NST 1

1. A set A of integers is *admissible* if it has the following property: if $x, y \in A$ (and possibly $x = y$), then $x^2 + kxy + y^2 \in A$ for every integer k. Determine all pairs m, n of non-zero integers such that the only admissible set containing m and n is the set of integers.

2. Let $n \geq 1$ be an integer. What is the maximum number of disjoint pairs of elements of the set $\{1, 2, 3, \ldots, n\}$ such that the sums of the different pairs are different integers not exceeding n? *A pair consists of two **different** elements of $\{1, 2, 3, \ldots, n\}$.*

3. In an acute triangle ABC, the feet of the altitudes through A, B and C are D, E and F respectively. The incentres of triangles AFE and BDF are I_1 and I_2 respectively. The circumcentres of triangles AI_1C and BCI_2 are O_1 and O_2 respectively. Prove that I_1I_2 and O_1O_2 are parallel.