Accept quadratic in t multiplied out and mark blunders and slips as in **(c) (i)** above.

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Each different error in transposition.

B3 Deals with only one value of t, but if only one value of x in **(c) (i)** then no further penalty.

B4 No final transposition, but penalise once only.

Slips (-1)

S1 Numerical errors to a max of 3

Attempts (3 marks)

A1 Correct removal of either bracket and stops.

A2 $x = \frac{1}{t}+2$ and stops.

Worthless (0)

W1 $t = 9$ or $t = 4$ and evaluates.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

✍ List the solution set of the inequality

$$-3x - 3 > x - 12, \quad x \in \mathbf{N}.$$

(a) **10 marks** **Att 3**

$-3x - 3 > x - 12 \Rightarrow -3x - x > 3 - 12 \Rightarrow -4x > -9 \Rightarrow x < 2{\cdot}25.$

Solution set $\{0, 1, 2\}$

* Accept $x \in \mathbf{N}$ or $x \in \mathbf{N}_0$.

* Accept correct indication on number line.

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Each different error in transposition.

B3 Mishandles inequality, e.g. $-4x > -9 \Rightarrow x > 2{\cdot}25$.

B4 No final set given.

B5 Plots $x < 2{\cdot}25$, $x \in \mathbf{R}$ on number line.

Slips (-1)

S1 Numerical errors to a max of 3.

S2 $x \in \mathbf{Z}$.

S3 Gets $x < 2{\cdot}25$ but solution set {3, 4, 5, etc}

S4 Earlier error results in $x <$ "a natural number",
and candidate treats as $x \leq$, e.g. $x < 4 \Rightarrow$ {4, 3, 2, 1 , 0}

Attempts (3 marks)

A1 Solves equation $-3x - 3 = x - 12$ to get $x = 2 \cdot 25$.

A2 Tests for one correct value of x and stops.

A3 Any correct transposition and stops.

A4 Treats as two inequalities, with some correct work.

Part (b) **20 (10 + 10) marks** **Att 3, 3**

(i) Factorise $4x^2 - 49$.

(ii) ✍ Factorise $ab - cb + ac - c^2$.

(b) **10 marks** **Att 3**

(i) $4x^2 - 49 = (2x-7)(2x+7)$

* Accept correct answer and no work – **no** ✍ .

Blunders (-3)

B1 Errors in sign.

B2 Stops at $2x(2x+7) - 7(2x+7)$.

B3 Incorrect factorisation of one term, e.g. $(4x-7)(4x+7)$.

Attempts (3 marks)

A1 Any correct factors of $4x^2$ and / or 49.

A2 $(4x-49)(4x+49)$.

A3 Indicates some knowledge of the difference of two squares.

(b) **10 marks** **Att 3**

(ii) $ab - cb + ac - c^2 = b(a-c)+c(a-c) = (a-c)(b+c)$

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Each different error in sign.

B3 Stops after first step, i.e. $b(a-c)+c(a-c)$ or similar.

B4 Answer given as $(a-c) + (b+c)$, but **Note:** answer such as $(a-c)$ *and* $(b+c)$ merits 10 marks.

Attempts (3 marks)

A1 Some effort at factorising – groups off or pairs.

Part (c) **20 (5 + 5 + 10)marks** **Att 2, 2, 3**

A cinema takes in €400 each time that all seats are sold.
Next week, eight seats will be removed to make room for a new emergency exit.
The price per seat will have to be increased by €2·50 in order to keep the takings at €400.

(i) Taking x to be the number of seats now in the cinema,
write an equation in x to represent the above information.

(ii) ✍ Solve the equation to find the number of seats now in the cinema and
the price per seat now.

(c) One correct expression **5 marks** **Att 2**

(i) Cost per seat now: $\frac{400}{x}$; **or** Cost per seat next week: $\frac{400}{x-8}$

* Accept other correct methods.

Blunders (-3)

B1 Cost per seat inverted, e.g. $\frac{x}{400}$ or $\frac{x-8}{400}$, but penalise once only.

B2 Uses $x + 8$.

Attempts (2 marks)

A1 Any correct relevant step, e.g. getting $x - 8$.

(c) Second expression & Setting up equation 5 marks **Att 2**

(i) $\frac{400}{x-8} - \frac{400}{x} = 2\cdot5$ or equivalent.

Blunders (-3)

B1 Sign error in setting up equation, e.g. $\frac{400}{x} - \frac{400}{x-8} = 2\cdot5$.

Attempts (2 marks)

A1 No equation, but expression for other cost per seat correct.

(c) **10 marks** **Att 3**

(ii) $\dfrac{400}{x-8} - \dfrac{400}{x} = \dfrac{5}{2}$

$\Rightarrow 2[400x - 400x + 3200] = 5x(x-8)$

$\Rightarrow 5x^2 - 40x - 6400 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow (x-40)(x+32) = 0$

$\Rightarrow x = 40$ or $x = -32$.

Number of seats is 40. Price per seat €10.

Blunders (-3)

B1 Each different error in distributive law.

B2 Errors in transposition.

B3 Mathematical / sign errors.

B4 One solution where there should be two, i.e. where there are 2 positive solutions.

B5 Correct factors and stops, but **note** will also incur B7 below.

B6 Incorrect factors each time and continues, but **note** $(x + 40)(x - 32)$ is only **one** Blunder.

B7 Failure to find the price per seat.

Slips (-1)

S1 Numerical errors to a max of 3.

Attempts (3 marks)

A1 No quadratic due previous errors, merits attempt at most.

A2 Incorrect factors and stops.

A3 $x = 40$ without work and verifies.

Worthless (0)

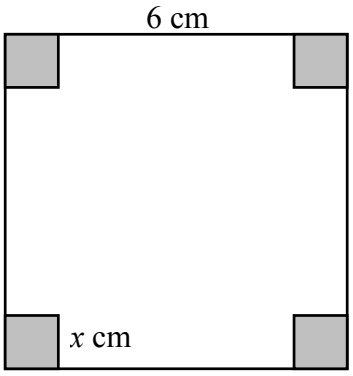
W1 $x = 40$ without work and stops.

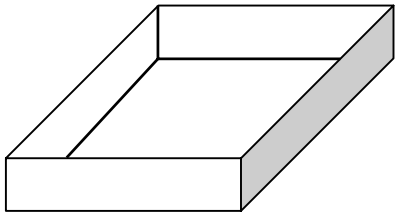
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

A square sheet of cardboard measures 6 cm by 6 cm.
A square of side x cm is removed from each corner.
The remaining piece of cardboard is folded to form an open box as shown.





✍ Show that the area, in cm^2, of each side of the box is $6x - 2x^2$.

(a) **10 marks** **Att 3**

Length $6 - 2x$, Height x. Area $x(6 - 2x) = 6x - 2x^2$

Blunders (-3)

B1 Length taken as $6 - x$ or similar.

B2 Errors in distributive law.

B3 Calculates surface area of box, i.e. gets $36 - 4x^2$.

Attempts (3 marks)

A1 Length taken as $6 - 2x$ or $6 - x$ and stops.

A2 Any correct area but **Note:** B3 above.

A3 Correct factors of $6x - 2x^2$ with the exception of $x(6 - 2x)$.

Part (b) **20 marks** **Att 7**

✍ Let f be the function $f : x \to 6x - 2x^2$. [Irish Version $f : x \to 6x - x^2$.]

Evaluate $f(x)$ when $x = 0, 1, 2, 3, 4$.

Hence, draw the graph of f for $0 \le x \le 4, \quad x \in \mathrm{R}$.

(b) **20 marks** **Att 7**

[Irish Version $f : x \to 6x - x^2$.]

$f(x) = 6x - 2x^2$

$f(0) = 6(0) - 2(0)^2 = 0 - 0 = 0$

$f(1) = 6(1) - 2(1)^2 = 6 - 2 = 4$

$f(2) = 6(2) - 2(2)^2 = 12 - 8 = 4$

$f(3) = 6(3) - 2(3)^2 = 18 - 18 = 0$

$f(4) = 6(4) - 2(4)^2 = 24 - 32 = -8$

$f(x) = 6x - x^2$

$f(0) = 6(0) - (0)^2 = 0 - 0 = 0$

$f(1) = 6(1) - (1)^2 = 6 - 1 = 5$

$f(2) = 6(2) - (2)^2 = 12 - 4 = 8$

$f(3) = 6(3) - (3)^2 = 18 - 9 = 9$

$f(4) = 6(4) - (4)^2 = 24 - 16 = 8$

or

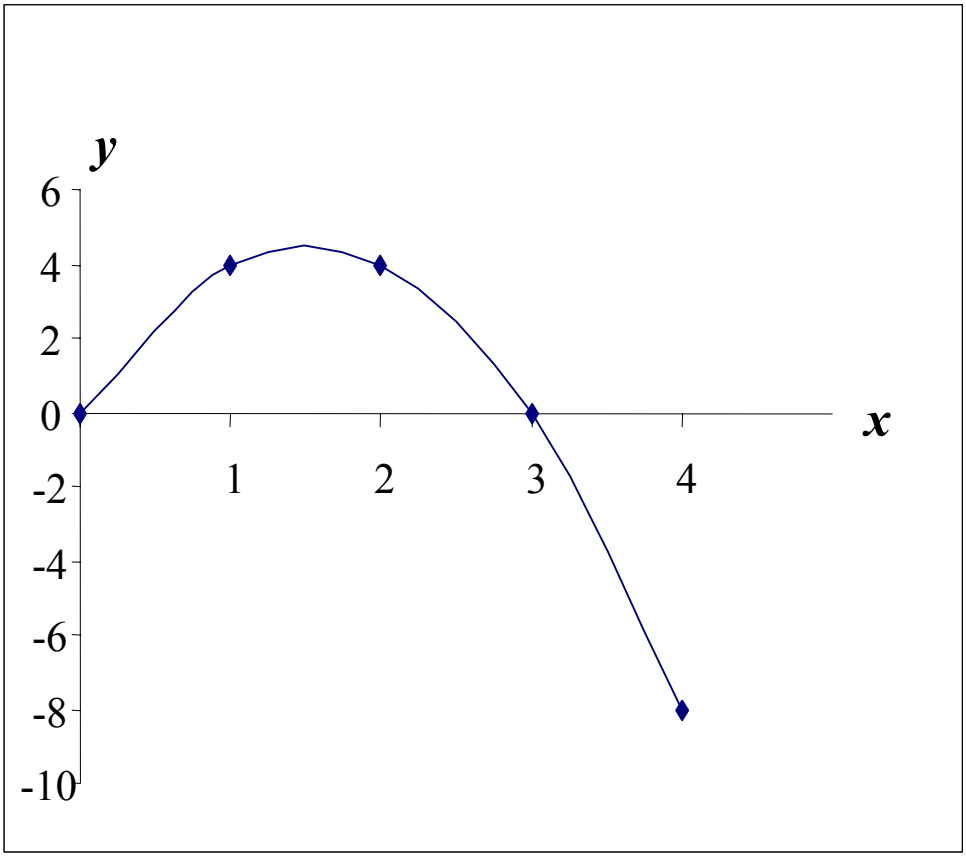
x	0	1	2	3	4
$-2x^2$	0	-2	-8	-18	-32
$+6x$	0	6	12	18	24
$f(x)$	0	4	4	0	-8

[Irish Version $f : x \to 6x - x^2$.]

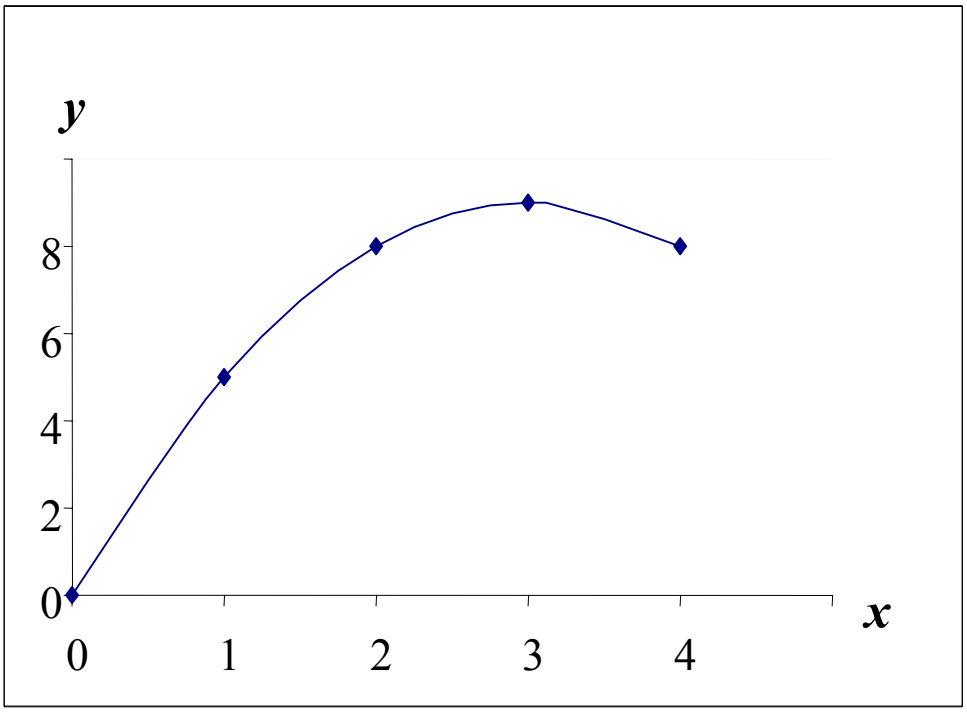
x	0	1	2	3	4
$-x^2$	0	-1	-4	-9	-16
$+6x$	0	6	12	18	24
$f(x)$	0	5	8	9	8

* Allow either set of answers for Irish scripts.

Graph of $f : x \to 6x - 2x^2$.



[Irish Version: Graph of $f : x \to 6x - x^2$.]



Values for quadratic graph

Blunders (-3)

B1 Each incorrect $f(x)$ without work.

B2 x row added in, i.e. top row, or adds in extra row.

B3 Omits $6x$ row.

B4 Treating the domain as $0 < x < 4$, can incur 2 Blunders if both omitted.

B5 Each different blunder which yields an incorrect row (full or part), e.g. $(2x)^2$ for $2x^2$.

B6 Avoids square for some (not all) values.

B7 Mathematical errors in tots, e.g. $-32 + 24 = 8$, but apply once only.

B8 Uses graph of $f : x \to 2x^2 - 6x$.

Slips (-1)

S1 Numerical slips to a max. of 3.

Attempts (7 marks)

A1 Omits $-2x^2$ or does not treat as x^2 (Treats as linear expression).

A2 Correct or partly correct table / values but no graph drawn.

Plotting the quadratic graph

* Accept candidate's values from the table.

Blunders (-3)

B1 Points not joined to form a reasonable graph.

B2 (x, y) plotted as (y, x), but apply once only.

B3 Scale not reasonably uniform. 1 x (–3) each axis.

B4 Each different blunder in plotting points from candidate's table / values.

B5 Each point omitted, if graph does not go reasonably close to where point should be.

B6 Points joined with straight lines.

Attempts (7 marks)

A1 Scaled axis drawn.

Part (c) **20 (5 + 5 + 5 + 5) marks** **Att 2, 2, 2, 2**

Use your graph from part **(b)** to estimate:

(i) ✍ the area of a side when $x = 0{\cdot}5$

(ii) ✍ the maximum possible area of a side

(iii) ✍ the value of x that gives sides of maximum area

(iv) the length and height of a side of maximum area.

(c) **5 marks** **Att 2**

(i) $2{\cdot}5 \text{ cm}^2$ [Irish Version: $2{\cdot}75 \text{ cm}^2$]

* Accept answer consistent with candidate's curve (within tolerance of $\pm\ 0{\cdot}4$).

Blunders (-3)

B1 Calculates area using $x = 0{\cdot}5$ in the function.

B2 Correct indication on graph, but no value given.

B3 Uses y or $f(x) = 0{\cdot}5$

Slips (-1)

S1 No indication on graph.

S2 Outside tolerance using candidate's graph

Attempts (2 marks)

A1 Point indicated on graph only.

Worthless (0)

W1 Answer inconsistent with candidate's graph.

(c) **5 marks** **Att 2**

(ii) Max area $= 4{\cdot}5 \text{ cm}^2$ [Irish Version: Max area $= 9 \text{ cm}^2$]

Blunders (-3)

B1 Correct indication on graph, but no value given.

Slips (-1)

S1 No indication on graph.

Attempts (2 marks)

A1 Point indicated on graph only.

(c) **5 marks** **Att 2**

(iii) $x = 1 \cdot 5$ cm [Irish Version: $x = 3$ cm]

* Accept answer consistent with candidate's graph.

Slips (-1)

S1 No indication on graph.

(c) **5 marks** **Att 2**

(iv) Length $= 6 - 2x = 6 - 2(1 \cdot 5) = 6 - 3 = 3$; Height $= x = 1 \cdot 5$

[Irish Version: $6 - 2x = 6 - 2(3) = 0$]

NB Full marks for candidates using Irish version of paper for this part even if candidate stops after part **(iii)**.

* Accept correct answer and no work – **no** ✍ .

* Accept answer from part **(ii)** divided by answer from part **(iii)** for full marks.

Blunders (-3)

B1 Gives area and height (i.e. max point on candidate's curve).

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

✍ Solve $3(x-2) + 1 = 19$ and verify your answer.

(a) **10 marks** **Att 3**

$3(x-2) + 1 = 19 \Rightarrow 3x - 6 + 1 = 19 \quad \Rightarrow \quad 3x = 19 + 5 \quad \Rightarrow \quad 3x = 24 \quad \Rightarrow \quad x = 8$

Verification: $3(8 - 2) + 1 = 3(6) + 1 = 18 + 1 = 19$

Blunders (-3)

B1 Correct answer, but no work shown (✍), but **Note:** may also incur B4 below.

B2 Each different error in transposition.

B3 Each different error in distributive law.

B4 Failure to verify.

Slips (-1)

S1 Numerical errors to a max of 3

Attempts (3 marks)

A1 Any correct transposition and stops.

A2 Testing incorrect value.

Part (b) **20 (15 + 5) marks** **Att 5, 2**

(i) ✍ Solve the simultaneous equations:

$$3x + 4y = -1$$
$$2x + 9 = -6y$$

(ii) ✍ By graphing the two lines on a single co-ordinate diagram, check your answer to part **(i)**.

(b) **15 marks** **Att 5**

(i) $3x + 4y = -1 \quad \Rightarrow \quad 6x + 8y = -2$

$2x + 9 = -6y \quad \Rightarrow \quad \underline{6x + 18y = -27}$

$-10y = 25 \quad \Rightarrow \quad y = -2 \cdot 5.$

$3x + 4y = -1 \quad \Rightarrow \quad 3x + 4(-2 \cdot 5) = -1 \Rightarrow 3x = 9 \quad \Rightarrow \quad x = 3.$

Blunders (-3)

B1 Each different error in transposing or signs.

B2 Does not multiply every term of equation, each time, e.g. $6x + 8y = -1$,
but $6x + 4y = -1$, → 2 Blunders.

B3 Mathematical error, e.g. $6x + 6x = 0$.

B4 Calculates the value of x **or** y correctly and stops.

Slips (-1)

S1 Finds $x = 3$ but subs in some other value of x to find y, e.g. $x = -3$.

S2 Numerical to a max. of 3.

Attempts (5 marks)

A1 $x = 3$ **and** $y = -2 \cdot 5$ without work.

A2 Some correct relevant work, e.g. $2x + 6y = -9$ and stops.

A3 Writes x in terms of y or vice-versa, e.g. $x = \dfrac{-6y - 9}{2}$

A4 Graphical solution.

Worthless (0)

W1 $x = 3$ **or** $y = -2 \cdot 5$ without work.

W2 Invents value for 1 variable and continues, e.g. $x = 1$ to get $y = -1$.

(b) **5 marks** **Att 2**

(ii) Line 1 $3x + 4y = -1$ $\Rightarrow$ $(0, -\frac{1}{4})$ and $(-\frac{1}{3}, 0)$

Line 2 $2x + 6y = -9$ $\Rightarrow$ $(0, -1\frac{1}{2})$ and $(-4\frac{1}{2}, 0)$

Point of intersection $(3, -2\frac{1}{2})$

* Allow reasonable tolerance if point of intersection close to required value .

Values for two lines

Blunders (-3)

B1 Mathematical errors.

Slips (-1)

S1 Numerical slips to a max. of 3.

Attempts (2 marks)

A1 At least one value correct but no graph drawn.

Plotting the two lines

* Accept candidate's values calculated above.

* Accept correct graph with point named without work (5 marks + Att 5 for **b(i)**).

Blunders (-3)

B1 (x, y) plotted as (y, x), but apply once only.

B2 Scale not reasonably uniform. 1 x (–3) each axis.

Slips (-1)

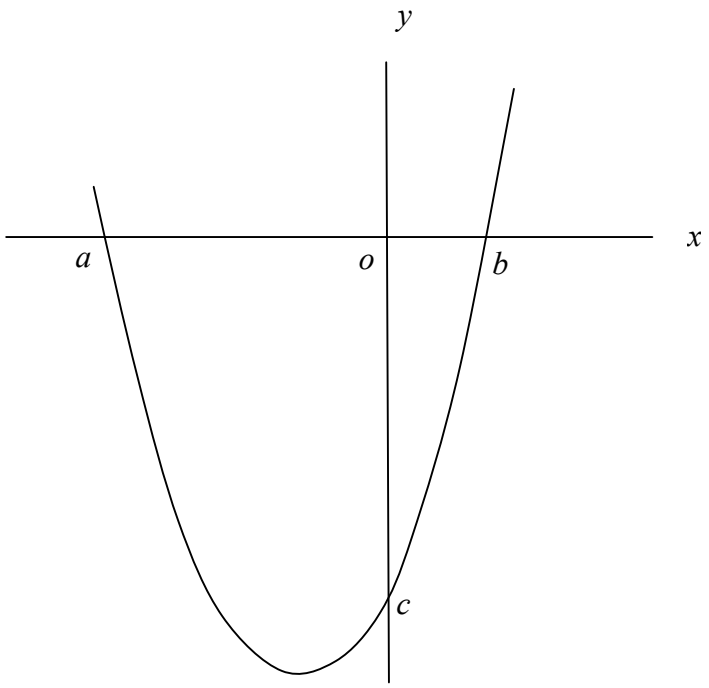
S1 Point of intersection not named.

Attempts (2 marks)

A1 Scaled axis drawn.

Part (c) **20 (10 + 5 + 5)marks** **Att 3, 2, 2**

The diagram shows part of the graph of the function $f : x \to x^2 + 2x - 8, \quad x \in \mathbf{R}$.



(i) ✍ The graph intersects the x-axis at a and b and the y-axis at c.
Find the co-ordinates of a, b, and c.

(ii) Hence, write down the range of values of x for which $x^2 + 2x - 8 \leq 0$.

(c) points *a* & *b* **10 marks** **Att 3**

(i) x-axis: $x^2 + 2x - 8 = 0 \Rightarrow (x+4)(x-2) = 0 \Rightarrow x = -4, x = 2.$

Hence, $a(-4, 0)$, $b(2, 0)$

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Incorrect factors each time and continues, but **Note:** incorrect factors containing 4 & 2 gives 1 Blunder .

B3 Correct factors of quadratic and stops, i.e. $(x+4)(x-2) = 0$ and stops, but **Note:** can also incur B6 below.

B4 Each error in using formula.

B5 Transposition error.

B6 Finds values of x but fails to state coordinates.

Slips (-1)

S1 Numerical errors to a max of 3.

Attempts (3 marks)

A1 Incorrect factors and stops.

A2 Correct test on one or more correct values.

A3 Gives $a = (?, 0)$ and / or $b = (?, 0)$.

(c) point *c* **5 marks** **Att 2**

(i) *y*-axis: $f(0) = 0 + 0 - 8 = -8.$

Hence, $c(0, -8)$

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Finds value of y but fails to state coordinates.

Attempts (2 marks)

A1 Gives $c = (0, ?)$.

(c) **5 marks** **Att 2**

(ii) $x^2 + 2x - 8 \leq 0$ for $-4 \leq x \leq 2.$

* Accept correct answer and no work – **no** ✍ .

* Answer must be consistent with **c (i)** above.

Blunders (-3)

B1 Error in writing down inequality, e.g. $x \leq -4$ or similar.

Slips (-1)

S1 Writes down $-4 < x < 2$ or states "between -4 and 2".

Attempts (2 marks)

A1 Correct shading.

A2 Gives integral values, i.e. -4, -3, -2, -1, 0, 1, 2.