THE INSTITUTION OF ENGINEERS, SRI LANKA

IESL ENGINEERING COURSE

PART I EXAMINATION – MARCH / APRIL 2006

106 APPLIED MECHANICS

TIME ALLOWED : THREE HOURS DATE : 10TH APRIL 2006

This question paper has two sections, SECTION A and SECTION B. Answer six (06) questions selecting a maximum of three (03) questions from each of the two parts. All questions carry equal marks.

Also, write the question No. appropriately in the cage appearing on the cover page.

Wherever relevant, use density of water as 1,000 kg/m³ and acceleration due to gravity as 10 m/s².

SECTION A

Question 1

a) Explain the following:

i) Instantaneous center of velocity

ii) Instantaneous center of acceleration

iii) Coriolisis component of acceleration

b) The ends of a link *AB* of length 600 mm slide in horizontal and vertical guides *Ox* and *Oy* as shown in Fig. Q1.

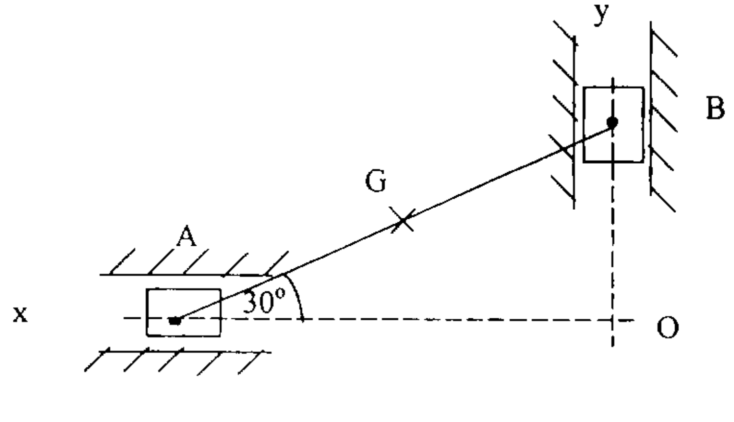

Fig. Q1

For the given configuration *A* has a velocity of 1.5m/s towards O and acceleration of 15 m/s^2 in the same direction.

Draw the given configuration, and velocity and acceleration diagrams clearly and neatly on graph papers to suitable scales. Hence determine:

i) the angular velocity and acceleration of *AB*.

ii) the velocity and acceleration of the end *B*.

iii) the velocity and acceleration of the mid-point *G* of *AB*.

Indicate the scales of each diagrmme.

Question 2

The torque exerted on the crankshaft of an engine is given by the equation,

T(Nm) = 10;500 + 1;620 Sin 2θ - 1,340 Cos 2θ,

where θ is the crank-angle displacement from the inner dead center. The resisting torque is constant.

Show that the mean torque is 10,500 Nm

Hence determine the following:

a) the power of the engine when the speed is 150 rev/min.
b) the moment of inertia of the flywheel if the speed variation is not to exceed ± 0.5 % of the mean speed,
c) the angular acceleration of the flywheel when the crank has turned through 30° from the inner dead center.

Question 3

Fig. Q3 shows a compound pendulum comprising a non-uniform bar having a mass of 24 kg pivoted at a point *O* to swing in a vertical plane. The distance *OG* from pivot to mass center is 0.64 m. The moment of inertia of the bar about a transverse axis through *O* is 12.6 kg m^2. The bar is swung to one side until *G* is level with *O* and released from rest. It comes to rest on the opposite swing 7° below the horizontal. There is a resistance to the motion of the pendulum as a friction torque at *O* of constant magnitude.

a) Determine the angular velocity of the pendulum at the instant OG is first vertical.

b) If OG makes an angle θ_3 with the horizontal when the pendulum again comes instantaneously to rest, show that

$$150.7 \sin \theta_3 + 0.1062\,\theta_3 - 36.73 = 0 \quad (\theta_3 \text{ is in degrees})$$

Assume the air resistance is negligible. Use work and energy equation.

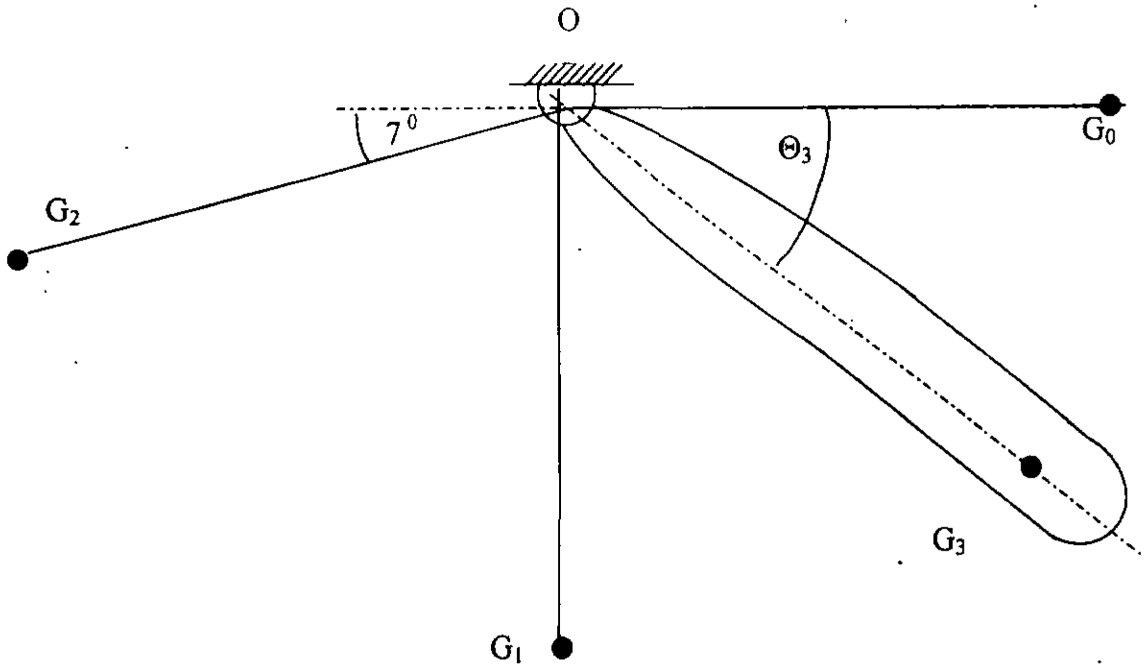

Fig Q3

Question 4

A platform is supported by a set of springs and a dashpot that provides viscous damping. The platform has a mass of 500 kg, springs have an effective stiffness of 72 kN/m and dashpot provides damping 20% of the critical value. The motion of the system is further subjected to a force of 720 N.

a) Show that the motion of the system can be represented by the equation

$$\ddot{x} + 4.8\dot{x} + 144x = 1.44$$

b) The platform is depressed downwards a distance x = 200 mm fr its mid-equilibrium position and then released from rest. Determine the position of the platform when it comes to rest for the first time.

Question 5

a) Explain

(i) Linear momentum

(ii) Angular momentum

(iii) Impulse

b) The winding drum shown in Fig Q5 has a moment of inertia, I = 840 kg m^2 about the transverse axis through the center of rotation and a winding radius, R = 1.9 m.

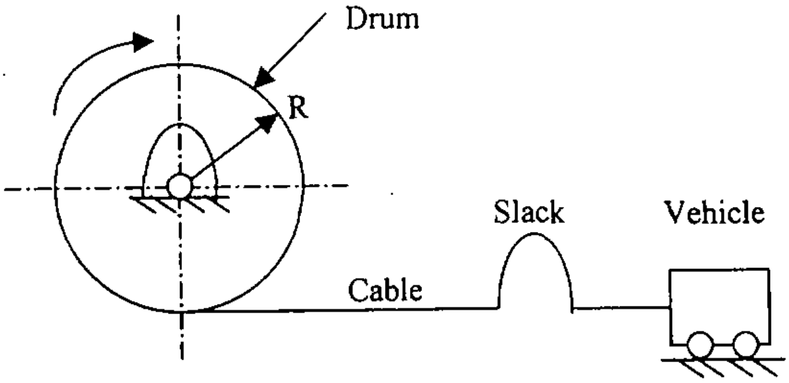

Fig. Q5

It winds a cable turning at a steady angular velocity of 12 ms^{-1}. The free end of the cable is connected to a vehicle of 184 kg mass, which is stationary on a horizontal plane and the cable being initially slack. Once the slack portion diminishes, the cable suddenly tightens.

Calculate

(iv) the speeds of the vehicle and the drum.

(v) the energy lost, and

(vi) the magnitude of the impulse in the cable.

SECTION B

Question 6

(a) From first principles prove that the hydrostatic force on a plane surface placed in a liquid at an angle of inclination α to the liquid level is $\rho g \bar{z}$, where ρ is the density of the liquid and $\bar{z}$ is the vertical distance to the centre of gravity of the plane surface from the liquid level.

Hence show that the centre of pressure locates at a distance $\bar{z}_p$ vertically below the liquid level given by

$$\bar{Z}_p = \bar{Z} + \frac{I_c \, Sin^2\alpha}{A\bar{Z}}$$

where I_c = Second moment of area of the plane about the transverse axis through centre of gravity and A is the area of the plane.

(b) Fig.Q6 shows a tipping arrangement, in which the gate is initially at rest at an angle 60 ^{0} to the horizontal and it starts tipping about the hinge at O when the water level reaches a certain height.

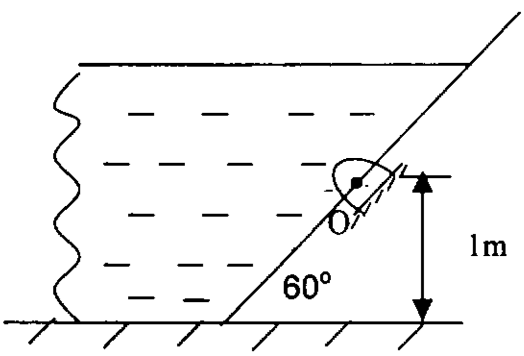

Fig. Q6

What is the depth of water for tipping to commence?

Question 7

a) Define clearly a streamline.

b) At a given time the velocity vector of a stream line
$\underline{U} = u\,\underline{i} + v\,\underline{j} + w\,\underline{k}$

Considering an elementary displacement given as
$\delta \underline{s} = \delta x\, \underline{i} + \delta y\, \underline{j} + \delta z\, \underline{k}$, where $\underline{I}$, $\underline{j}$ and $\underline{k}$ are unit vectors in the directions of x, y and z, show that

$$\frac{\delta x}{u} = \frac{\delta y}{v} = \frac{\delta z}{w}$$

c)

i) State the continuity equation for three–dimensional flow. If this flow is incompressible, explain the change of the above equation.

ii) The velocity in an incompressible fluid flow is given by

$$\underline{U} = (x^3 - y^3)\,\underline{i} + (z^3 - y^3)\,\underline{j} + w\,\underline{k}$$

If this flow has a stagnation point at the origin, determine the third velocity component (w).

Question 8

a) Water flows over a flat plate with the velocity at the leading edge being U and that at the trailing edge increasing linearly from zero to U over a height *h*. The profile of this velocity variation is shown in the Fig. Q8.

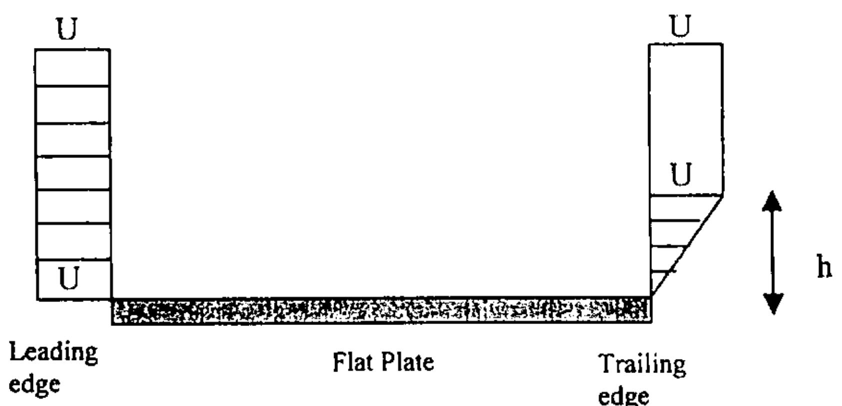

Fig.Q8

Show that the force exerted by the fluid on the plate is $(\rho U^2 h/6)$.

b) A flat plate is towed longitudinally at 10 m/s in water, which is stationary otherwise. It is observed that the velocity at the trailing edge is linearly varying from the edge so that it is zero at the edge and 10m/s at a a level vertically 100 mm above the edge. Calculate the drag on the plate. Assume that the pressure is same everywhere in the flow field.

Question 9

a) A fluid flows steadily with a velocity U through a pipe of constant diameter d. From first principles show that the pressure drop over a length l, is

$$h_f = \frac{fl}{d}\frac{U^2}{2g},$$ where f is the friction factor for pipe roughness.

b) A pipe of diameter D has an arrangement as shown in Fig.Q9 to discharge the flow across its surface at a rate of q per unit length. The up stream flow rate is Q and at the end of the entire length of the pipe the flow rate is zero.

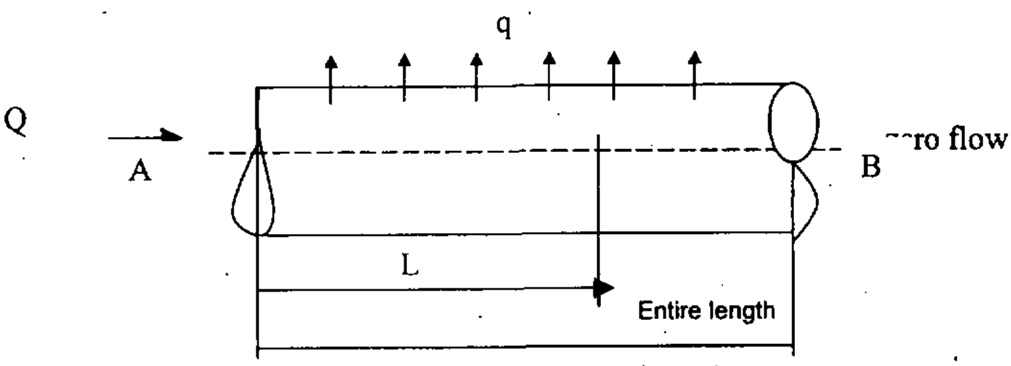

Fig. Q9

Name :- D.C. Wijewardena
Designation :- Lecturer
I.D No. 691332756V

Name :- V.R Jayasekara
Designation :- Engineering Teaching Assistant
I.D No. 742531880V

Prove that the head loss over a length L.

$$h_{fL} = \frac{fLQ^2}{12D^5}\left[1 + \frac{q^2L^2}{3Q^2} - \frac{qL}{Q}\right]$$

Hence show that head loss for entire length (AB).

$$h_f = \frac{fQ^3}{36\,qD^5}$$

Question 10

a) $$\frac{du}{dr} = \frac{1}{2\mu}\frac{dp}{dx}r$$

is a relationship that can be established for laminar flow of a fluid with viscosity μ through a pipe with radius R ($2R=D$). $\frac{dp}{dx}$ is the pressure gradient and u is the velocity at a radius r ($r<R$).

Show that

i) The velocity profile is a paraboloid of revolution.

ii) The total discharge through the pipe

$$Q = -\frac{\pi}{128\mu}D^4\left(\frac{dp}{dx}\right)$$

(iii) The drop in pressure over a finite length L

$$\Delta p = \frac{8\mu U}{R^2}L$$, where U is the average velocity

b) A liquid having a viscosity of 0.098 Ns/m^2 and a specific gravity of 1.59 flows through a horizontal pipe of 50 mm diameter with a pressure drop of 6 kN/m^2 per meter length of the pipe.

Determine the following:

i) The rate of flow in kg per minute
ii) The maximum shear stress
iii) The total drag for 100 m length of the pipe
iv) Power required to maintain the flow through 100 m length of the pipe

Assume that shear stress, $\tau = -\frac{dp}{dx}\frac{r}{2}$
