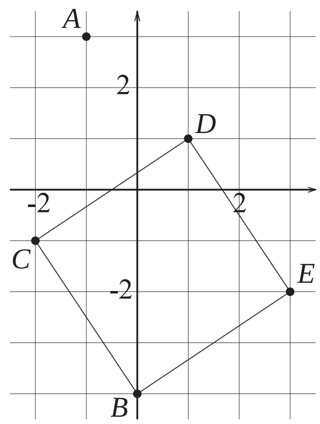
Solutions to the European Kangaroo Pink Paper

1. **B** The calculation can be approximated as follows:

$$\frac{17 \times 0.3 \times 20.16}{999} \approx \frac{17 \times 3 \times 2}{1000} = \frac{51 \times 2}{1000} \approx \frac{100}{1000} = 0.1.$$

2. **A** By plotting the points, it is easy to check that *BCDE* is a square. Since any three vertices of a square determine the fourth vertex, it is impossible to make a square using three of these points and the point *A*.



3. **B** The number x is 3 more than a multiple of 6, so $x = 6k + 3$ for some non-negative integer value of k. And then $3x = 3(6k + 3) = 18k + 9 = 6(3k + 1) + 3$. Hence $3x$ is 3 more than a multiple of 6, so leaves remainder 3 when divided by 6.

4. **D** By dividing by 4 and then by 6, we see that $2016 = 4 \times 504 = 4 \times 6 \times 84 = 24 \times 84$. So 2016 hours is 84 days and $84 = 7 \times 12$ so 2016 hours is 12 weeks.

5. **C** In Lucas's notation, the first zero represents the minus sign, and the number of other zeroes is the magnitude of the number, so 000 is -2 and 0000 is -3. Then $000 + 0000$ is $-2 + -3 = -5$, which is 000000 in Lucas's notation.

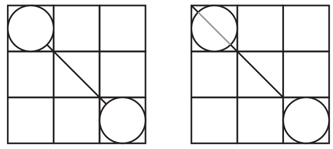
6. **D** Any odd score must be the sum of an even (positive) number and an odd (negative number). The largest odd total will be the largest even number added to the smallest odd number, which is $6 + -1 = 5$. Hence Marie cannot achieve 7. The others are achievable: $3 = 4 + -1$; $4 = 2 + 2$; $5 = 6 + -1$; $8 = 4 + 4$.

7. **C** In order to get from TEAM to MATE, at some point M will have to pass each of T, E and A. Likewise Angelo will have to move A in front of T and E. So at least 5 swaps are required. The list: TEAM, TEMA, TMEA, MTEA, MTAE, MATE shows it can be done in five swaps.

8. **E** Sven can choose from {1, 2, 3, 4, 5, 6, 7, 8, 9}, but can choose at most one from each of the pairs that add to 10: {1, 9}, {2, 8}, {3, 7}, {4, 6}. Since this gives him a maximum of 4 integers, he must always pick the digit 5.

9. **D** Note that $d = c^2 + 7$, so $d > c$; $d = b^2 + 3$, so $d > b$; and $d = a + 9$, so $d > a$. Hence d is the largest.

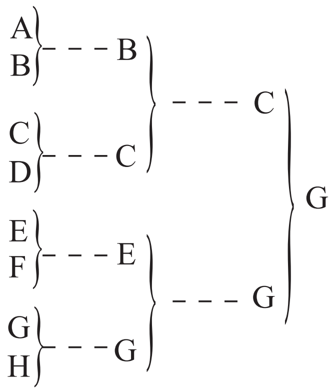
10. **A** Each square has side-length 1 unit, so by Pythagoras' Theorem the diagonals have length $\sqrt{1^2 + 1^2} = \sqrt{2}$. The distance between the two circles consists of a whole diagonal and two part diagonals (from the corner of a square to the circle). This is the same as the length of two whole diagonals, minus a diameter, that is $2\sqrt{2} - 1$.



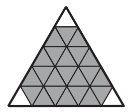
11. **E** The most matches that any player could play is three. Any player who wins twice will play in the final. Hence Celine and Gina must be the two finalists, and Gina beat Celine. This means Gina must have won three matches altogether, but only two are recorded. Hence the missing result must be that Gina beat someone.

All the other players must have lost once, but Emma has no loss recorded, so the missing result must be that Gina beat Emma.

Indeed, from the given information we can deduce that the pairings must have been as shown (using players' initials instead of their full names):



12. **C** By dissecting the triangle into smaller, identical triangles, we see that the shaded area is 22/25 of the larger triangle, which corresponds to $88/100 = 88\%$.



13. **B** Since each of the nine numbers appears exactly once in the three rows, the product of each of the three rows is equal to the product of all nine numbers, that is $1 \times 2 \times 4 \times 5 \times 10 \times 20 \times 25 \times 50 \times 100$. This equals $1 \times 2 \times 2^2 \times 5 \times 2 \times 5 \times 2^2 \times 5 \times 5^2 \times 2 \times 5^2 \times 2^2 \times 5^2 = 2^9 \times 5^9$.

Since the products of each of the three rows are equal, these products are equal to $\sqrt[3]{2^9 \times 5^9} = 2^3 \times 5^3$. So the ‘magic product’ is 1000.

By considering the top row, we can see that the top right cell must contain $1000 \div (20 \times 1) = 50$.

The only other way to get 1000 as the product of 1 and two of the remaining numbers is $1 \times 10 \times 100$. So 10 and 100 must fill the two spaces below 1. We cannot put the 100 in the centre since the products of the numbers on each diagonal would then be too large. So 10 goes in the centre and 100 below it. From the diagonal we see that the bottom right entry is 5 and so the middle number on the right is 4.

Note: *It may interest readers to know that in a* 3×3 *multiplicative magic square the centre number is always the cube root of the magic total.*

14. **D** The sum of the numbers in the envelopes is 255. Let E be the sum of Evie's numbers. Then Alie's numbers will have a total of $E - 31$. Hence we have $E + (E - 31) = 255$, giving $2E - 31 = 255$, so $2E = 286$ and $E = 143$. The only way to add up powers of two to get 143 is $128 + 8 + 4 + 2 + 1$, so Evie took 5 envelopes.

15. C Peter needs three different colours for the top row, say colours A, B, C. The central cell must be different from each of these (as it lies on the same diagonal as A and also of C, and in the same column as B), say colour D.

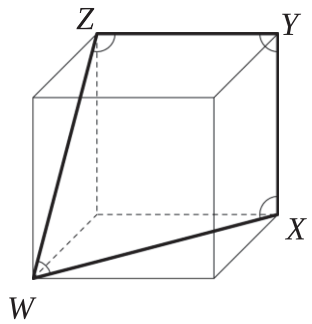
A	B	C
	D	
B		?

A	B	C
C	D	B
B	C	E

Suppose it is possible to use only these four colours. Note that the bottom left cell must be different from A (same column), and different from C and D (same diagonal), hence it must be colour B. But then the bottom right cell must be different from A and D (same diagonal), from B (same row) and from C (same column). Hence a fifth colour is needed. The arrangement above shows that five colours are sufficient.

16. B The lengths of WX, XZ and ZW are all equal (each being the diagonal of a square face), hence triangle WXZ is equilateral and angle ZWX is 60°.

The other angles are all 90°, so the total of all four angles is $90° + 90° + 90° + 60° = 330°$.



17. A Let G be the number of grey kangaroos, and P the number of pink kangaroos, so $G + P = 2016$. For each grey kangaroo we calculate the fraction as $\frac{P}{G}$; and there are G of these, so the total of the fractions for grey kangaroos is $G \times \frac{P}{G} = P$. Similarly the total of the fractions calculated for the pink kangaroos is G. Thus the total of all the fractions is $P + G = 2016$.

18. C Since the remainder of a division is always less than the divisor used, we can begin our search for the largest possible remainder by looking at the largest possible divisor, that is the largest possible sum of the digits of a 2-digit number.

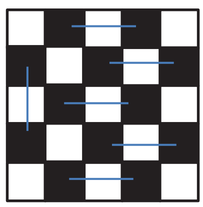
The largest possible sum of digits is $9 + 9 = 18$. And $99 \div 18$ has remainder 9.

The next largest is 17, which could come from 89 or 98. Doing the division, we see $89 \div 17$ has remainder 4 and $98 \div 17$ has remainder 13.

The next largest sum of digits is 16, which could come from 88, 97, 79. And division shows that $88 \div 16$ has remainder 8; $97 \div 16$ has remainder 1, and $79 \div 16$ has remainder 15 (the largest remainder so far).

Any sum of digits below 16 will have remainder below 15, so the remainder of 15 that we have achieved must be the largest possible.

19. B Note that, for each black square that we wish to produce, there will need to be a move which makes it black. This move will not change the colour of any of the other squares which we wish to make black (since the desired black cells are not neighbouring). Since there are 12 such squares, we must necessarily make at least 12 moves.



However, we can show that 12 moves are sufficient. Consider a pair of black cells with a white cell between them. This colouring can be made in two moves as follows: Starting with WWW, change the colours of two adjacent cells to obtain BBW, then change the middle cell and the one on its right to obtain BWB. By pairing off the 12 black cells into 6 pairs as shown, it is possible to create 12 black cells in $6 \times 2 = 12$ moves.

20. E Let D be the distance from X to Y, and let u be the speed of the boat and v be the speed of the current. When travelling downstream the overall speed of the boat in the current is $u + v$, and travelling upstream against the current it is $u - v$. Then using time $= \dfrac{\text{distance}}{\text{speed}}$, we get $\dfrac{D}{u + v} = 4$ for the journey downstream and $\dfrac{D}{u - v} = 6$ for the journey upstream. Inverting the equations gives $\dfrac{u + v}{D} = \dfrac{1}{4}$ or $4u + 4v = D\ldots$ (1) and $\dfrac{u - v}{D} = \dfrac{1}{6}$ or $6u - 6v = D\ldots$ (2). Multiplying (1) by 3 and (2) by 2, we get $12u + 12v = 3D\ldots$ (3) and $12u - 12v = 2D\ldots$ (4). Subtracting, (3) $-$ (4) gives $24v = D$ which rearranges to $\dfrac{D}{v} = 24$ (hours), which is the time taken for the log to float downstream at the speed of the current alone.

21. A Every multiple of 6 is a holiday. For $n > 0$, the days in between $6n$ and $6n + 6$ will contain three consecutive non-primes $6n + 2$ (divisible by 2), $6n + 3$ (divisible by 3) and $6n + 4$ (divisible by 2). Using H to represent a holiday, W a working day and ? for an unknown day, we see the numbers $6n$ to $6n + 6$ form the pattern H?WWW?H. We are searching for the pattern HWH but this will not fit into the pattern shown above. Hence, the only days that can possibly give HWH must occur in the first week of the month. The days 1, 2, 3, 4, 5, 6 have the pattern WHHWHH so contain one occurrence of HWH.

The first day of the month could possibly be a working day between two holidays, but a quick check shows that of course day 40 is also a working day.

22. C Let n be the smallest of the integers. The four consecutive integers have sum $n + n + 1 + n + 2 + n + 3 = 4n + 6$. Then the four possible sums formed by taking three of these at a time are

$4n + 6 - n = 3n + 6$ (divisible by 3 so not prime)

$4n + 6 - (n + 1) = 3n + 5$ (if n is odd then this is even, so is not prime)

$4n + 6 - (n + 2) = 3n + 4$

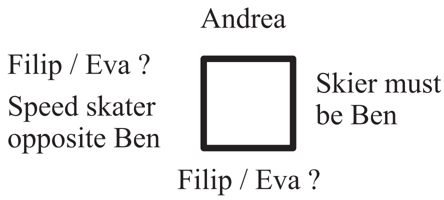
$4n + 6 - (n + 3) = 3n + 3$ (divisible by 3 so not prime)

Hence we are looking for the smallest n for which neither $3n + 4$ nor $3n + 5$ is prime.

n	$3n + 4$	$3n + 5$
1	7 prime	8 not prime
2	10 not prime	11 prime
3	13 prime	14 not prime
4	16 not prime	17 prime
5	19 prime	20 not prime
6	22 not prime	23 prime
7	25 not prime	26 not prime

When $n = 7$, none of the four sums is prime.

23. **A** Put Andrea at the top of the table. Since Eva and Filip are next to each other, Ben must be next to Andrea. If Ben was to her right then he would be opposite the skier, whom we know is to her left. But Ben is opposite the speed skater. So Ben is to Andrea's left and he is the skier. We are now at the stage shown on the diagram.



The hockey player is not opposite Ben, but has a woman to the left. Therefore the hockey player must be at the bottom of the table and Eva is on the left side of the table. So Eva is the speed skater.

24. **B** The smallest possible first digit in the year is 2. This eliminates December from the month, leaving only months that start with 0, or 10 (11 cannot be used because it has a repeated digit). So we will definitely use 0 for the month. Not using 0 or 2 for a day leaves us with 13 to 19, or 31, all of which use a 1. Since we are using 0 and 1 for the month and day, the earliest year we could have is 2345. The earliest month is then 06, and the earliest day that can be made is 17. This gives 17.06.2345. So the month is June.

25. **D** P2015 shakes hands with each of the other 2015 candidates (including P2016). In particular he shakes hands with P1, leaving P1 with no more handshakes to perform. P2014 then shakes hands with each of the candidates, not including P1. But then P2 has used up his two handshakes (once with P2015 and once with P2014). Proceeding in this way, we see that P2013 uses up the third shake for P3, P2012 uses up the fourth shake for P4, and so on, until we get towards the half-way point. The handshakes of P1009 again include P2016 and use up the 1007th shake for P1007. By this point, P2016 has shaken hands with each of P1009 to P2015, and now must provide the 1008th shake for P1008, a total of 1008 shakes for P2016.