THE INSTITUTE OF ENGINEERS-SRI LANKA

PART I EXAMINATION- NOVEMBER 2010

MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) For each of the following matrices, find the appropriate elementary row operations to describe the transformation from one matrix to the next. Also continue the row reduction until the matrix is in row echelon form.

(i) $\left(\begin{array}{ccc|c} 1 & 4 & 2 & 3 \\ 2 & 6 & 3 & 0 \\ 4 & -2 & 5 & 4 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 4 & 2 & 3 \\ 0 & -2 & -1 & -6 \\ 0 & -18 & -3 & -8 \end{array}\right)$ and $\left(\begin{array}{ccc|c} 1 & 4 & 2 & 3 \\ 0 & -2 & -1 & -6 \\ 0 & -18 & -3 & -8 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 4 & 2 & 3 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 6 & 46 \end{array}\right)$

(ii) $\left(\begin{array}{ccc|c} 3 & 4 & 1 & 3 \\ 2 & 8 & 0 & 2 \\ 0 & 8 & 3 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -4 & 1 & 1 \\ 1 & 4 & 0 & 1 \\ 0 & 8 & 3 & 0 \end{array}\right)$

(b) Hence or otherwise solve the following system of linear equations;

$$x + 4y + 2z = 3$$

$$2x + 6y + 3z = 0$$

$$4x - 2y + 5z = 4$$

Question 2

(a) A particle moves in space so that at time t its position vector is stated as $x=2t+3$, $y=t^2+3t$, $z = t^3+2t^2$. Find the components of its velocity and acceleration in the direction of the vector $2\underline{i}+3\underline{j}+4\underline{k}$ when $t = 1$

(b) Expand $\underline{a}\times(\underline{b}\times\underline{c})$ and $\underline{a}\cdot(\underline{b}\times\underline{c})$ in terms of vectors $\underline{a}$, $\underline{b}$, and $\underline{c}$.

(c) Show that $\underline{a}\times(\underline{b}\times\underline{c})+\underline{b}\times(\underline{c}\times\underline{a})+\underline{c}\times(\underline{a}\times\underline{b})=\underline{0}$, where $\underline{a}$, $\underline{b}$, and $\underline{c}$ are constant vectors.

(2)

Question 3

(a) Find the symmetrical form of the line of intersection of the two planes $x+2y+3z+4=0$ and $x+y+z+1=0$.

(b) Find the coordinates of the foot of the perpendicular from the point (2,-1,3) to the plane $3x-2y-z-9=0$

(c) Find the equation of line through the point (-2,3,4) and parallel to the planes 2x+3y+4z = 5 and 3x + 4y +5z = 6

Question 4

Solve the following differential equations:

(a). $yx^2+(y+7)\frac{dy}{dx}=0$

(b) $(y^2+2xy)dx+(2x^2+3xy)dy=0$

(c) A body of mass m is thrown vertically upwards into the air with an initial velocity v_0 .Assuming that the body encounters air resistance proportional to its velocity, according to the Newton's law, the equation of motion is

$$m\frac{dv}{dt}=-mg-tv\,.$$

(i). Find an expression for the velocity of the body at time t .

(ii). Find the time at which the body reaches its maximum height.

Question 5

Using the method of D operators, obtain the solutions for the following differential equations:

(a). $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=e^{3x}$

(a) $\frac{d^2y}{dx^2}+4y=x^2+Sin2x$

(c) Let $y=(1+4x^2)\tan^{-1}(2x)$

Show that

(i) $(1+4x^2)\frac{dy}{dx}-8xy=2(1+4x^2)$, and

(ii) $(1+4x^2)\frac{d^2y}{dx^2} - 8y = 16x$

Hence, find $\left(\frac{d^3y}{dx^3}\right)_{x=0}$

Question 6

(a) Use mathematical induction to establish the formula $1+z+z^2+....+z^n = \frac{1-z^{n+1}}{1-z}$ $(z \neq 1)$

(b) By setting $z = e^{i\theta}$ and taking real parts of (a), derive the formula

$$1+\cos\theta+\cos 2\theta+...+\cos n\theta = \frac{1}{2}+\frac{\sin\left(\left(n+\frac{1}{2}\right)\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)}$$

(c) Let $\omega_n = \cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}$ prove that $1+\omega_n+\varpi_n^2+....+\varpi_n^{n-r} = 0$

(d) If $\sin(u+iv) = x+iy$, prove that

$$\frac{x^2}{\cosh^2 v}+\frac{y^2}{\sinh^2 v} = 1$$

Question 7

(a) Find the vector equation of the line through the point with position vector $2\underline{i}-\underline{j}-3\underline{k}$ which is parallel to the vector $\underline{i}+\underline{j}+\underline{k}$. Determine the points corresponding to $\lambda = 3,\ 0,\ 2$ in the resulting equation.

(b) Show that the equations of planes of the bisectors of the angles between planes $a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$ are given by

$$\left|\frac{a_1x+b_1y+c_1z-d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}\right| = \pm\left|\frac{a_2x+b_2y+c_2z-d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right|.$$

(c) Hence or otherwise find the equations of the bisecting planes of $2x-y+2z=-3$ and $3x-2y+6z=-8$.

Question (8)

(a) If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$

(b) Show that matrix A satisfies $A^3 - 6A^2 + 6A - 11I_3 = 0$, where I_3 is an identity matrix of order 3.

(c) Hence, deduce that $A^{-1} = \frac{1}{11}\begin{pmatrix} 5 & -1 & -7 \\ -4 & 3 & 10 \\ 3 & -5 & -2 \end{pmatrix}$

Question 9

(a) Find the approximate value correct to three places of decimals of the real root which lies between -2 and -3 of the equation $x^3 - 3x + 4 = 0$, using Newton –Raphson method.

(b) Using Taylor's series method, obtain the solution of the initial value problem: $\frac{dy}{dx} = 3x + y^2$, $y(0) = 1$. Find the value of y for x = 0.1 correct to four places of decimals.

-0-0-0-0-0-0-0-