EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘PINK’
Thursday 19th March 2015

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 60 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

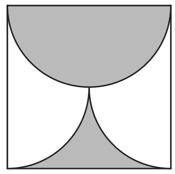
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB non-propelling pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the units digit of the number $2015^2 + 2015^0 + 2015^1 + 2015^5$?

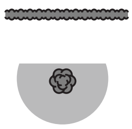
A 1 B 5 C 6 D 7 E 9

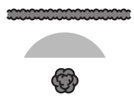
2. The diagram shows a square with sides of length a. The shaded part of the square is bounded by a semicircle and two quarter-circle arcs. What is the shaded area?

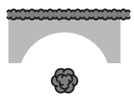


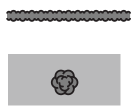
A $\frac{\pi a^2}{8}$ B $\frac{a^2}{2}$ C $\frac{\pi a^2}{2}$ D $\frac{a^2}{4}$ E $\frac{\pi a^2}{4}$

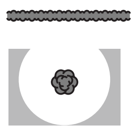
3. Mr Hyde can't remember exactly where he has hidden his treasure. He knows it is at least 5 m from his hedge, and at most 5 m from his tree. Which of the following shaded areas could represent the largest region where his treasure could lie?

A 

B 

C 

D 

E 

4. Three sisters bought a packet of biscuits for £1.50 and divided them equally among them, each receiving 10 biscuits. However, Anya paid 80 pence, Berini paid 50 pence and Carla paid 20 pence. If the biscuits had been divided in the same ratios as the amounts each sister had paid, how many more biscuits would Anya have received than she did originally?

A 10 B 9 C 8 D 7 E 6

5. Each of the children in a class of 33 children likes either PE or Computing, and 3 of them like both. The number who like only PE is half as many as like only Computing. How many students like Computing?

A 15 B 18 C 20 D 22 E 23

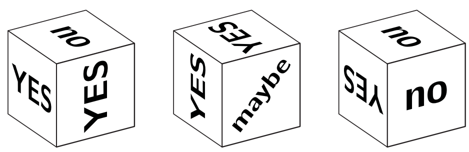
6. Which of the following is neither a square nor a cube?

A 2^9 B 3^{10} C 4^{11} D 5^{12} E 6^{13}

7. Martha draws some pentagons, and counts the number of right-angles in each of her pentagons. No two of her pentagons have the same number of right-angles. Which of the following is the complete list of possible numbers of right-angles that could occur in Martha's pentagons?

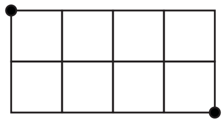
A 1, 2, 3 B 0, 1, 2, 3, 4 C 0, 1, 2, 3 D 0, 1, 2 E 1, 2

8. The picture shows the same die in three different positions. When the die is rolled, what is the probability of rolling a 'YES' ?



A $\frac{1}{3}$ B $\frac{1}{2}$ C $\frac{5}{9}$ D $\frac{2}{3}$ E $\frac{5}{6}$

9. In the grid, each small square has side of length 1. What is the minimum distance from 'Start' to 'Finish' travelling only on edges or diagonals of the squares?

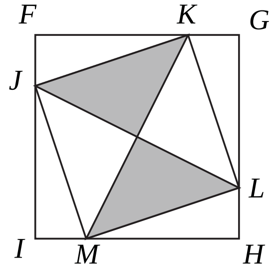


A $2\sqrt{2}$ B $\sqrt{10}+\sqrt{2}$ C $2+2\sqrt{2}$ D $4\sqrt{2}$ E 6

10. Three inhabitants of the planet Zog met in a crater and counted each other's ears. Imi said, "I can see exactly 8 ears"; Dimi said, "I can see exactly 7 ears"; Timi said, "I can see exactly 5 ears". None of them could see their own ears. How many ears does Timi have?

A 2 B 4 C 5 D 6 E 7

11. The square $FGHI$ has area 80. Points J, K, L, M are marked on the sides of the square so that $FK = GL = HM = IJ$ and $FK = 3KG$. What is the area of the shaded region?

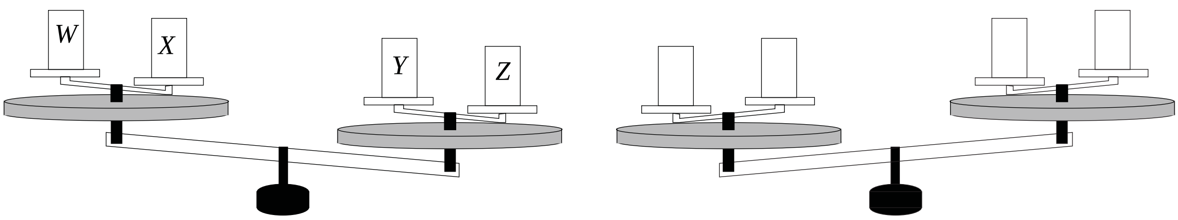


A 40 B 35 C 30 D 25 E 20

12. The product of the ages of a father and his son is 2015. What is the difference between their ages?

A 29 B 31 C 34 D 36 E None of these

13. A large set of weighing scales has two identical sets of scales placed on it, one on each pan. Four weights W, X, Y, Z are placed on the weighing scales as shown in the left diagram.



Then two of these weights are swapped, and the pans now appear as shown in the diagram on the right. Which two weights were swapped?

A W and Z B W and Y C W and X D X and Z E X and Y

14. The two roots of the quadratic equation

$$x^2 - 85x + c = 0$$

are both prime numbers. What is the sum of the digits of c?

A 12 B 13 C 14 D 15 E 21

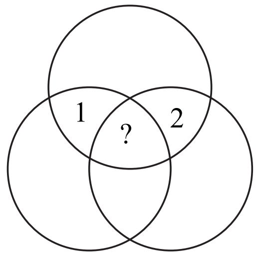
15. How many three-digit numbers are there in which any two adjacent digits differ by 3?

A 12 B 14 C 16 D 18 E 20

16. Which of the following values of n is a counterexample to the statement, 'If n is a prime number, then exactly one of $n - 2$ and $n + 2$ is prime' ?

A 11 B 19 C 21 D 29 E 37

17. The figure shows seven regions enclosed by three circles. We call two regions neighbouring if their boundaries have more than one common point. In each region a number is written. The number in any region is equal to the sum of the numbers of its neighbouring regions. Two of the numbers are shown. What number is written in the central region?



A -6 B 6 C -3 D 3 E 0

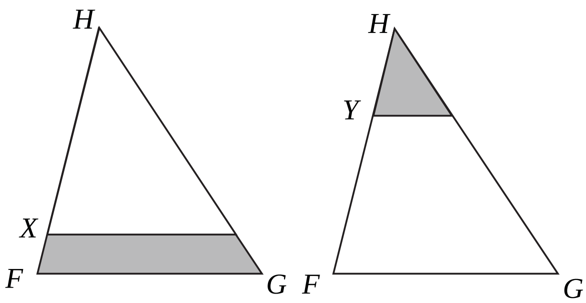
18. Petra has three different dictionaries and two different novels on a shelf. How many ways are there to arrange the books if she wants to keep the dictionaries together and the novels together?

A 12 B 24 C 30 D 60 E 120

19. How many 2-digit numbers can be written as the sum of exactly six different powers of 2, including 2^0 ?

A 0 B 1 C 2 D 3 E 4

20. In the triangle FGH, we can draw a line parallel to its base FG, through point X or Y. The areas of the shaded regions are the same. The ratio $HX : XF = 4 : 1$. What is the ratio $HY : YF$?



A $1:1$ B $2:1$ C $3:1$ D $3:2$ E $4:3$

21. In a right-angled triangle, the angle bisector of an acute angle divides the opposite side into segments of length 1 and 2. What is the length of the bisector?

A $\sqrt{2}$ B $\sqrt{3}$ C $\sqrt{4}$ D $\sqrt{5}$ E $\sqrt{6}$

22. We use the notation $\overline{ab}$ for the two-digit number with digits a and b. Let a, b, c be different digits. How many ways can you choose the digits a, b, c such that $\overline{ab} < \overline{bc} < \overline{ca}$?

A 84 B 96 C 504 D 729 E 1000

23. When one number was removed from the set of positive integers from 1 to n, inclusive, the mean of the remaining numbers was 4.75. What number was eliminated?

A 5 B 7 C 8 D 9 E impossible to determine

24. Ten different numbers (not necessarily integers) are written down. Any number that is equal to the product of the other nine numbers is then underlined. At most, how many numbers can be underlined?

A 0 B 1 C 2 D 9 E 10

25. Several different points are marked on a line, and all possible line segments are constructed between pairs of these points. One of these points lies on exactly 80 of these segments (not including any segments of which this point is an endpoint). Another one of these points lies on exactly 90 segments (not including any segments of which it is an endpoint). How many points are marked on the line?

A 20 B 22 C 80 D 85 E 90