2017 Pink Kangaroo Solutions

1. **B** The left-hand cell in the middle row is $2039 - 2020 = 19$. The middle cell in the bottom row is $2020 - 2017 = 3$, so the left-hand cell in the bottom row is $19 - 3 = 16$.

2. **E** As the wheel goes over the top it pivots around the peak so the midpoint travels through a circular arc. At the troughs the wheel changes directions in an instant from down-right to up-right, so the midpoint travels through a sharp change of direction. This gives the locus in diagram E.

3. **D** Antonia is fifth to the left of Bianca, so there are four girls in between. Similarly there are seven between them to the right. Hence there are 4 + 7 + 1 + 1 = 13 girls.

4. **D** The circumference is 2π, so every time the circle rolls 2π it has turned 360° and looks the same as it did at K. After 11π, it has turned 5 ½ turns, which is picture D.

5. **C** If she wins five more games, then she will have won 14 out of 20, which is equivalent to $\frac{7}{10}$ or 70%.

6. **A** Seven-eighths of all the guests were adults, of which three-sevenths were men, so the fraction of guests who were adult women equals $\frac{7}{8} \times \frac{4}{7} = \frac{1}{2}$.

7. **D** If the student has taken six buttons, he may already have three of the same colour, but it is possible that he has exactly two of each. However, if he takes a seventh button, he is guaranteed to have three of the same colour.

8. **C** Let x be the length FJ, and h be the height of the trapezium. Then the area of triangle FJI is $\frac{1}{2}xh$ and the area of trapezium $FGHI$ is $\frac{1}{2}h(20 + 50) = 35h$. The triangle is half the area of the trapezium, so $\frac{1}{2}xh = \frac{1}{2} \times 35h$, so $x = 35$.

9. **E** If exactly one of N and $N + 20$ has four digits, then the other has either 3 or 5 digits. If N has 3 digits and $N + 20$ has 4 digits, then $980 \leqslant N \leqslant 999$, giving 20 possibilities. If N has 4 digits and $N + 20$ has 5 digits, then $9980 \leqslant N \leqslant 9999$, giving 20 possibilities. Overall there are 40 possibilities for N.

10. **C** The three squares will be approximately a third of 770, so roughly 250. Adding up $15^2 = 225$, $16^2 = 256$ and $17^2 = 289$ gives the total 770.

Alternatively, using algebra we could let n be the middle integer and then add the squares $(n - 1)^2 + n^2 + (n + 1)^2 = n^2 - 2n + 1 + n^2 + n^2 + 2n + 1 = 3n^2 + 2 = 770$. This gives $n^2 = 256$ and hence $n = 16$, so the largest integer is 17.

11. **B** To compare wheels K and M, we need to find the lowest common multiple of 4 and 6, which is 12. When wheel L makes 12 turns, wheel K makes 15 turns and wheel M makes 14 turns. When wheel L makes 24 turns, wheel K makes 30 turns and wheel M makes 28 turns, so the ratio of the circumferences of wheel K to wheel M is 28:30.

12. **B** Any day when Tycho jogs is immediately followed by a day without a jog. Therefore any period of seven days has three pairs of ‘jog, no-jog’ days and one extra no-jog day. There are seven possibilities for this extra non-jog day, so seven distinct schedules.

13. **A** Let k cm be the difference between the heights.

Then the heights in cm are: Tobias 184, Victor $184 + k$, Peter $184 - k$, and Oscar $184 - 2k$. The mean is 178 so $\frac{1}{4}(184 + 184 + k + 184 - k + 184 - 2k) = 178$. Hence $4 \times 184 - 2k = 4 \times 178$, giving $2k = 4 \times 184 - 4 \times 178 = 4 \times 6 = 24$. Hence $k = 12$.

Therefore Oscar's height in cm is $184 - 2 \times 12 = 160$ cm.

14. **C** Let m be the number of days with sunny mornings and wet afternoons. Let n be the number of days with sunny mornings and sunny afternoons. There were 5 sunny mornings so $m + n = 5$… (1). Since there are seven wet days, the number of days with wet mornings and sunny afternoons must be $7 - m$. There are 6 sunny afternoons so $n + (7 - m) = 6$, which rearranges to $m = n + 1$… (2).

Equations (1) and (2) together give $m = 3$, $n = 2$, so Johannes had 3 days with sunny mornings and wet afternoons, 2 days sunny all day, and 4 days with wet mornings and sunny afternoons, a total of 9 days (not counting any cloudy days he may have had!).

15. **A** Let the numbers around the top left cell be a, b and c as shown. Then the sum of the top left 2×2 square (and hence *all* the 2×2 squares) is $a + b + c + 3$. The top right 2×2 square already contains a and b and 1, so the middle right cell must contain $c + 2$. The bottom left 2×2 square contains $b + c + 2$ so the bottom middle cell is $a + 1$. The bottom right 2×2 square already contains $a + b + c + 3$ so the missing value is zero. There are many ways to complete the grid; one way is shown here.

3	a	1
c	b	$c+2$
2	$a+1$	?

3	7	1
4	5	6
2	8	0

16. **A** Each number a, b, c, d, e, f, g differs from its neighbour by one, so they alternate odd and even. To obtain an odd total, we must have an odd number of odd numbers in the list. Hence b, d, f are odd and cannot be equal to 286.

If $c = 286$, then the biggest total possible is $288 + 287 + 286 + 287 + 288 + 289 + 290 = 2015$ which is too small. By reversing this list, we can also rule out $e = 286$.

We can obtain the total 2017 if we start with $a = 286$ since $286 + 287 + 288 + 289 + 290 + 289 + 288 = 2017$. By reversing this, we could also end with $g = 286$.

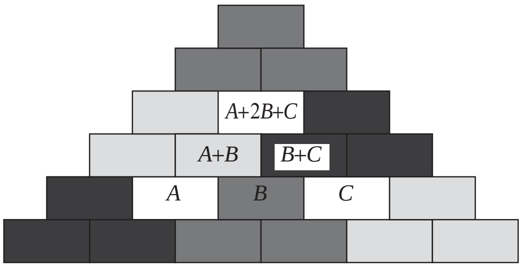
17. **D** The prime factor decomposition of 882 is $2 \times 3^2 \times 7^2$. The ages must be under 18, so cannot be $3 \times 7 = 21$ or $7 \times 7 = 49$. Hence, the only way to create two different numbers using 7 are: 7 and $2 \times 7 = 14$. This leaves only 3^2 which can create the two ages 1 and 9. The sum of the ages is then $1 + 9 + 7 + 14 = 31$.

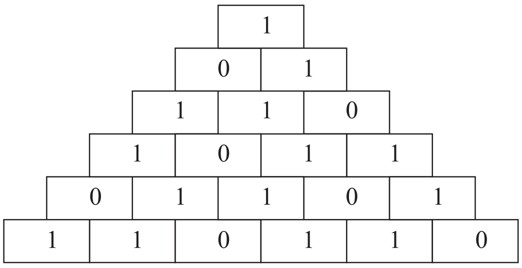
18. **C** We can get a negative product if the first die is negative and the second positive, with probability $\frac{3}{6} \times \frac{2}{6} = \frac{6}{36}$, or if the first die is positive and the second is negative, with probability $\frac{2}{6} \times \frac{3}{6} = \frac{6}{36}$. Together this gives a probability of $\frac{12}{36} = \frac{1}{3}$.

19. **C** Let '*ab*' be the 2-digit number with digits a and b, then the 6-digit number '*ababab*' = '*ab*' × 10101 = '*ab*' × 3 × 7 × 13 × 37 so is always divisible by 3, 7, 13 and 37. But it is only divisible by 2, 5, 9 or 11 if '*ab*' is.

20. **E** The password has length 7 so the different digits making it up must add to 7. The possibilities are: {7}, {6, 1}, {5, 2}, {4, 3}, {4, 2, 1}. Using only the digit 7 produces just one password, 7777777. Using two digits gives two possibilities, depending on which digit goes first, so the three pairs give $2 \times 3 = 6$ phone numbers. Three different digits can be arranged in six ways. This gives $1 + 6 + 6 = 13$ possibilities.

21. B





Each 'triple' consisting of a cell and the two cells immediately below can have at most two odds (for if the bottom two are both odd, the one above is even, so they cannot be all odd). The whole diagram can be dissected into six of these (shaded) triples as shown in the top diagram, with three other (white) cells left over. These six triples have at most $6 \times 2 = 12$ odds between them. Moreover, the three remaining white cells cannot all be odd; if we assign the values A and C to the lowest of these white cells, and B to the cell between them, then the cells above have values $A + B$ and $B + C$. The top white cell then contains $A + 2B + C$, which is even when A and C are both odd. Hence the three white cells have at most two odds, giving the whole diagram at most $12 + 2 = 14$ odds.

The second diagram shows one possible way of achieving this maximum of 14 odds.

22. E Let $x°$ be the missing angle. The correct total of the angles is then $(2017 + x)°$. The interior sum of angles in a polygon with n sides is $180\,(n - 2)°$, so we require $2017 + x$ to be a multiple of 180. The larger multiples of 180 are 2160 plus any multiple of 180. Hence $x = 143$ plus any multiple of 180. However, the polygon is convex so $x = 143$.

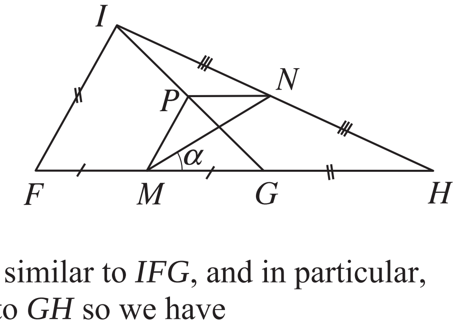
23. B The total mass is 621g so any three masses with total mass over 310.5g could be in the heavier pan. There are eight of these triples that include the 106g mass: (106, 105, 104), (106, 105, 103), (106, 105, 102), (106, 105, 101), (106, 104, 103), (106, 104, 102), (106, 104, 101), and (106, 103, 102).

Without 106, there are 2 ways to make a set over 310.5g: (105, 104, 103) and (105, 104, 102).

Hence the probabililty that the 106g mass is included in the heavier pan is $\frac{8}{8+2} = \frac{8}{10}$ or 80%.

24. D Let x be the length FG and let r be the radius. Then $FI = x + 6$ and $GH = HI = r$. Angle FIH is a right angle (the tangent and radius are perpendicular) so $FI^2 + HI^2 + FH^2$, which gives $(x + 6)^2 + r^2 = (x + r)^2$. Expanding this gives $x^2 + 12x + 36 + r^2 = x^2 + 2rx + r^2$, which simplifies to $12x + 36 = 2rx$. Halving this gives $6x + 18 = rx$, which rearranges to $r = 6 + \frac{18}{x}$. Since r is an integer, x must be a (positive) factor of 18, namely 1, 2, 3, 6, 9, 18; each of these six factors give a different value of r (or HI) as required.

25. A We start by drawing the line segment IG. Let P be the point on IG such that PN is parallel to FH. The angle PNM is alternate to NMH so $\angle PNM = \alpha$. Also, the triangle PNI is similar to the triangle GHI (the angles of each triangle are clearly the same); moreover since N is the midpoint of HI, $PN = \frac{1}{2}GH$. Also $IP = \frac{1}{2}IG$,



so $PG = \frac{1}{2}IG$. Since $MG = \frac{1}{2}FG$, the triangle PMG is similar to IFG, and in particular, $PM = \frac{1}{2}IF$. However, we know IF is equal in length to GH so we have $PN = \frac{1}{2}GH = \frac{1}{2}IF = PM$, so triangle MNP is isosceles and $\angle PMN = \angle PNM = \alpha$.

Since triangles PMG and IFG are similar, we have $\angle IFG = \angle PMG = \alpha + \alpha = 2\alpha$.