Solutions to the European Kangaroo Grey Paper 2015

1. **E** In diagrams A, C and D, the letters ‘N’, ‘R’ and ‘G’ respectively have been reversed. In diagram B, the letters are not in the order they appear on the original umbrella. Hence only option E shows part of the original umbrella.

(This is immediately clear if you turn the question paper round so the handles are pointing up rather than down.)

2. **E** Round each number in the product to one significant figure to give $2 \times 500 = 1000$. Hence 1000 is closest to the given product.

3. **B** From the diagram, the length of a small rectangle is twice the width. Hence the length of a small rectangle is 20 cm. Therefore the length of the large rectangle, in cm, is $20 + 2 \times 10 = 40$.

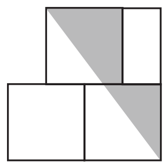
4. **D** The numbers in options A, B and E are clearly integers. In option C, 2 + 0 + 1 + 3 = 6 so, using the divisibility rule for divisibility by 3, $\frac{2013}{3}$ is also an integer. However, while 2000 is divisible by 4, 14 is not so only $\frac{2014}{4}$ is not an integer.

5. **D** The perimeter of the equilateral triangle is (6 + 10 + 11) cm = 27 cm. Hence the length of the sides of the equilateral triangle is 27 cm ÷ 3 = 9 cm.

6. **D** In five seconds, the cyclist will have travelled 5 × 5 m = 25 m. Hence the wheels will have made 25 ÷ 1.25 = 20 complete turns.

7. **B** Suppose another child were to join the class. The question tells us that then one of the two conditions would no longer be true. For this to happen, there must be no day of the week available for a new boy to have been born on and no month of the year available for a new girl to have been born in. Hence there must be 7 boys and 12 girls currently in the class and so there are 19 children in total in the class.

8. **C** The centre of the top square is directly above the common edge of the lower two squares. Hence a rectangle half the size of the square, and so of area $\frac{1}{2}$ cm^2, can be added to the diagram to form a right-angled triangle as shown. The area of the shaded region and the added rectangle is equal to $\left(\frac{1}{2} \times 2 \times 1\frac{1}{2}\right)$ cm^2 $= 1\frac{1}{2}$ cm^2. Hence the area of the shaded region, in cm^2, is $1\frac{1}{2} - \frac{1}{2} = 1$.



9. **B** The sum of the digits on the left-hand side of the equation is 24. Hence the equation formed by inserting + and − signs must be equivalent to $12 - 12 = 0$. The smallest number of the given digits required to make 12 is three (2 + 5 + 5). Therefore the smallest number of asterisks that can be replaced by + is two and they would be placed in front of two of the three 5s.

10. **D** One litre is equivalent to 1000 cm^3. Hence 15 litres falling over an area of one square metre is equivalent to 15 000 cm^3 falling over an area of 10 000 cm^2. Therefore the amount the water in Michael's pool would rise by, in cm, is 15 000 ÷ 10 000 = 1.5.

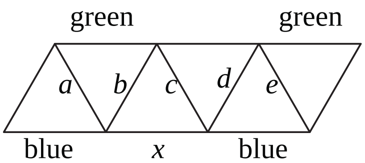
11. **E** The maximum number of leaves the bush could have is $10 \times 5 = 50$. Each branch that has two leaves and a flower instead of five leaves reduces the number of leaves the bush has by three. Therefore the total number of leaves the bush has is of the form $50 - 3n$ where n is the number of branches with two leaves and a flower. It is straightforward to check that none of options A to D has that form and so the answer is E.

12. **C** Let the mean score of the 40% of the students who failed the test be x. The information in the question tells us that $0.6 \times 8 + 0.4 \times x = 6$. Hence $0.4x = 1.2$ and so $x = 3$.

13. **C** The area of the darker triangle in the diagram is $\frac{1}{8}$ of the area of the whole square and this also represents the difference between the area of the square and the area of the pentagon. Hence $\frac{1}{8}$ of the area of the square is equal to 1 unit and so the area of the whole square is 8 units.

14. **B** Let the length of the rectangle be x cm and let the width be y cm. The information in the question tells us that $2x + y = 44$ and $x + 2y = 40$. Add these two equations to obtain $3x + 3y = 84$. Hence $x + y = 28$ and so the perimeter, which is equal to $2(x + y)$ cm, is 56 cm.

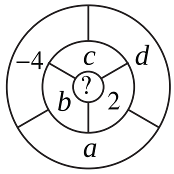
15. **A**



Label the internal sides of the diagram a, b, c, d and e as shown. The side labelled a is in a triangle with a green side and in a triangle with a blue side and so is to be coloured red. This is also the case for the side labelled e. Hence, the side labelled b is in a triangle with a red side and a green side and so is to be coloured blue while the side labelled d is in a triangle with a red side and a blue side and so is to be coloured green. Finally, the side labelled c is in a triangle with a green side and in a triangle with a blue side and so is to be coloured red. Hence the side labelled x is in a triangle with a side that is to be coloured blue (side b) and with a side that is to be coloured red (side c). Therefore the side labelled x is to be coloured green.

16. **B** All the students have given different answers to the questions so only one, at most, can be telling the truth. Suppose no student is telling the truth; but then Daniel is telling the truth, contradicting this. Hence exactly one student, Ellen, is telling the truth and so only one student had done their homework.

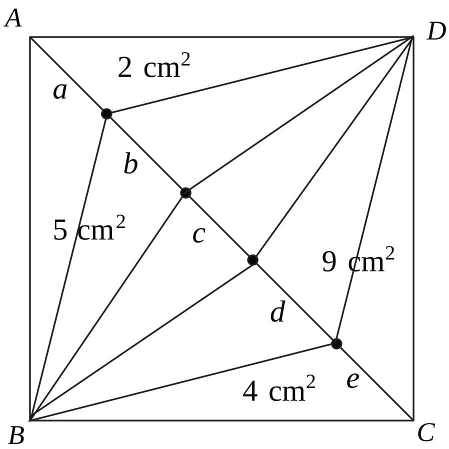
17. **E** Let the numbers in the four regions that are neighbours to -4 be a, b, c and d as shown in the diagram. The question tells us that $a + b + c + d = -4$. However, we also know that $a + b + c + d + ? = 2$ and hence $? = 6$.



(*Note*: The values $a = d = -4$ and $b = c = 2$ give a complete solution to the problem).

18. **C** Let the five integers be a, b, c, d and e with $a \leqslant b \leqslant c \leqslant d \leqslant e$. The smallest total is 57, which is an odd number so $b \neq a$. Similarly, the largest total is 83, which is also an odd number so $d \neq e$. Hence we now have $a < b \leqslant c \leqslant d < e$ and $a + b = 57$ and $d + e = 83$. Only one possible total remains and so $b = c = d$ with $c + d = 70$. This gives $c = d (= b) = 35$ and therefore e, the largest integer, is $83 - 35 = 48$ (whilst $a = 22$).

19. D Label the corners of the square A, B, C and D going anticlockwise from the top left corner. Draw in the lines from each marked point on the diagonal to B and to D. All the triangles with a base on the diagonal and a vertex at B or D have the same perpendicular height. Hence their areas are directly proportional to the length of their bases. The two triangles with e as their base both have area 4 cm^2. Hence the two triangles with base d both have area 5 cm^2. Similarly the two triangles with base a have area 2 cm^2, and so the two triangles with base b have area 3 cm^2. Finally, since the area of the square is 30 cm^2, the two triangles with base c have area 1 cm^2. Since the triangles with largest area are those of area 5 cm^2, the longest base is that part labelled d.

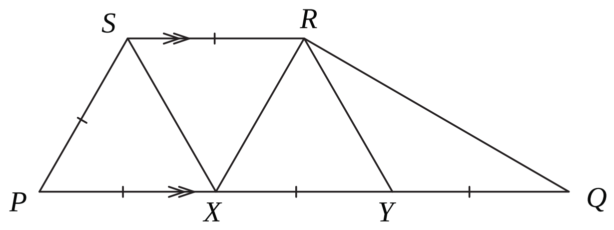


20. A The remaining kangaroos weigh $(100-25-60)\% = 15\%$ of the total weight. However, this cannot be made up of the weights of more than one kangaroo since the information in the question tells us that the lightest two weigh 25% of the total. Hence there are $2 + 1 + 3 = 6$ kangaroos in the mob.

21. D Since there is no overlap of wires, each vertex of the cube requires at least one end of a piece of wire to form it. A cube has eight vertices and each piece of wire has two ends, so the minimum number of pieces of wire required is $8 \div 2 = 4$.

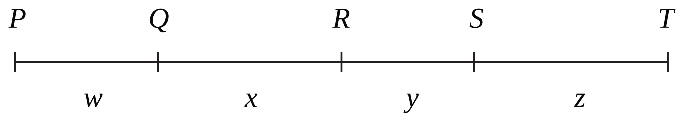
Such a solution is possible, for example with wires of length 1 cm, 2 cm, 4 cm and 5 cm, the arrangement of which is left for the reader.

22. D

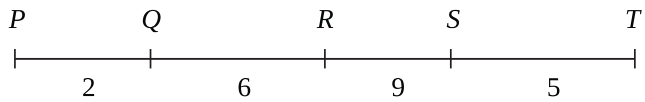


The diagram shows the trapezium described with points X and Y added on PQ so that $PX = XY = YQ = PS = SR$. Since angle $RSP = 120°$, angle $SPQ = 60°$ using co-interior angles and so triangle SPX is equilateral. Similarly, it can easily be shown that triangles SXR and RXY are also equilateral. In triangle RYQ we then have $RY = YQ$ and angle $RYQ = 120°$ using angles on a straight line adding to 180°. Hence triangle RYQ is isosceles and so angle $YQR = \frac{1}{2}(180° - 120°) = 30°$. Therefore angle PQR is 30°.

23. A Let the five points be P, Q, R, S and T with individual distances between them of w, x, y and z as shown.



The maximum distance between any two points is 22 so $PT = 22$. The next largest distance is 20 and, since no distance is 1, this is either PS or QT. Assume $QT = 20$ so that $PQ = w = 2$. The next largest distance is 17 and, since no distance is 3 this cannot be QS or RT and so is PS. Hence $ST = z = 5$ and $QS = 15$. The remaining distances are 6, 8, 9 and k which represent the lengths of PR, QR, RS and RT in some order with $QR + RS = 15$. Since $k > 9$, the only possible pair of distances adding to 15 is 6 and 9 and so $QR = x = 6$ and $RS = y = 9$ (since $QR = 9$ and $RS = 6$ would mean two distances are 11) leaving $PR = 8$ and $RT = k = 14$. Hence the value of k is 14 with the distances between the points taking the values shown below.



(*Note*: The same distances would result but in the reverse order if we assumed $PS = 20$.)

24. C Suppose we obtain the remainder r when we divide 2015 by the positive integer d. Then $r \leqslant d - 1$. Also, with $d \leqslant 1000$ the quotient must be at least 2. This suggests that to get the largest possible remainder we should aim to write 2015 in the form $2d + (d - 1)$. The equation $2015 = 2d + (d - 1)$ has the solution $d = 672$. So we obtain the remainder 671 when we divide 2015 by 672. If we divide 2015 by an integer $d_1 < 672$, the remainder will be at most $d_1 - 1$ and so will be less than 671. If we divide 2015 by an integer d_2, where $672 < d_2 \leqslant 1000$ and obtain remainder r_2, we would have $r_2 = 2015 - 2d_2 < 2015 - 2 \times 672 = 671$. Hence 671 is the largest remainder we can obtain.

25. **D** There are just six ways to colour the positive integers to meet the two conditions:

(a) All positive integers are coloured red.

(b) 1 is coloured red and all the rest are coloured green.

(c) 1 is coloured red, 2 is coloured green and all the rest are coloured red.

(d) All positive integers are coloured green.

(e) 1 is coloured green and all the rest are coloured red.

(f) 1 is coloured green, 2 is coloured red and all the rest are coloured green.

Note that (d), (e) and (f) can be obtained from (a), (b) and (c) respectively by swapping round the colours red and green.

It is easy to see that these colourings meet the two conditions given. We now need to show that no other colouring does so.

We first show there is no colouring different from a), b) and c) in which the number 1 is coloured red. In such a colouring, since it is different from b), there must be some other number coloured red. Let n be the smallest other number coloured red. Then, as 1 and n are red, so is $n + 1$. Hence $(n + 1) + 1 = n + 2$ is also red, and so on. So all the integers from n onwards are red. Since such a colouring is different from a), $n \neq 2$, and since it is different from c) $n \neq 3$. So $n \geqslant 4$ and therefore 2 and 3 are green. But then $2 + 3 = 5$ is green, so $2 + 5 = 7$ is green and so on. So all positive integers of the form $2k + 3$ are green. In particular $2n + 3$ is green, contradicting the fact that all integers from n onwards are red. Hence no such colouring exists.

A similar argument shows that there is no colouring meeting the given conditions other than (d), (e) and (f) in which 1 is coloured green.