GREY 'KANGAROO' MATHEMATICAL CHALLENGE

Thursday 15th March 2018

Organised by the United Kingdom Mathematics Trust

SOLUTIONS

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1. **E** When we simplify the calculation, we obtain $\dfrac{3 \times 2018}{4 \times 2018}$ which can then be cancelled down to give $\dfrac{3}{4}$.

2. **E** When the letters in each word are examined in turn, it can be seen that the letters R, B and L do not have a vertical line of symmetry while the letters M, O and T do. Hence the only word whose letters all have a vertical line of symmetry is TOOT.

3. **B** Each of the nets shown has two faces of each colour. The question tells us that the two faces are opposite each other, so they cannot have an edge in common. This eliminates all the nets except net B.

4. **D** Write each number in the expression as a product of prime factors to obtain $2 \times 2 \times 3 \times 3 \times 2 \times 7 = 2 \times 3 \times * \times 7$. It can then be seen that $2 \times 2 \times 3 = *$ and hence the value of * is 12.

5. **D** The largest possible value of $a - b$ comes from subtracting the smallest possible value of b from the largest possible value of a. Hence the largest possible value is $10 - (-5) = 10 + 5 = 15$.

6. **C** Since the horizontal lengths of the small rectangles are 10 cm, the length of the large rectangle is 2×10 cm = 20 cm. Also, since the sum of the heights of five small rectangles is equal to the length of the large rectangle, the height of a small rectangle is 20 cm ÷ 5 = 4 cm. Therefore, the perimeter of the large rectangle is $(2 \times 20 + 2 \times 10 + 4 \times 4)$ cm = 76 cm.

7. **E** From the diagram, it can be seen that the distance between the centres of the two circles is 11 cm – 2 × the radius of a circle or equivalently 11 cm – the diameter of each circle. Since this diameter is 7 cm, the distance between the centres is 11 cm – 7 cm = 4 cm.

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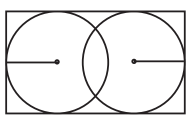
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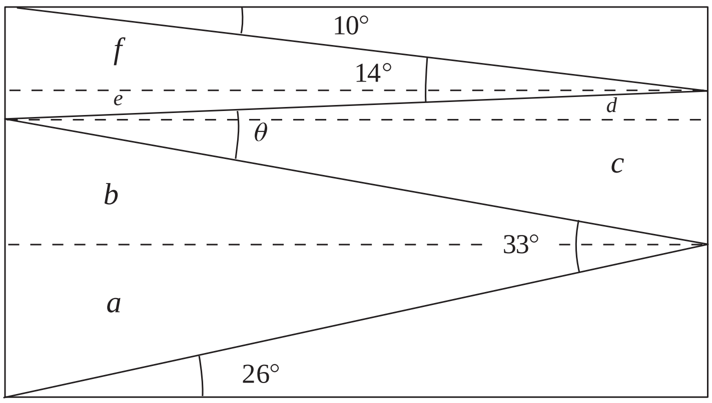
8. **D** The area of square *ABCD* is (3×3) cm^2 = 9 cm^2. Hence the area of each piece is $\frac{1}{3} \times 9$ cm^2 = 3 cm^2. Since the area of a triangle is equal to half its base multiplied by its perpendicular height, we have $\frac{1}{2} \times DM \times DC = 3$ cm^2. Therefore $\frac{1}{2} \times DM \times 3\,\text{cm} = 3\,\text{cm}^2$ and hence the length of DM is 2 cm.

9. **B** Let the three missing digits from left to right be a, b and c. Consider the final digit ‘2’ of the answer. This is the last digit of $3 \times b$ and hence $b = 4$. Also, note that if a were 2 or more, then the answer would be more than 400. Therefore $a = 1$. Hence the multiplication is $13 \times 24 = 312$, giving $c = 1$. Therefore the sum of the digits scribbled out is $a + b + c = 1 + 4 + 1 = 6$.

10. **C** The possible rectangles containing 40 equal squares have dimensions (length × height) 1 × 40, 2 × 20, 4 × 10, 5 × 8, 8 × 5, 10 × 4, 20 × 2 and 40 × 1. The question tells us that when it is divided into squares, the rectangle contains more than one row and also that it has a middle row and hence has an odd number of rows. The number of rows of squares the rectangle contains is related to its height which must be an odd value. Therefore the rectangle has dimensions 8 × 5. Hence, when Andrew coloured the middle row, he coloured eight squares. Therefore the number of squares he did not colour is 40 – 8 = 32.

11. **C** Consider the three possible positions of the lion in turn. If the lion were in room 1, the notes on both room 1 and room 2 would be true. Hence the lion is not in room 1. If the lion were in room 2, the notes on all three rooms would be false. Hence the lion is not in room 2. However, if the lion were in room 3, the notes on room 1 and room 3 would be false but the note on room 2 would be true. Hence the lion is in room 3.

12. **A** Add to the diagram three lines parallel to two of the sides of the rectangle, creating angles a, b, c, d, e and f as shown.



Since alternate angles formed by parallel lines are equal, we have $a = 26°$ and $f = 10°$. Since $a + b = 33°$ and $e + f = 14°$, we have $b = 7°$ and $e = 4°$. Similarly, since alternate angles are equal, we have $c = b = 7°$ and $d = e = 4°$. Therefore $\theta = c + d = 7° + 4° = 11°$.

13. **D** First note that none of 2, 4 or 5 can occur as the final digit of a two-digit prime. Also 21 and 51 are not prime since they are divisible by 3. Therefore the primes which might occur in Alice's list are 2, 3, 5, 13, 23, 31, 41, 43 and 53. Since 1 and 4 are not prime, they must occur in Alice's list as digits in some two-digit prime. If 41 is not used, the only two-digit primes which could use the 4 and the 1 are 43 and one of 31 or 13. However, this would repeat the digit 3. Hence 41 must be in Alice's list together with one of the groups 2, 3, 5 or 2, 53 or 5, 23.

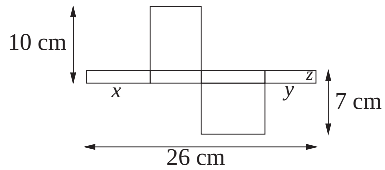
14. **D** The slogan suggests that there will be at most 15 days without sun in 2018. In this case it is possible to have a run of 31 days without having two consecutive days of sun by alternating a sunny day with a non-sunny day, starting with a sunny day. Hence Will Burn has to stay for one further day, or 32 days in total, to be certain of having two consecutive days of sun.

15. **A** The total of the sums of the three rows of the table is equal to the sum of all the digits from 1 to 9, which is 45. Similarly, the total of the sums of the three columns of the table is also equal to 45. Hence James' six answers add to 45 + 45 = 90. The sum of the five answers given is 12 + 13 + 15 + 16 + 17 = 73 and hence his sixth answer is 90 – 73 = 17. [*It is left as an exercise to find a possible arrangement of the digits 1 to 9 that actually gives these six sums.*]

16. **B** Let the distance between the first two points be x cm. Consider the other nine points. The sum of their distances from the first point is $(2018 - x)$ cm and the sum of their distances from the second point is $(2000 - x)$ cm. Also, for each of these nine points, its distance from the second point is x cm less than its distance from the first point. Therefore, when we total the distances over the nine points, we obtain $9x = (2018 - x) - (2000 - x)$ and hence $9x = 18$. Therefore $x = 2$ and so the distance between the first and second points is 2 cm.

17. **E** The number of votes left to be cast is $130 - (24 + 29 + 37) = 40$. Let the number of these votes Alan receives be x. Since Katie is the closest challenger to Alan, for Alan to be certain of having the most votes, $37 + x > 29 + 40 - x$. Therefore $2x > 32$ and hence $x > 16$. Therefore Alan needs at least 17 more votes to be certain to finish with the most votes.

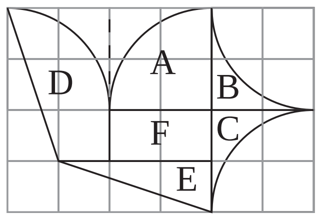
18. **C** Let the dimensions of the box be x cm by y cm by z cm as indicated. From the diagram, we have $2x + 2y = 26$, $x + z = 10$ and $y + z = 7$. When we add the last two of these, we obtain $x + y + 2z = 17$ and, when we then double this, we obtain $2x + 2y + 4z = 34$. Therefore $4z = 34 - 26 = 8$ and hence $z = 2$. Therefore $x = 8$ and $y = 5$ and hence the volume of the box in cm^3 is $8 \times 5 \times 2 = 80$.



19. **E** Let the amount Chloe spent be £x. Therefore Amy spent £$1.6x$ and Becky spent £$0.15x$. The total amount spent is £55 and hence $x + 1.6x + 0.15x = 55$. Therefore $2.75x = 55$, which has solution $x = 20$. Hence Amy spent £$(1.6 \times 20) =$ £32.

20. **B** Let the width of the pool be x m. Therefore the total distance Ruth runs is $5(2 \times 50 + 2x)$ m $= (500 + 10x)$ m. The total distance Sarah swims is 6×50 m $= 300$ m. Since Ruth runs three times as fast as Sarah swims, $500 + 10x = 3 \times 300$. Therefore $10x = 400$ and hence $x = 40$.

21. **D** Let each of the small squares in the grid have side-length x cm. Remove the shading and divide the dove into regions as shown.



It can be seen that the regions marked A and B combine to make a square of side $2x$ cm and hence of area $4x^2$ cm^2. Similarly, regions C, D and E combine to make a rectangle with sides $2x$ cm and $3x$ cm and hence area $6x^2$ cm^2. Finally, region F is a rectangle with sides $2x$ cm and x cm and hence area $2x^2$ cm^2. Since the total area of the dove is 192 cm^2, we have $4x^2 + 6x^2 + 2x^2 = 192$ and hence $12x^2 = 192$. Therefore $x^2 = 16$ and hence $x = 4$. Hence the flag has length (6×4) cm = 24 cm and height (4×4) cm $= 16$ cm.

22. **C** The dominoes in the line contain three ends with four spots and three ends with six spots, as shown in diagram 1. Therefore, a correctly arranged set of these dominoes will have four spots at one end and six spots at the other, as is currently the case. Hence, Paul does not need to move either of the end dominoes.

If he swaps the third and the fifth dominoes from diagram 1, he obtains the row shown in diagram 2 which has the same number of spots in the adjacent ends of the fourth, fifth and sixth dominoes. Next, if he swaps the second and third dominoes from diagram 2 to obtain the line shown in diagram 3, he has matched the spots at the adjacent ends of the first and second dominoes. Finally, rotating the third domino in diagram 3, he obtains the correctly arranged line as shown in diagram 4. This shows that it is possible to arrange the dominos correctly in three moves.

To see that two moves is not sufficient, note that, whatever else needs to happen, the two

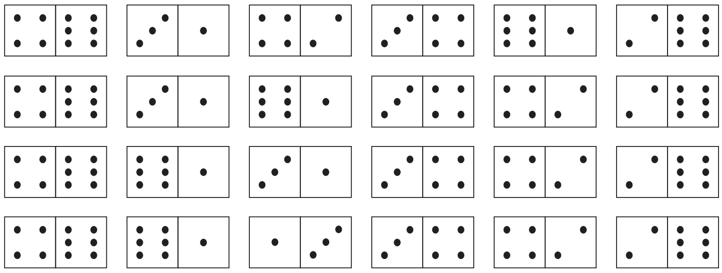
1s must be correctly placed next to each other. To do that requires one of the dominos with a 1 to be rotated and then one pair of dominos to be swapped so that the two 1s are now next to each other. A similar argument applies to the two dominos with a 3. However, this is not possible in only two moves. Therefore the smallest number of moves he needs to make is 3.

DIAGRAM 1

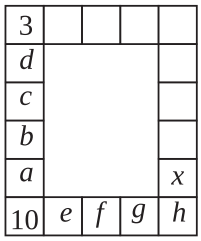
DIAGRAM 2

DIAGRAM 3

DIAGRAM 4



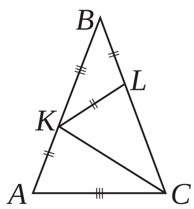
23. **B** Let the values she writes in some of the cells be as shown in the diagram. Since the number in any cell is equal to the sum of the numbers in the two cells that border it, we have $a = 10 + b$ and hence $b = a - 10$. Also we have $b = c + a$ giving $c = -10$, $c = b + d$ giving $d = -a$ and $d = c + 3$ giving $d = -7$. Therefore $a = 7$ and $b = -3$ and it is now possible to work out the remaining unknown values. Since $10 = 7 + e$, we obtain $e = 3$.



Similarly, $e = 10 + f$ giving $f = -7$ and then $f = e + g$ giving $g = -10$. Also $g = f + h$ giving $h = -3$ and $h = g + x$ giving $x = 7$.

24. **C** Let the number of jumps Viola has already made be n and let the total distance she has jumped in these n jumps be T m. Since the total distance jumped is equal to the average distance jumped multiplied by the number of jumps, the information in the question tells us that $T = 3.8n$ and that $T + 3.99 = 3.81(n + 1)$. Therefore $3.8n + 3.99 = 3.81n + 3.81$ and hence $0.18 = 0.01n$, which has solution $n = 18$. In order to increase her average distance to 3.82 m with her final jump, she must jump x m, where $T + 3.99 + x = 3.82(n + 2)$. Therefore $3.8n + 3.99 + x = 3.82n + 7.64$ and hence $x = 0.02n + 3.65$. Since we have already shown $n = 18$, we have $x = 0.36 + 3.65 = 4.01$. Hence the distance she needs to jump with her final jump is 4.01 m.

25. **A** Since triangle ABC is isosceles with $AB = BC$ and we are given that $LB = AK$, the other parts of the equal sides must themselves be equal. Hence $LC = BK = AC$. Draw in line KC as shown to form triangles ACK and LCK. Since $AK = KL$, $AC = LC$ and KC is common to both, triangles ACK and LCK are congruent and hence $\angle KAC = \angle CLK$.



Let the size of $\angle LBK$ be $x°$. Since $KL = LB$, triangle KLB is isosceles and hence $\angle BKL = x°$. Since an exterior angle of a triangle is equal to the sum of the interior opposite angles, $\angle KLC = 2x°$ and hence $\angle KAC = 2x°$. Since the base angles of an isosceles triangle are equal, $\angle ACL = 2x°$. Therefore, since angles in a triangle add to 180°, when we consider triangle ABC, we have $x° + 2x° + 2x° = 180°$ and hence $x = 36$. Therefore the size of $\angle ABC$ is 36°.