The United Kingdom Mathematics Trust
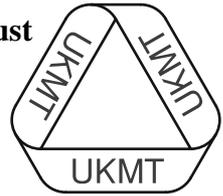
Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 19th March 2015

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.

2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.

3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.

4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.

5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.

6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

1. A train travelling at constant speed takes five seconds to pass completely through a tunnel which is 85 m long, and eight seconds to pass completely through a second tunnel which is 160 m long.

    What is the speed of the train?

2. The integers *a*, *b*, *c*, *d*, *e*, *f* and *g*, none of which is negative, satisfy the following five simultaneous equations:

    $$a + b + c = 2$$
    $$b + c + d = 2$$
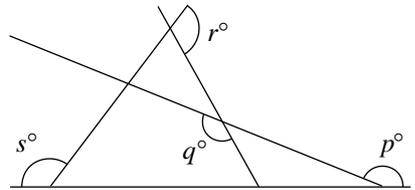 $$c + d + e = 2$$
    $$d + e + f = 2$$
    $$e + f + g = 2.$$

    What is the maximum possible value of $a + b + c + d + e + f + g$?

3. Four straight lines intersect as shown.

    What is the value of $p + q + r + s$?

4. Ten balls, each coloured green, red or blue, are placed in a bag. Ten more balls, each coloured green, red or blue, are placed in a second bag.
    In one of the bags there are at least seven blue balls and in the other bag there are at least four red balls. Overall there are half as many green balls as there are blue balls.

    Prove that the total number of red balls in both bags is equal to either the total number of blue balls in both bags or the total number of green balls in both bags.

5. The diagram shows a right-angled triangle and three circles. Each side of the triangle is a diameter of one of the circles. The shaded region *R* is the region inside the two smaller circles but outside the largest circle.
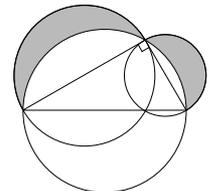
    Prove that the area of *R* is equal to the area of the triangle.

6. I have four identical black beads and four identical white beads.

    Carefully explain how many different bracelets I can make using all the beads.