THE INSTITUTE OF ENGINEERS, SRI LANKA

PART 1 EXAMINATION –MARCH 2008

MATHEMATICS **Time allowed: Three Hours**

Answer FIVE Questions only .

Question 1.

(a) If $A = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$, Show that $A^2 = A$.

(b) Write down the augmented matrix corresponding to the equations:

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 4 \\ 2x_1 - 5x_2 + 2x_3 &= 1 \\ 5x_1 + 16x_2 - 7x_3 &= 10 \end{aligned}$$

Show by reducing the matrix to its echelon form, that these equations are consistent and solve them.

Question 2

(a) Find the approximate value of decimals, of the real root which lies between -2 and -3 of the equation $x^2 - 3x + 4 = 0$, using the method of bisection four times in succession.

(b). Determine approximately the smallest positive root of the equation $x^3 + 2x - 2 = 0$ correct to two significant figures using Newton's-Raphson iterative method.

Question 3

(a) In estimating the cost of a pile of bricks measures as 2m, 2m and 1.2m length, width and height respectively. The tape is stretched 1 percent beyond the standard length. If the count is 450 bricks to $1m^3$ and brick cost Rs 7,000.00 per thousand bricks. Find the approximate error in the cost.

(b) If A and B aretwo events and P(A) = 0.6. P(B) = 0.3 and P(AU B) = 0.8. find:

(a) $p(A \cap B)$ (b) $p(A' \cap B)$ (c) $p(A \cap B')$

(d) $p(A' \cap B')$ (e) $P(AUB')$ (f) $P(A'UB)$.

(2)

Question 4

Solve by the Gauss- Seidel iterative method the system of equations

$$5x_1 + x_2 - x_3 = 4$$
$$x_1 + 4x_2 + 2x_3 = 15$$
$$x_1 - 2x_2 + 5x_3 = 12.$$

Question 5

(a) A rectangle is constructed in the complex plane with its sides parallel to the axes and its centre situated at the origin. If one of the vertices of the rectangle is $1 + i\sqrt{3}$, find the complex numbers representing the other three vertices of the rectangle. Find also the area of the rectangle.

(b) What domain of the z- plane is represented by

(1) $2 \le |z+3| < 4$ (11) $\pi/3 < \text{amp}(z) < \pi/2$ (111) $|z+2| + |z-2| < 4$

Question 6

(a) Find the modulus and amplitude of $\dfrac{(3-\sqrt{2i})^2}{1+2i}$

(b)Express the following in the form a+ib;

(i) $\dfrac{2-\sqrt{3i}}{1+i}$ (ii) $\dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2}$

(c) If $\alpha + i\beta = 1/(a+ib)$, prove that $(\alpha^2+\beta^2)(a^2+b^2) = 1$.

Question 7

(a) Find the vector equation of the line that passes through the point A with position vector $\underline{a} = 2i - j + k$ and is normal to both the vectors $\underline{b}$ and $\underline{c}$ where $\underline{b} = i + 2j + k$ and $\underline{c} = -i + j - k$.

(b) Masses 1, 3 and 2 units at the point specified by the position vectors $3i - k$, $2i - 3j + k$, and $i + j + k$ relative to point O have accelerations represented by $3j + \underline{k}$, $3i + j + 2k$, and $i - j + k$ respectively. Determine the vector sum of their moments of each of these masses about O.

Question 8

Solve the following differential equations

(i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 3e^{2x}$

(ii) $\frac{d^3y}{dx^3} + 9\frac{dy}{dx} = 0$

(iii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 2 + 5x$

Question 9

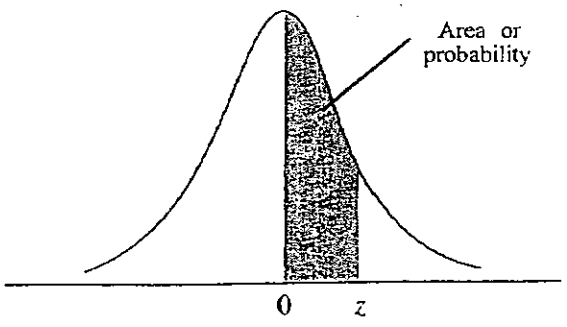
Find the general solution of the following equations:

(i). $(1+2x)\frac{dy}{dx} = 1 - x$

(ii). $(1-y)\frac{dy}{dx} = 1 + y^2$

(iii). $(x+y+1)\frac{dy}{dx} = 1$

Areas for the Standard Normal Distribution



Entries in the table give the area under the curve between the mean and z standard deviations above the mean.
For example, for $z = 1.25$ the area under the curve between the mean and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4578	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4849	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4986	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990