INSTITUTION OF ENGINEERS, SRI LANKA
PART I EXAMINATION – MARCH 2008

106 – APPLIED MECHANICS

Time allowed: Three Hours **Date:** 11 April 2008

Before answering the question paper, read and adhere to the instructions given below,

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- This question paper has **two sections**, SECETION A and SECTION B. Answer **only six (06)** questions selecting **maximum three (03)** questions from each section.
- All questions carry equal marks.
- Start answering each question from a new page. Write the relevant question No. on the starting page.
- It is extremely important that you write the question No. to which you answer appropriately in the cage appearing on the cover page
- Pay special attention to the Units. You should answer in SI Units only.
- Strike out any rough work and contents that you do not wish to draw the attention of the examiner.
- Wherever relevant, use free hand sketches to explain your answer.
- Use density of water as 1,000 kg/m^3 and acceleration due to gravity as 10 m/s^2, unless otherwise specified.
- **Marks will be deducted if the above instructions are not adhered to as above.**

SECTION A

Question 1

Figure 1 shows a toggle mechanism. The crank OA rotates anticlockwise. The slider D moves horizontally. Links are smoothly pivoted at A, B and D. In the given configuration, crank OA is rotated at 210 rpm anticloskwise, while the velocity increasing at the rate of 60 rad/s^2. For the same configuration,

(i) list the known and unknown velocity component of each member of the mechanism and determine the velocity of slider D and angular velocity of the link BD.

(ii) list the known and unknown components of tangential and centripetal accelerations of each and every member of the mechanism and determine the angular acceleration of slider D and angular acceleration of the link BD.

OA = 200 mm, AB = 400 mm, QB = 300 mm and BD = 500 mm. X-X and Y-Y are parallel and 400 mm apart. Use a graph paper and draw the acceleration and velocity diagramme to solve this problem.

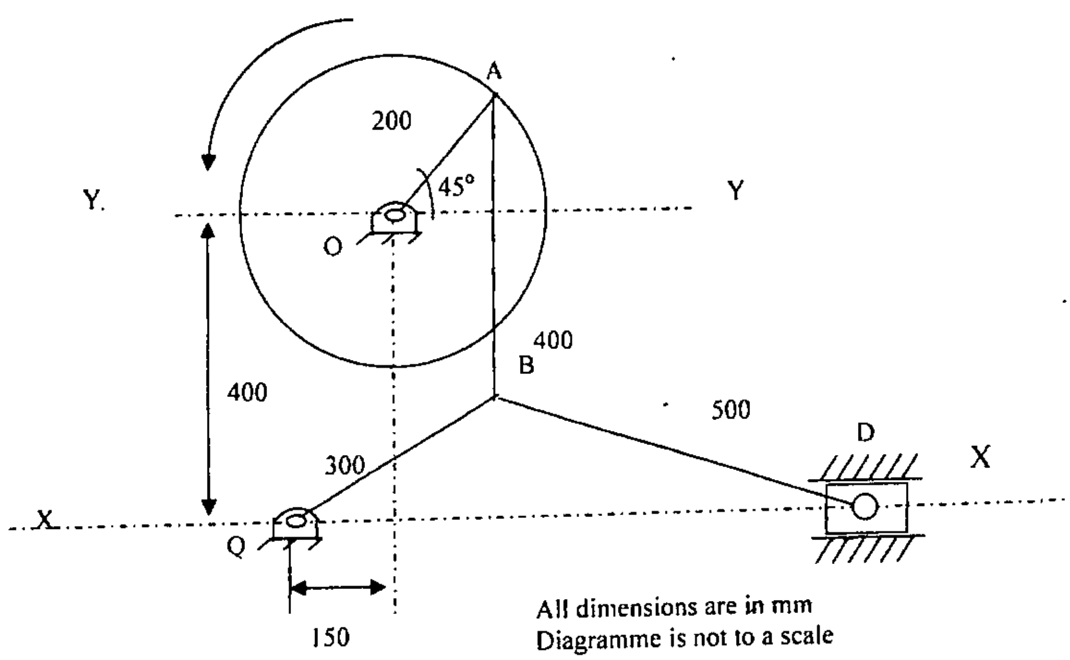

Figure 1

Question 2

(i) An object, weighing **W**, rests on an inclined plane making an angle α [to the] horizontal. A force **F** acts horizontally upwards on the weight. When [the] weight is in equilibrium show that $F = W \tan(\alpha + \phi)$, where ϕ is the angle of friction.

(ii) When the object moves downwards with F acting downwards, express F in terms of W, α and ϕ.

(iii) Explain how the configurations in (i) and (ii) above are analogous to a lifting and lowering a load vertically using a pair of screw and nut with a square thread, of which the helix angle is α and angle of friction is ϕ.

(iv) A load of 20 kN is raised by means of a screw jack. The mean diameter of this square thread is 40 mm and the pitch is 8 mm. A force of 125 Nm is applied at the end of a lever to raise the load. Determine the length of the lever to be used and the mechanical advantage obtained. Is the screw self-locking? Take $\mu = 0.2$.

Question 3

(i) *"Two masses in different planes are necessary to rectify the problem of dynamic imbalance."* Comment.

(ii) A power transmission system consisting of four pulleys A, B, C and D and a shaft that carries the four pulleys is found to be completely balanced. The revolving radii of mass centres of pulleys C and D make angles 90^0 and 195^0 respectively with that of the pulley B measured in the counterclockwise direction. The pulleys have following parameters.

Pulley	Mass (kg)	Distance from axis of rotation to the centre of mass (mm)
A	Unknown	150
B	25	200
C	40	100
D	35	180

B and C are 250 mm apart.

Under the given conditions, determine the following:

(a) .Mass of the pulley A, and the angular position of its plane [relative to] that of B

(b) Position of all the planes on the shaft relative to the plane of the pulley B.

Use **graphical or analytical** method to answer this problem.

Question 4

(i) A circular body of radius r and mass M rolls down on an inclined plane, the plane makes an angle α to the horizontal. The body starts from rest at O which is vertically at a height H from the reference line and the motion of its centre of gravity is referred to axes OX and OY as shown in the Figure Q4(i). The body when at the position C, which is vertically at a height h above the reference line, has a linear velocity V and an angular velocity ω.

(a) Write the three kinematic equations.

(b) Show that these three equations can be deduced to

$$\frac{d}{dt}\left(\frac{1}{2}MV^2\right)+\frac{d}{dt}\left(\frac{1}{2}I_o\omega^2\right)=MgSin\alpha-F(V-r\omega)$$

(c) If the motion is pure rolling and $I = Mk^2$, show that the equation in (b) can be deduced to $\frac{1}{2}MV^2+(1+\frac{k^2}{r})+Mgh=MgH$, and it satisfies the Law of Conservation of Momentum: Kinetic Energy + Potential Energy = Constant

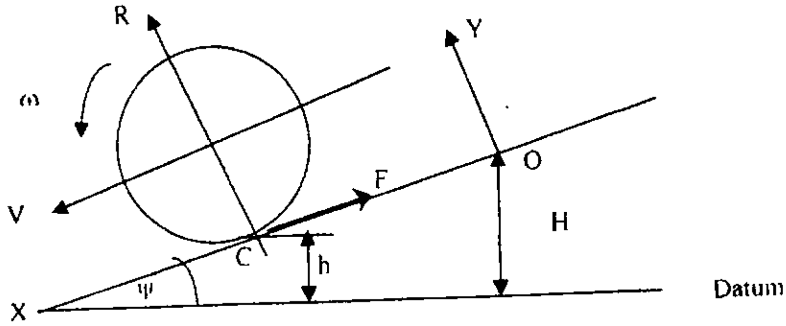

Figure Q 4(i)

(iii) A uniform solid disc of flywheel with mass M, and diam... m has a ***light*** uniform spindle of diameter 12.5 mm. An inexte... is attached to each end of the spindle symmetrically and woun... as shown in the Figure 4 (ii). The upper end of each cord is attach... fixed point, so that the spindle axis is initially at the horizontal posi... When the disk is released from rest, calculate the acceleration of th... centre of gravity in terms of 'g', and the pull P of the cord in terms of M and 'g'.

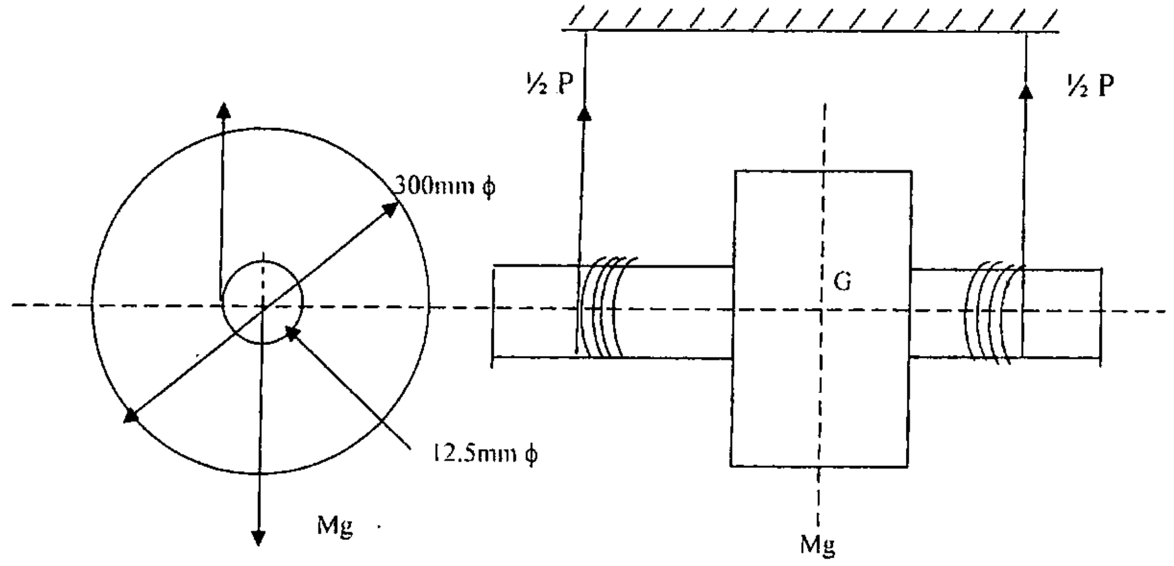

Figure Q4 (ii)

Question 5

(i) What is meant by the damping ratio of a vibrating system?

(ii) Illustrate the standard model of a damped vibrating system (without periodic function) and obtain an equation that represents the vibratory motion of the model.

(iii) Write the solution/s of the equation in (ii) and explain the following types of vibration:

(a) Undamped
(b) Under-damped
(c) Critically damped
(d) Over-damped

(iv) A vibrating system consists of a mass of 50 kg, a spring w[illegible]ess of 30 kN/m and a damper. The damping provided is only 20% [illegible]ical damping. Determine:

(a) the damping factor
(b) the critical damping coefficient
(c) the natural frequency of damped vibration
(d) the logarithmic decrement
(e) the ratio of two consecutive amplitudes

SECTION B

Question 6

(i) A and B are two closed chambers containing SAE 30 oil with specific gravity 0.89 and carbon tetrachloride with specific gravity 1.59 respectively. Air is trapped above the oil and carbon tetrachloride surfaces. The two chambers are connected by a differential manometer as shown in Figure Q 6(i) and it was found that the level difference of the mercury column in the manometer is 0.3 m. Determine the pressure difference between chambers A and B. Assume the specific gravity of mercury as 13.6.

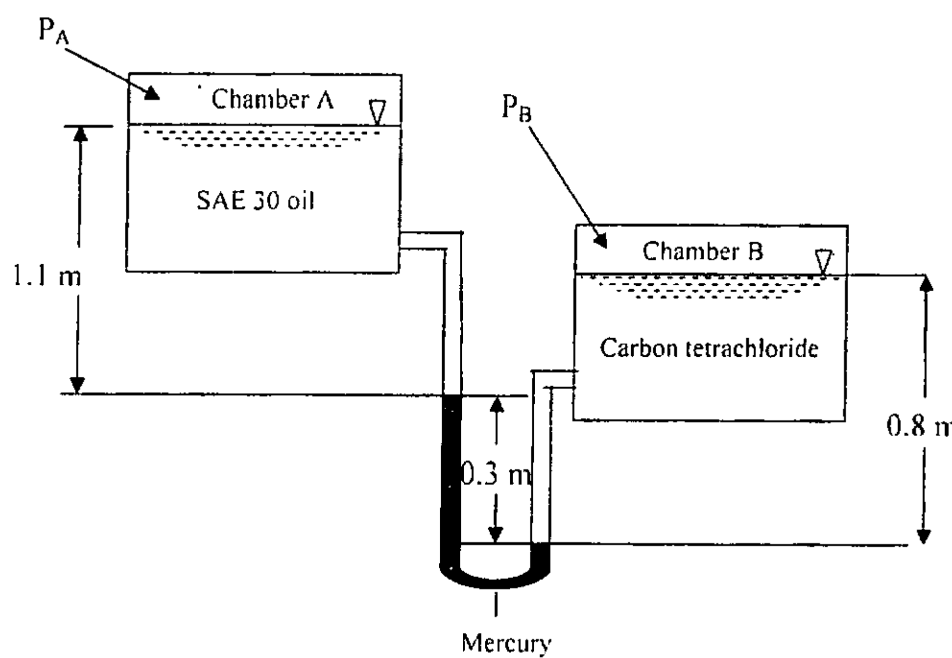

Figure Q6 (i)

(ii) In the arrangement shown in Figure Q6 (ii), the two gl[...]ers, A and B completely filled with water are connected by a[...] U shaped glass tube which contains oil with specific gravity 0.8[...] level difference of the oil column is 0.70 m and the centre [...] container B is vertically 1.5 m above that of the container A. Ass[...] that there is no air trapped in the tube and the containers are complete[...] filled with water, calculate the pressure difference between the containers A and B.

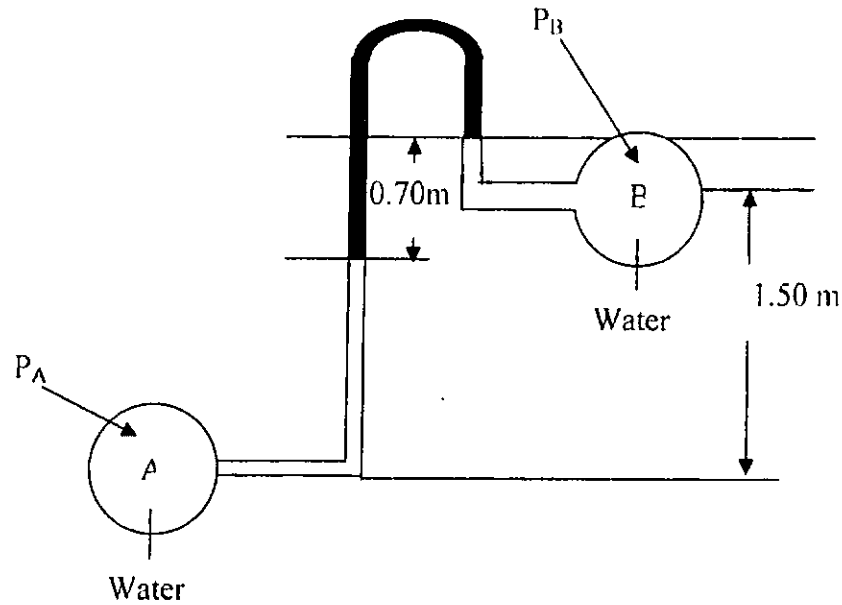

Figure Q6 (ii)

Question 7

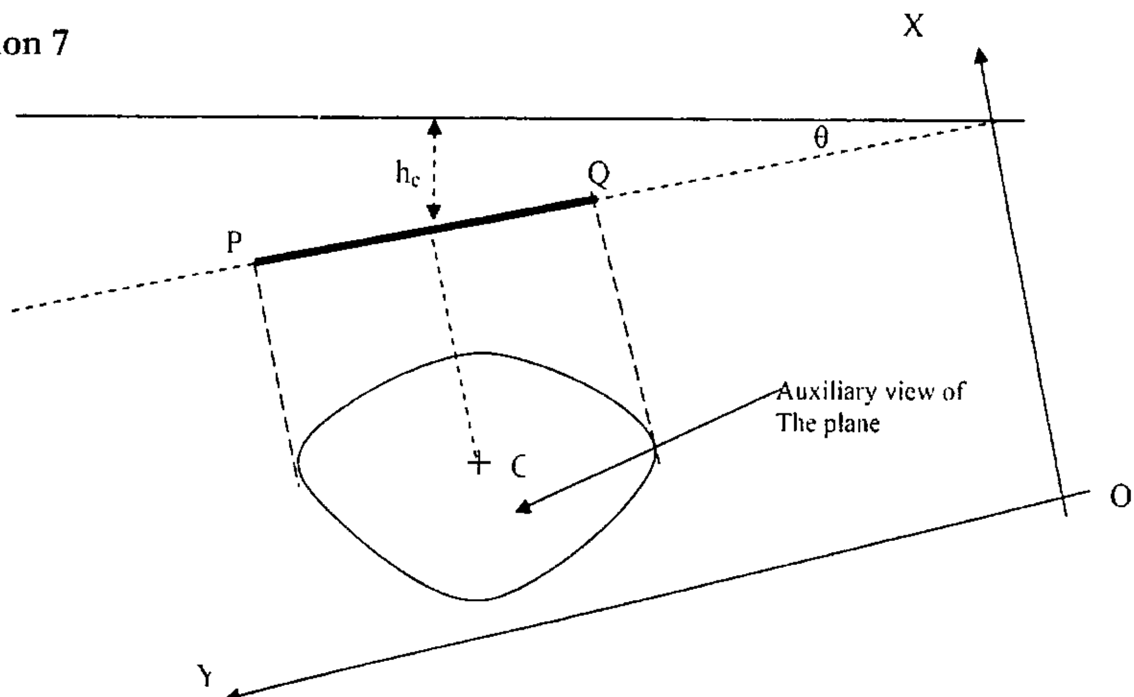

Figure Q7(i)

(i) A plane PQ having an area A is submerged in a liquid, such that ... an angle θ with the liquid surface and its centre of area lying at a dep... from the liquid surface as shown in Figure Q7(i). If the plane has secon... moment area I_x about an axis parallel to OX and passing through the centre of area, derive an expression for the centre of pressure in terms of I_x, A and h_c.

(ii)

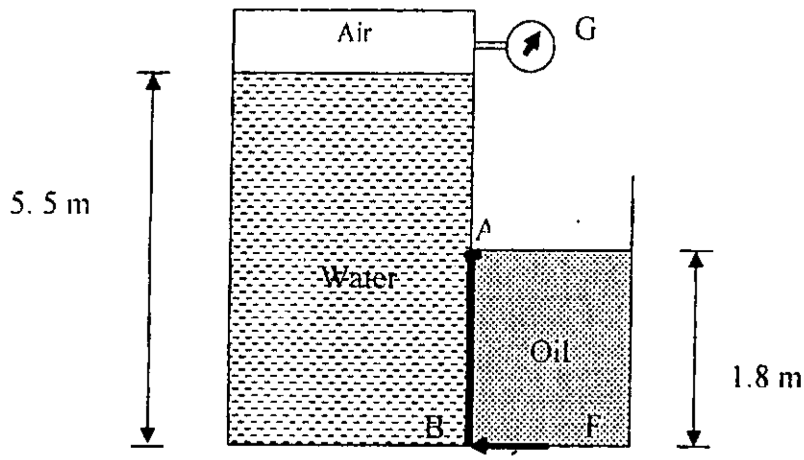

Figure Q7 (ii)

The rectangular gate, AB hinged at A separates two tanks. The left one, which is closed, contains water and the right one, which is open to the atmosphere, contains oil with specific gravity 0.75. The depth of water is 5.5 m and that of oil is 1.8 m. The gate is 1.2 m wide and 1.8 m high. This arrangement is shown in Figure Q7 (ii). The gauge pressure of the air entrapped by the water surface and the container lid is (-)15 kN/m^2 (negative pressure). Under the given conditions, determine the horizontal force required at B for gate to be kept closed.

Question 8

(i) Explain the terms Metacentre, Centre of Buoyancy and Centre of Gravity of a submerged body. What is the significance of these centres in deciding the stability of a submerged body?

(ii) A block of wood having a volume of 0.034 m^3 and we... 0 N is suspended in a reservoir by a rod AB as shown in the Figure Q8... B of the rod is smoothly hinged to the vertical wall of a reservoir, of whi... r level is 310 mm below the hinge. The rod, which weighs 16 N has a l... 3.4 m and a cross section of 2,000 mm^2. When the block and the rod ... equilibrium state, what is the angle of inclination of the rod to the horizontal?

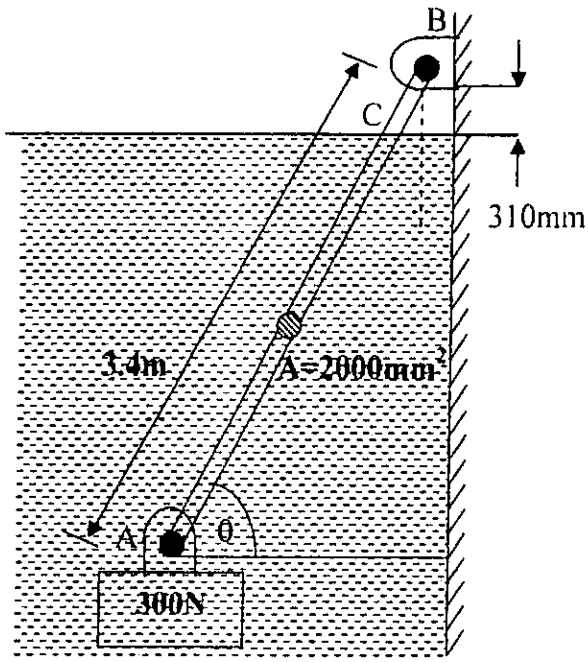

Figure Q8

Question 9

i) A two dimensional velocity field is given by

$$u = y^2,$$
$$v = 3x. \ and$$
$$w = 0$$

At (x, y, z) ≡ (1, 2, 0), compute the following parameters.

(a) Velocity vector
(b) Local acceleration vector
(c) Convective acceleration
(d) The acceleration component parallel to the velocity vector
(e) The acceleration component normal to the velocity vector

ii) A two dimensional velocity field is given by
$\underline{V} = (x^2 - 2y^2 + 2x)\underline{i} - (3xy + y)\underline{j}$. At (x, y) ≡ (2, 2), compute [illegible]ng

(a) X and Y component of acceleration.
(b) Velocity component in the direction of $\theta = 32^0$ with the X axis.
(c) Direction of maximum acceleration and maximum velocity.

Question 10

(i) Starting from first principles, show that the energy loss per unit weight of fluid in laminar flow through a horizontal circular tube can be expressed by

$$\frac{\Delta p}{\gamma} = \frac{32\mu v L}{\rho g d^2}, \text{ where}$$

μ = viscosity, v = velocity, ρ = density, and γ = specific weight for the fluid and

L = length and d = diameter for the pipe.
Δp = Pressure drop over the length L

(iii) Show that the equation derived in (i) above can also be expressed in the following manner:

$$h_L = \frac{fLV^2}{2dg}, \text{ where } f = \frac{64}{\rho V \frac{d}{\mu}} = \frac{64}{R}, \; f \text{ is the friction factor for the pipe.}$$

(iv) A pump delivers oil of viscosity 0.85 N.s/m^2 and specific gravity of 0.92 at a rate of 1,200 litres /minute through a 1,000 m long pipe. The pipe, of which the diameter is 100 mm, has its first 300 m laid along the ground sloping upward at 10^0 to the horizontal and the next 700 m laid in the same way, but sloping upwards at 15^0 to the horizontal.

(a) Investigate whether the flow is laminar, or turbulent.
(b) Neglecting losses due to any bends, and at the pump-inlet and exit, determine the pressure required to be developed by the pump and the power of the driving motor if the pump efficiency is 70%
