The United Kingdom Mathematics Trust

# Intermediate Mathematical Olympiad and Kangaroo (IMOK)

## Olympiad Maclaurin Paper

Thursday 10th March 2016

**All candidates must be in *School Year 11* (England and Wales), *S4* (Scotland), or *School Year 12* (Northern Ireland).**

## READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.

   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   ***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as $\pi$, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.

   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**


The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

*http://www.ukmt.org.uk*

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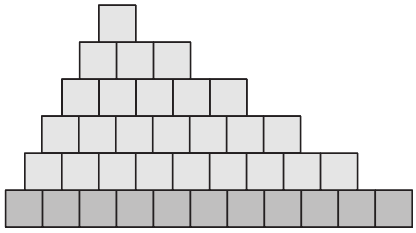
**M1.** The positive integer $N$ has five digits.

The six-digit integer $P$ is formed by appending the digit 2 to the front of $N$.

The six-digit integer $Q$ is formed by appending the digit 2 to the end of $N$.

Given that $Q = 3P$, what values of $N$ are possible?

**M2.** A ‘stepped’ shape, such as the example shown, is made from $1 \times 1$ squares in the following way.



(i) There are no gaps or overlaps.

(ii) There are an odd number of squares in the bottom row (eleven in the example shown).

(iii) In every row apart from the bottom one, there are two fewer squares than in the row immediately below.

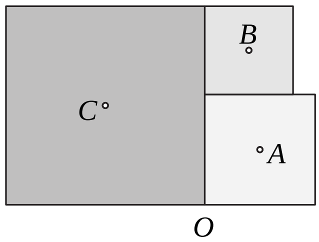
(iv) In every row apart from the bottom one, each square touches two squares in the row immediately below.

(v) There is one square in the top row.

Prove that $36A = (P + 2)^2$, where $A$ is the area of the shape and $P$ is the length of its perimeter.

**M3.** The diagram shows three squares with centres $A$, $B$ and $C$. The point $O$ is a vertex of two squares.

Prove that $OB$ and $AC$ are equal and perpendicular.




**M4.** What are the solutions of the simultaneous equations:

$$3x^2 + xy - 2y^2 = -5;$$
$$x^2 + 2xy + y^2 = 1 ?$$

**M5.** The number of my hotel room is a three-digit integer. I thought that the same number could be obtained by multiplying together all of:

(i) one more than the first digit;

(ii) one more than the second digit;

(iii) the third digit.

Prove that I was mistaken.

**M6.** The diagram shows two squares $APQR$ and $ASTU$, which have vertex $A$ in common. The point $M$ is the midpoint of $PU$.

Prove that $AM = \frac{1}{2}RS$.