THE INSTITUTE OF ENGINEERS-SRI LANKA

PART I EXAMINATION- APRIL 2009

MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) If A is a non singular matrix of order nxn, show that
(i) A is a square matrix,
(ii) A^{-1} is an unique matrix, and
(iii) $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$

(b) (i) Express the matrix $A=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ -3 & 1 & 4 \end{pmatrix}$ as the sum of Symmetric and skew Symmetric matrices.

(ii) Find the adj(A) or adjoint of A, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ and verify that

$A(\text{adj}A) = (\text{adj}A)\,A = |A|I_3$. Where I_3 is an identity matrix of order 3

Question 2

(a)Show that there is only one value of k for which the system of equations

$$\begin{aligned} 2x + y - z &= 0 \\ (k-2)x + ky + 2z &= 0 \\ 6x + 3y + (k-1)z &= 0 \end{aligned}$$

has non trivial solution. Solve the system of equations for this value of k.

(b) Find the rank of the coefficient matrix and the augmented matrix of the system of the system of equations

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + y - z &= 16 \\ x + 5y + 8z &= -3 \end{aligned}$$

Hence find the solution of the system of equations.

Question 3

(a) Define the modules and argument (amplitude) of a complex number.

(b)Two complex numbers z_1 and z_2 are represented by polar form as $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ respectively. Show that

(i) $|z_1 z_2| = |z_1||z_2|$.

(ii) $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$

(c)Express following complex numbers in polar form:

(i) 1+i (ii) 3+4i , hence simplify $\dfrac{(1+i)^4}{(3+4i)^3}$

Question 4

(a) Find the equation of the plane which passes through the point (1,2, -1) and which contains the line. $\dfrac{x+1}{2} = \dfrac{y-1}{3} = \dfrac{z+2}{-1}$

(b) Prove that the lines $L_1 := \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $L_2 := \dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ are coplanar and find the equation of the plane containing them.

(c) Find the angle between the planes $2x + y + z + 3 = 0$ and $2x - y + z + 5 = 0$.

Question 5

Solve the following differential equations;

(i) $\dfrac{d^2y}{dx^2} + 4y = x^2 + \sin 2x$

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = e^x + x^2$

(iii) $\dfrac{dy}{dx} = \dfrac{(x^2+2)}{(y-2)}$

Question 6

(a) The table gives the distances in nautical miles of the visible horizon for the given heig[hts in] feet above the earth's surface.

X (height):	100	150	200	250	300	350	400
Y (distance):	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when x = 120 ft .

(b) Find a root of a equation $x^3 - 5x - 9 = 0$, using the bisection method correct to two decimal places.

Question 7

Solve the equations

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

(a) by Jacob's method

(b) by Gauss Seidel method.

Question 8

(a)If $x = u + v + w$, $y = uv + vw + wu$, $z = uvw$ and $f = f(x, y, z)$ prove that

$$x\frac{\partial f}{\partial x} + 2y\frac{\partial f}{\partial y} + 3z\frac{\partial f}{\partial z} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v} + w\frac{\partial f}{\partial w}$$

(b) The focal length of a mirror is given by the formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$. If equal errors k is made in the dimension of u and v show that the percentage error in f is $100k\left(\frac{1}{u} + \frac{1}{v}\right)$

Question 9

(a) Find $\frac{du}{dx}$ if $u = log(x^2 + y^2)$, $x = \sqrt{1+t}$, $y = \sqrt{1-t}$

(b) If $y = e^{\alpha \sin^{-1} x}$ show that,

(i) $(1-x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx} - \alpha^2 y = 0$,

(ii) $(1-x^2)\frac{d^{n+2}y}{dx^{n+2}} - (2n+1)x\frac{d^{n+1}y}{dx^{n+1}} - (n^2+\alpha^2)\frac{d^n y}{dx^n} = 0$, for positive integer n

Hence, or otherwise, expand y into a series in ascending powers of x , as far as the term containing x^5.