Solutions to the European Kangaroo Grey Paper

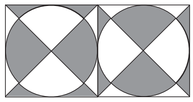
1. **C** Since angles in a triangle add to 180° and one angle is given as 90°, the two blank angles in the triangle add to 90°. Since angles on a straight line add to 180°, the sum of the two marked angles and the two blank angles in the triangle is $2 \times 180^\circ = 360^\circ$. Therefore the sum of the two marked angles is $360^\circ - 90^\circ = 270^\circ$.

2. **D** Jenny subtracted 26 instead of adding 26 and obtained −14. Therefore to obtain the answer she should have obtained, she must add two lots of 26 to −14. Therefore the number she should have obtained is $-14 + 2 \times 26 = -14 + 52 = 38$.

3. **B** When the card is turned about its lower edge, the light grey triangle will be at the top and the dark grey triangle will be on the left. When this is turned about its right-hand edge, the light grey triangle will be at the top and the dark grey triangle will be on the right. Therefore Joanna will see option B.

4. **C** The percentage of teachers coming to school by car is one fifth of the percentage of teachers coming to school by bicycle. Therefore, the number of teachers coming to school by car is $\frac{1}{5} \times 45 = 9$.

5. **C** The area of the whole rectangle is 200 cm^2. Suppose the rectangle is cut in two by a vertical cut joining the midpoints of its longer edges and the right-hand half is then given a quarter turn about its centre to produce the arrangement as shown. It can then be seen that every grey region has a corresponding white region of the same shape and size. Hence, the total area of the grey regions is half the area of the rectangle and so is 100 cm^2.

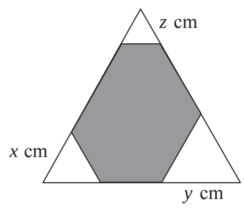


6. **B** Since all the parts cut are of the same length, Alex will obtain twice as many parts from his 2 metre piece of rope as he does from his 1 metre piece of rope. Hence, the total number of parts he obtains will always be a multiple of 3, and he can make it any multiple of 3. Of the options given, only 8 is not a multiple of 3 and so could not be obtained.

7. **C** Any route starts by going from S to P or S to Q or S to R. Any route starting S to P must then go to Q and then has the choice of going clockwise or anticlockwise round triangle QSR, giving two possible routes. By a similar argument, there are two routes that start S to R. For those routes that start by going from S to Q, there is then the choice of going clockwise or anticlockwise round quadrilateral $QPSR$, giving two more routes. Therefore there are six possible routes in total.

8. **E** For the one red bead to be 10% of the final number of beads, there must be nine blue beads representing 90% of the final number of beads. Therefore the number of beads Petra must remove is $49 - 9 = 40$.

9. **A** Let the lengths of the sides of the equilateral triangles that are cut off be x cm, y cm and z cm, as shown in the diagram. The length of a side of the large equilateral triangle is $\frac{1}{3} \times 60\text{ cm} = 20\text{ cm}$. The perimeter of the irregular hexagon is 40 cm. Therefore we have

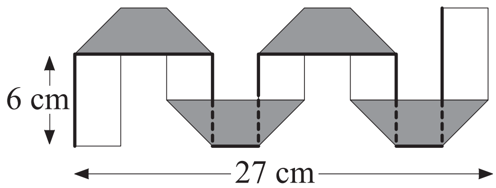


$40 = x + (20 - x - y) + y + (20 - y - z) + z + (20 - z - x)$. Hence $40 = 60 - (x + y + z)$ and therefore $x + y + z = 20$. Therefore the sum of the perimeters of the triangles cut off is $(3x + 3y + 3z)\text{ cm} = 60\text{ cm}$.

10. D Let x be the age of each of Tim, Tom and Tam. Therefore the age of both Jon and Jim is $x - 3$. Therefore the sum of all their ages is $3x + 2(x - 3) = 5x - 6$ and $5x - 6$ can also be written as $5(x - 1) - 1$. Hence the sum of their ages is always one less than a multiple of 5. Of the options given, the only number for which this is true is 89 (when Tim, Tom and Tam are 19 and Jon and Jim are 16).

11. D Let the length of the shorter of the two parallel sides of the grey trapeziums be x cm. Since the folded shape is 27 cm long and the strip is 3 cm wide, we have $3 + x + 3 + x + 3 + x + 3 + x + 3 = 27$ which has solution $x = 3$. Hence the length of the longer of the two parallel sides of the grey trapezium is $(3 + x + 3)$ cm $= 9$ cm. Also, since the height of each trapezium is equal to the width of the strip, the height is 3 cm and hence the height of each of the small rectangles is $(6 - 3)$ cm. Therefore the total length of the strip (along the edge marked in the diagram) is

$$(6 + 9 + 3 + 3 + 3 + 3 + 3 + 9 + 3 + 3 + 3 + 3 + 6) \text{ cm} = 57 \text{ cm}.$$



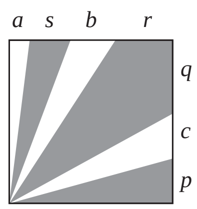
12. B Since the two kangaroos jump at the same time and in the same direction, Ing will catch Bo when they have jumped the same distance. After n jumps, Bo has jumped $6n$ metres whereas Ing has jumped $(1 + 2 + 3 + \ldots + n)$ metres. The sum of the whole numbers from 1 to n is given by the formula $\frac{1}{2}n(n + 1)$ (it is left as an exercise for the reader to prove this) and hence $\frac{1}{2}n(n + 1) = 6n$. Hence $n + 1 = 12$, which has solution $n = 11$. Therefore Ing will catch Bo after 11 jumps.

13. A The pair who play in the final will have played three matches. Only Glen and Carl play three matches so they are the pair who play in the final.

14. D A standard die has a total of 21 dots on its faces. The faces that are glued together have the same number of dots. Since the die in the centre of the solid has all its faces glued to other dice, the sum of the dots that are *not* on the surface of the solid is 2×21. Therefore, the number of dots on the surface of the solid is $7 \times 21 - 2 \times 21 = 5 \times 21 = 105$.

15. B Let the number of boys in the class be x and let the number of girls be y. Since one third of the boys sit with a girl and one half of the girls sit with a boy, we have $\frac{1}{3}x = \frac{1}{2}y$ and hence $\frac{2}{3}x = y$. The total number of students in the class is 20. Therefore $x + \frac{2}{3}x = 20$. Hence $\frac{5}{3}x = 20$, which has solution $x = 12$. Therefore there are 12 boys in the class.

16. D Since the area of the square is 36 cm^2, the length of a side of the square is 6 cm. Since the shaded area is 27 cm^2, the area not shaded is $(36 - 27)$ cm^2 $= 9$ cm^2. Let a cm, b cm and c cm be the lengths of the parts of the sides shown on the diagram. The area of a triangle is $\frac{1}{2} \times$ base $\times$ height.



Therefore $\frac{1}{2} \times a \times 6 + \frac{1}{2} \times b \times 6 + \frac{1}{2} \times c \times 6 = 9$ and hence $a + b + c = 3$. Therefore, since $(a + b + c) + (p + q + r + s)$ is the sum of the lengths of two sides of the square and so is equal to 12 cm, the value of $p + q + r + s$ is 9 cm.

17. D Since Theo thinks it is 12:00, his watch shows 12:05. Therefore, the correct time is 12:15 since Theo's watch is 10 minutes slow. Since Leo's watch is five minutes fast, at 12:15 his watch will show 12:20. However, Leo thinks his watch is 10 minutes slow so he thinks the time is 12:30.

18. E The total number of cupcakes eaten by the girls is $12 \times 1\frac{1}{2} = 18$. Two girls ate no cakes so the 18 cupcakes were eaten by 10 girls. Since no-one ate more than two cupcakes, the maximum number of cupcakes the 10 girls could have eaten is $10 \times 2 = 20$. For every girl who eats only one cupcake, the total of 20 is reduced by 1. Hence the number of girls who ate only one cupcake is $20 - 18 = 2$ and hence the number of girls who ate two cupcakes is 8.

19. D Let x be the number of waffles Little Red Riding Hood delivered to each of the grannies. She delivered x waffles to the third granny and so, since the Big Bad Wolf eats half of the waffles in her basket just before she enters each granny's house, she must have arrived with $2x$ waffles. Therefore she had $3x$ waffles before giving the second granny her waffles and hence had $2 \times 3x = 6x$ waffles when she arrived. Therefore she had $6x + x = 7x$ waffles before giving the first granny her waffles and hence had $2 \times 7x = 14x$ waffles in her basket when she arrived. Hence, since we do not know the value of x, the only numbers that definitely divide the number of waffles she started with are the numbers that divide 14, namely 1, 2, 7 and 14. Therefore, of the options given, only 7 definitely divides the number of waffles she started with.

20. E The diagram below shows the day on which certain cubes turned grey.

2	1	2	
1	0	1	2
2	1	2	
	2		

top layer

	2		
2	1	2	
	2		

second layer

	2		

third layer

As can be seen, at the end of the second day there are $11 + 5 + 1 = 17$ grey cubes.

21. C The only ways to express 16 as the product of two different positive integers are 1×16 and 2×8. The only ways to express 225 as the product of two different positive integers are 1×225, 3×75, 5×45 and 9×25. Therefore, since both integers in the first pair must be smaller than both integers in the second pair, the only possible combination is for the two smallest integers to be 2 and 8 and for the two largest integers to be 9 and 25. There are no other integers written on the blackboard since they would need to be different from these four, be less than 9 and greater than 8. Hence the sum of the integers written on the blackboard is $2 + 8 + 9 + 25 = 44$.

22. A Let the radius of the circle with centre A be x cm. Therefore, since the circles drawn on each side of the pentagon touch, the radius of the circle with centre B is $(16 - x)$ cm. Similarly, the radius of the circle with centre C is $\big(14 - (16 - x)\big)$ cm $= (x - 2)$ cm, the radius of the circle with centre D is $\big(17 - (x - 2)\big)$ cm $= (19 - x)$ cm and the radius of the circle with centre E is $\big(13 - (19 - x)\big)$ cm $= (x - 6)$ cm. However, the radius of the circle with centre E is also equal to $(14 - x)$ cm since the circle with centre A has radius x cm. Therefore $14 - x = x - 6$, which has solution $x = 10$. Hence the radii of the circles centres A, B, C, D and E are 10 cm, 6 cm, 8 cm, 9 cm and 4 cm respectively. Therefore point A is the centre of the largest circle Sephideh draws.

23. D Let the integers written on the small cubes in the bottom layer be arranged as shown.

a	b	c
d	e	f
g	h	i

Hence, since the integers written on the cubes in the second and third layers are the sum of the integers on the four cubes underneath, the following is written on the cubes in the second layer.

$a + b + d + e$	$b + c + e + f$
$d + e + g + h$	$e + f + h + i$

Therefore the integer written on the top cube is

$$(a + b + d + e) + (b + c + e + f) + (d + e + g + h) + (e + f + h + i)$$
$$= (a + b + c + d + e + f + g + h + i) + (b + d + f + h) + 3e.$$

Since the sum of the integers on the bottom layer is 50, the integer written on the top cube is equal to $50 + (b + d + f + h) + 3e$. To maximise this, we first require e to be as large as possible which will be obtained when the other eight integers are as small as possible. Therefore $e = 50 - (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 14$. Secondly, $(b + d + f + h)$ should now be made as large as possible and hence b, d, f and h are 5, 6, 7 and 8 in any order. Therefore $(b + d + f + h) = 5 + 6 + 7 + 8 = 26$. Hence the greatest possible integer she can write on the top cube is $50 + 26 + 3 \times 14 = 118$.

24. C Let the numbers of passengers in the five carriages be p, q, r, s and t respectively with p, q, r, s and t all at least 1. Consider the neighbours of the passengers in the first and second carriages. Since each passenger has 5 or 10 neighbours, we have $p - 1 + q = 5$ or 10 and $p + q - 1 + r = 5$ or 10. Therefore $p + q = 6$ or 11 and $p + q + r = 6$ or 11. However, we know that $r \geqslant 1$ and hence $p + q \leqslant 10$ and therefore $p + q = 6$ and $r = 5$. Similarly, considering the neighbours of the passengers in the fourth and fifth carriages, we obtain $s + t = 6$ and (again) $r = 5$. Therefore, the total number of passengers in the train is $6 + 5 + 6 = 17$. (Note that while the total number of passengers in the train is uniquely determined, the arrangement of these passengers in all but the centre carriage is not unique.)

25. A Note first that a small cube in the centre of a face of the large cube will only appear on one face while a cube appearing on the edge of a face of the large cube will appear on two faces and a cube appearing at a corner of the face of the large cube will appear on three faces. Hence, the total number of white faces on the edge of the large cube is an even number and the total number of white faces on the corners of the large cube is a multiple of 3. The five faces shown contain 1 centre white face from 1 small white cube, 12 edge white faces and 5 corner white faces. Therefore, since the total number of white faces on the corners is a multiple of 3, the missing face contains 1 or 4 white faces at its corners. None of the options contains 4 white corners so the missing face contains one white corner as in options A and E, making 6 in total. These 6 faces come from $6 \div 3 = 2$ small white cubes. Both options A and E have two white faces on their edges, making 14 in total over the six faces from $14 \div 2 = 7$ white cubes. Hence the number of white cubes whose positions we know is $1 + 7 + 2 = 10$. The large cube is made with 12 small white cubes so there are still two more to be placed. One can be at the centre of the large cube and the only place the remaining cube can be is at the centre of the missing face. Therefore, the missing face contains one centre white face, two edge white faces and one corner white face. Hence, the missing face is A. (This proof shows that the only possible missing face for such a cube is face A. It is left to the reader to check that the five given faces and face A can indeed be fitted together consistently to form the faces of a cube.)