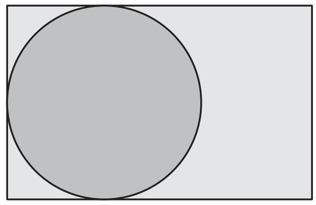
C1. The positive integer N has six digits in increasing order. For example, 124 689 is such a number.

However, unlike 124 689, three of the digits of N are 3, 4 and 5, and N is a multiple of 6.

How many possible six-digit integers N are there?

C2. A circle lies within a rectangle and touches three of its edges, as shown.



The area inside the circle equals the area inside the rectangle but outside the circle.

What is the ratio of the length of the rectangle to its width?

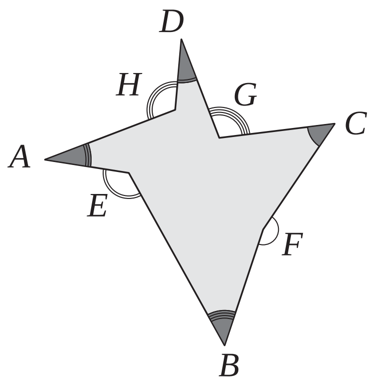
C3. The addition sum $XCV + XXV = CXX$ is true in Roman numerals.

In this question, however, the sum is actually the letter-sum shown alongside, in which: each letter stands for one of the digits 0 to 9, and stands for the same digit each time it occurs; different letters stand for different digits; and no number starts with a zero.

$$\begin{array}{r} XCV \\ +\,XXV \\ \hline CXX \end{array}$$

Find all solutions, and explain how you can be sure you have found every solution.

C4. Prove that the difference between the sum of the four marked interior angles A, B, C, D and the sum of the four marked exterior angles E, F, G, H of the polygon shown is 360°.

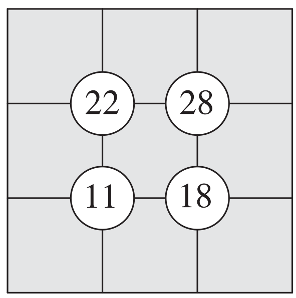


C5. In the expression below, three of the + signs are changed into − signs so that the expression is equal to 100:

$$0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$
$$+ 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20.$$

In how many ways can this be done?

C6. In the puzzle *Suko*, the numbers from 1 to 9 are to be placed in the spaces (one number in each) so that the number in each circle is equal to the sum of the numbers in the four surrounding spaces.

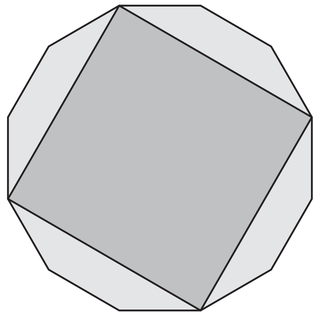


How many solutions are there to the *Suko* puzzle shown alongside?

H1. The positive integers m and n satisfy the equation $20m + 18n = 2018$. How many possible values of m are there?

H2. How many nine-digit integers of the form '*pqrpqrpqr*' are multiples of 24? (*Note that p, q and r need not be different.*)

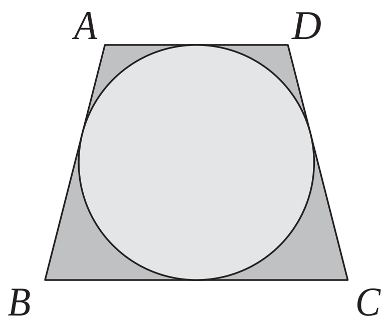
H3. The diagram shows a regular dodecagon and a square, whose vertices are also vertices of the dodecagon.



What is the value of the ratio

area of the square : area of the dodecagon?

H4. The diagram shows a circle and a trapezium $ABCD$ in which AD is parallel to BC and $AB = DC$. All four sides of $ABCD$ are tangents of the circle. The circle has radius 4 and the area of $ABCD$ is 72.

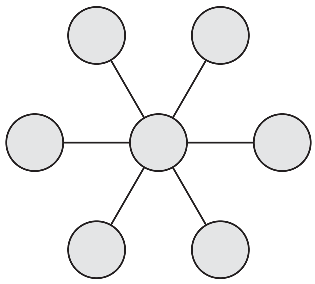


What is the length of AB?

H5. A two-digit number is divided by the sum of its digits. The result is a number between 2.6 and 2.7.

Find all of the possible values of the original two-digit number.

H6. The figure shows seven circles joined by three straight lines.



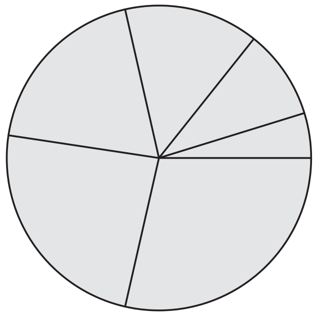
The numbers 9, 12, 18, 24, 36, 48 and 96 are to be placed into the circles, one in each, so that the product of the three numbers on each of the three lines is the same.

Which of the numbers could go in the centre?

M1. The sum of the squares of two real numbers is equal to fifteen times their sum. The difference of the squares of the same two numbers is equal to three times their difference.

Find all possible pairs of numbers that satisfy the above criteria.

M2. The diagram shows a circle that has been divided into six sectors of different sizes.



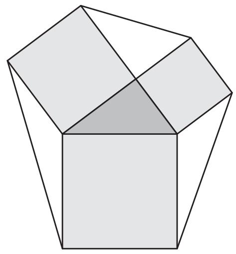
Two of the sectors are to be painted red, two of them are to be painted blue, and two of them are to be painted yellow. Any two sectors which share an edge are to be painted in different colours.

In how many ways can the circle be painted?

M3. Three positive integers have sum 25 and product 360.

Find all possible triples of these integers.

M4. The squares on each side of a right-angled scalene triangle are constructed and three further line segments drawn from the corners of the squares to create a hexagon, as shown. The squares on these three further line segments are then constructed (outside the hexagon).



The combined area of the two equal-sized squares is $2018\ \text{cm}^2$.

What is the total area of the six squares?

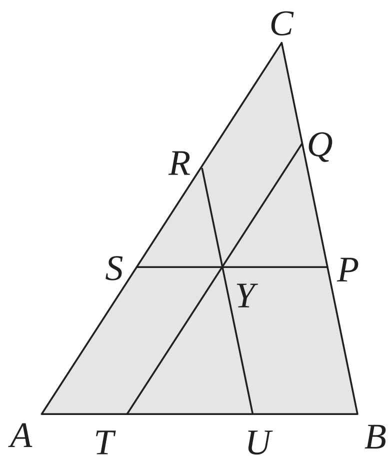
M5. For which integers n is $\dfrac{16(n^2 - n - 1)^2}{2n - 1}$ also an integer?

M6. The diagram shows a triangle ABC and points T, U on the edge AB, points P, Q on BC, and R, S on CA, where:

(i) SP and AB are parallel, UR and BC are parallel, and QT and CA are parallel;

(ii) SP, UR and QT all pass through a point Y; and

(iii) $PQ = RS = TU$.



Prove that

$$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{BC} + \frac{1}{CA}.$$