The United Kingdom Mathematics Trust
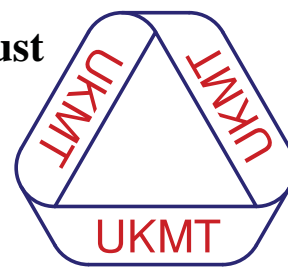
Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 10th March 2016

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.

2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.

3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.

4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.

5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.

6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

- *Do not hurry, but spend time working carefully on one question before attempting another.*

- *Try to finish whole questions even if you cannot do many.*

- *You will have done well if you hand in full solutions to two or more questions.*

- *Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.*

- *Give full written solutions, including mathematical reasons as to why your method is correct.*

- *Just stating an answer, even a correct one, will earn you very few marks.*

- *Incomplete or poorly presented solutions will not receive full marks.*

- ***Do not hand in rough work.***

C1. How many three-digit multiples of 9 consist only of odd digits?

C2. In a 6 × 6 grid of numbers:
- (i) all the numbers in the top row and the leftmost column are the same;
- (ii) each other number is the sum of the number above it and the number to the left of it;
- (iii) the number in the bottom right corner is 2016.

What are the possible numbers in the top left corner?

C3. All the telephone numbers in Georgetown have six digits and each of them begins with the digits 81. Kate finds the scrap of paper shown, with part of Jenny's telephone number on it.

How many different possibilities are there for Jenny's telephone number?

C4. The diagram shows an equilateral triangle ABC and two squares $AWXB$ and $AYZC$.

Prove that triangle AXZ is equilateral.
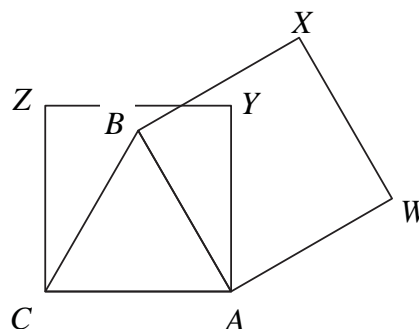
C5. Dean wishes to place the positive integers 1, 2, 3, …, 9 in the cells of a 3 × 3 square grid so that:
- (i) there is exactly one number in each cell;
- (ii) the product of the numbers in each row is a multiple of four;
- (iii) the product of the numbers in each column is a multiple of four.

Is Dean's task possible? Prove that your answer is correct.

C6. The diagram shows two regular heptagons $ABCDEFG$ and $APQRSTU$. The vertex P lies on the side AB (and hence U lies on the side GA). Also, Q lies on OB, where O is the centre of the larger heptagon.

Prove that $AB = 2AP$.