

INTERNATIONAL INDIAN SCHOOL - DAMMAM**FIRST TERMINAL EXAMINATION- JUNE -2013****CLASS : XII**
SUBJECT: MATHEMATICS**SET - A****Max Marks : 100****Time: 3 hours****General Instructions:**

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B, C, Section A contains 10 questions of 1 mark each, Section B is of 12 questions of 4 marks each and section C is of 7 questions of 6 marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

Section. A

- 1) Find the value for x and y , if $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$
- 2) Find the identity element in the set Q of all rational numbers for the binary operation $*$ defined by $a * b = \frac{ab}{5} \forall a, b \in Q$.
- 3) Find the value of $\cos(\sec^{-1}x + \cos^{-1}x)$, $|x| \geq 1$.
- 4) Evaluate $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.
- 5) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.
- 6) If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f[f(x)]$.
- 7) Differentiate $\sin[\log(x^3 - 1)]$, w.r.t. x .
- 8) Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

9) The radius of a circle is increasing uniformly at the rate of 3cm/s. Find the rate at which the area of the circle is increasing when the radius is 10cm.

10) Examine the continuity of the function $f(x) = x^2 + 5$ at $x = -1$

Section. B

11) If $f: R \rightarrow R$ be defined by $f(x) = 3x + 7$. Show that f is invertible and hence find f^{-1} .

12) Find the values of x , if $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

13) Show that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

OR

Using properties of determinants show that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

14) Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$.

OR

Express in the simplest form $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

15) Let $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and 'I' be the identity matrix of order 2. Show that

$$I + A = I - A \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

16) Differentiate $\sin^{-1} \frac{2^{x+1}}{1+4^x}$ with respect to x

- 17) Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cut the

X-axis.

- 18) A store in a mall has three dozen shirts with 'SAVE ENVIRONMENT' printed, two dozen 'SAVE TIGER' printed and five dozen shirts with 'GROW PLANTS' printed. The cost of each shirt is Rs. 595, Rs.610 and Rs.795 respectively. All these items were sold in a day. Find total collection of the store using matrix method. Which shirt you would like to buy and why ?

- 19) If $y = e^{a \cos^{-1} x}, -1 \leq x \leq 1$, show that $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

- 20) If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ show that $A^2 - 5A - 14I = 0$. Hence find A^{-1} .

- 21) If $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$, find the value of x .

- 22) Show that $f(x) = |x-3| \forall x \in R$, is continuous but not differentiable at $x=3$.

OR

If $x = a \left(\cos t + \log \tan \frac{t}{2} \right), y = a \sin t$ find $\frac{d^2 y}{dx^2}$.

Section -C

- 23) Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by

$(a, b) * (c, d) = (ac, b + ad); \forall (a, b), (c, d) \in A$, then with respect to $*$ on A

- (i) Is $*$ commutative (ii) Is $*$ associative

- (iii) Find the identity element of A (iv) Find invertible element of A

24) Find the product of matrices $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ and use it for solving

the equations $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 2$.

OR

Using elementary transformations find the inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

25) Two sign boards, one circular and one square are to be made using a wire of length 40 m and cutting it into two pieces. The sign boards are to depict "BE HONEST" and "BE PUNCTUAL" and these are to be displayed near the main gate of the school. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum? Do you think these values are important in life and why?

26) If $\tan\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$ then prove that $\frac{dy}{dx} = \frac{y}{x}$.

27) Differentiate with respect to x $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

28) Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$

29) Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

with its vertex at one end of the major axis.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere

of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.