THE INSTITUTION OF ENGINEERS, SRI LANKA

PART 1 EXAMINATION- AUGUST 2007

MATHEMATICS **Time Allowed: Three Hours**

Answer FIVE questions only.

INSTRUCTIONS TO CANDIDATES:

This paper contains 9 questions and 5 pages.

Answer **FIVE** questions and **NO MORE**.

This is a closed book examination.

Each question carry equal marks

Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions made on the script.

If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state it on the script.

Question 1

(a). Find inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{pmatrix}$$

(ii) Hence, or otherwise, Solve the linear system

$$x + y + z = 1$$
$$2x + 3y + 4z = 5$$
$$4x + 9y + 16z = 25$$

(iii) For what values of b is the following system consistent?

$$2x + 3y = 7$$
$$x - y = b$$
$$bx + 2y = 8$$

Question 2

(a) Given that,

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix},$$

find

(i) Minor of A

(ii) Co-factor of A

(iii)Adjoint of A

(iv)Echelon form of A

(v) Determinant of A

(vi)Rank of A

(b) Show that by applying suitable elementary operations to the matrix A we can transform A o the matrix B,

$$B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

Question 3

(a) Define the argument and modulus of a complex number.

(b) Two complex numbers z_1, and z_2 are represented by points P_1 and P_2 on Argand diagram. Show how to construct geometrically the points which represent $(z_1 + z_2)$ and $(z_1 - z_2)$.

(c) Simplify the following complex expressions

(i) $\frac{(2+3i)(5-6i)}{(1-i)^2}$ (ii) $\frac{(2+i)^3(1-2i)^3}{(1+i)^2}$

(c) Interpret geometrically, the following on the Argand diagram where $|z+ik|^2 + |z-ik|^2 = 10k^2$ $(k > 0)$ where $z = x + iy$.

Question 4

Solve the following differential equations;

(a) $\frac{dy}{dx} + ye^x = 5e^x$

(b) $\frac{dy}{dx} + \frac{y}{x} = 5$

(c) $(x+y)\frac{dy}{dx} + (x-y) = 0$

(d) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

Question 5

(a) Write down the position vector OP in terms of the unit vectors $\underline{i}$, $\underline{j}$, $\underline{k}$ given that o is the origin and the point P is (1,1,1), hence find direction ratio and direction cosines of line OP

(b) Evaluate the triple scalar products $\underline{a} \cdot (\underline{b} x \underline{c})$ and $\underline{b} \cdot (\underline{a} x \underline{c})$ given that:
$\underline{a} = -2\underline{i} + 3\underline{j} - \underline{k}$ $\underline{b} = 2\underline{i} - 3\underline{j} + \underline{k}$ and $\underline{c} = 4\underline{i} - \underline{j} - 3\underline{k}$

(c) Find the distance of the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ from the point (-1,-5,-10).

Question 6

(a) If $w = \cosh z$, where $z = x+iy$, express the real part u and imaginary part v of w, in terms of x and y.

Verify that

(i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, and $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$,

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$

(b) If $z = 1 + i\sqrt{3}$, express z in its polar form i.e., In the form $z = r(\cos\theta + i\sin\theta)$.

Hence, or otherwise,

show that (i) $z^2 = -2\bar{z}$

(ii) $\bar{z} = 2 - z$

(iii) $z^3 = -8$

Question 7

(a) Explain briefly the advantages of numerical methods for use solving non linear equations.

(b) Using an appropriate numerical method, find a root of the equation $xe^x - 2 = 0$, correct to three decimal places.

(c) Estimate from the following available data $f(22)$

x	20	25	30	35	40	45
f(x)	354	332	291	260	231	204

Question 8

(a)Find the constant p and q such that $(x-2)$ is a common factor of $(x^3 - x^2 - 2px + 3q)$ and $(qx^3 - px^2 + x + 2)$. Hence find factors of these two expressions completely.

(b) If $y = (1-x^2)^{1/2}\sin^{-1}x$, prove that

(i) $(1-x^2)\frac{dy}{dx} + xy = 1 - x^2$,

(ii) $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = -2x$,

Hence, or otherwise, expand y as a series of powers of x, as far as the term containing x^4.

Question 9

(a) Explain the chain rule of differentiation of an implicit differentiation

(b) Find $\frac{dy}{dx}$ if $x = \frac{t-2}{t+2}$, $y = \frac{2t}{t+1}$ $(-\alpha < t < 2)$, show that $\frac{dy}{dx}$ is always positive. Use th[e] result to sketch the curve of y as a function of x.

(c) Find all stationary points of $f(x,y) = x^3 + y^3 - 3(x+y)$, and determine, if possible, their nature.

(d) Determine the first and second order partial derivatives of the function $f(x,y) = e^{x-y}(\sin x + \cos x)$.

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