EXAMINATION

29 April 2009 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

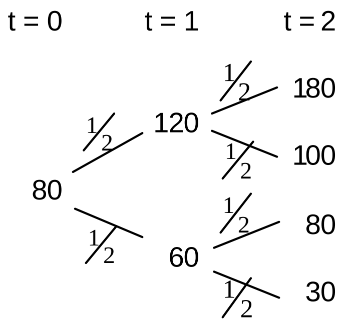
Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

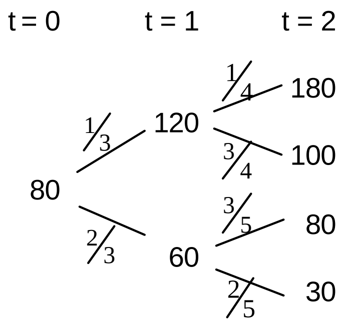
NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 Consider a tree of possible values for a stochastic process at times t = 1 and 2, and two different probability measures **P** and **Q** that could apply to the <u>same</u> tree, as shown below:

Tree of values with probabilities P



Tree of values with probabilities Q



(i) Show, with reasons, that:

(a) **P** and **Q** are equivalent measures.

(b) The process is a martingale under <u>only one</u> of the two measures.

[4]

(ii) (a) Under measure **P**, calculate the one-period drift μ_0 and standard deviation σ_0 (i.e. from t = 0 to 1).

(b) Calculate the values of the Radon-Nikodym derivative of **Q** with respect to **P** for both time periods.

[5]

(iii) (a) State the Cameron-Martin-Girsanov (CMG) theorem in continuous time.

(b) For the <u>first time period</u> only, evaluate as far as you can the continuous time Radon-Nikodym derivative using a (previsible) drift-adjusting process $\gamma = \dfrac{\mu_0}{\sigma_0}$.

[3]
[Total 12]

2 Let X be a random variable under the Normal distribution with zero mean and variance v^2, i.e. $X \sim N(0, v^2)$.

(i) Show that $E[\exp(\theta X)] = \exp(\frac{1}{2}\theta^2 v^2)$ for a constant scalar θ. [2]

[<u>*Note*</u>*:* E[...] *means "expectation with respect to the underlying probability distribution"*.]

Consider a stochastic process $B = \{B_t : t \geq 0\}$.

(ii) Write down the criteria for B to be a standard Brownian motion under a probability measure **P**. [2]

(iii) Derive an expression for $E[\exp(B_s + B_t + B_u)]$, where B is a standard Brownian motion and $0 < s < t < u$. [3]

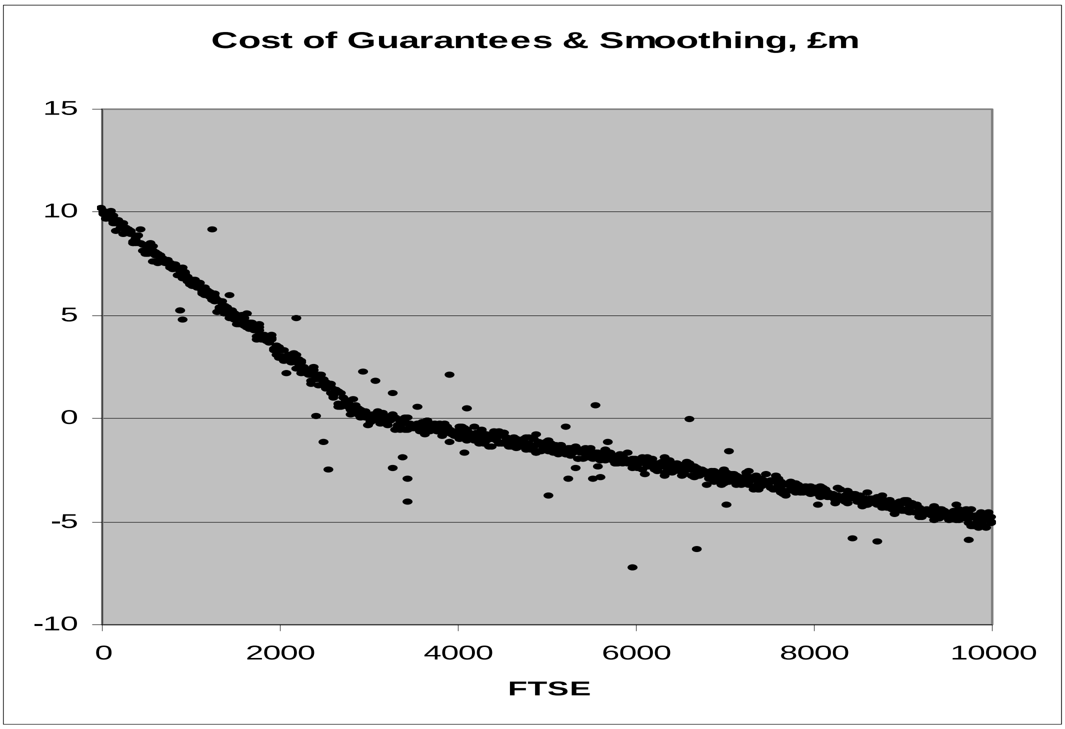
(iv) Explain how a standard Brownian motion process can be adapted to model the price of non-dividend paying equities. [3]

[Total 10]

 PLEASE TURN OVER

3 A life insurance company has written a tranche of with profits bonds that will mature in five years' time and is thinking about hedging the investment risks within the business.

An actuarial student has performed a large number of stochastic runs and has produced a scatter-plot profile of the cost of guarantees and smoothing in five years' time against possible levels of the FTSE 100 index at that time. The insurance company wishes to flatten this profile (i.e. remove both the upside and the downside) using a simple hedging strategy.



The FTSE 100 index currently stands at 4,000.

(i) Suggest a hedge that meets the firm's requirements, using only one plain vanilla FTSE option and one FTSE forward deal. You should describe how the hedge would be set up and perform any necessary calculations. [6]

(ii) Sketch a chart showing the cost of guarantees and smoothing net of the hedge payoff at the end of year five, assuming the hedge proposed in (i) is implemented from the outset. [2]

The FTSE 100 index is not a total return equity index.

(iii) Describe the difficulties that might arise for the life insurance company from this feature when operating the proposed hedge. [3]

[*Note: Tax can be ignored in answering this question.*]

[Total 11]

4 Consider a set of ten large corporate entities, each of whom regularly issues long-dated bonds. (These ten corporates are all based in the same country, and are representative of major companies active in various different parts of the consumer sector — retailers, manufacturers, etc.)

An investor in another country wishes to take exposure to bonds issued by these corporates over a period of five years. However, although the investor does not have precise views on the individual names, he is concerned that one or two of the companies might be particularly vulnerable to a prolonged recession, and is therefore looking to limit exposure to first loss.

(i) Describe the features of:

(a) an *n*th-to-default basket swap
(b) a Collateralised Debt Obligation (CDO) [6]

(ii) Compare the appropriateness, in respect of the investor's stated risk appetite, of using *n*th-to-default basket swaps on the ten corporate names as against buying a tranche of a CDO of bonds issued by each company. [5]

(iii) Explain the impact of an increase in correlation of defaults on the value of the CDO in (ii). [2]
[Total 13]

 PLEASE TURN OVER

5 (i) (a) Explain what LIBOR is, and how it operates in the money markets.

(b) Outline why interest rate swaps and government bonds are valued on different yield curves.

[5]

The UK money market has the following yield curve for par coupon Gilts and interest rate swaps:

	Maturity (years)					
	5	*10*	*15*	*20*	*25*	*30*
Gilt yield (%)	3.00	3.70	4.30	3.60	3.20	2.95
Swap rate (%)	4.30	4.75	5.10	4.15	3.50	3.00

These yields and rates are quoted as annual percentages, and can be interpolated linearly between maturities.

The swap rate is the fixed coupon on an annual fixed-to-floating interest rate swap quoted at par (i.e. with zero mark-to-market value).

In addition, the 25-year zero coupon Gilt yield is 2.95% and the equivalent swap zero yield is 3.05%.

You may ignore the fact that Gilts pay semi-annual coupons and, for the purposes of this question, treat them as if they were annual coupon bonds.

The yield curve has been constructed from the data given above, and below is an extract from years 25 to 30, with three values omitted. (The "discount rate" is the decimal price of a zero coupon bond of a given term.)

Term	*Gilt par yield*	*Swap par rate*	*Gilt discount rate*	*Swap discount rate*
25	3.20	3.50		
26	3.15	3.40	0.47650	0.47093
27	3.10	3.30	0.47023	0.47095
28	3.05	3.20	0.46461	0.47188
29	3.00	3.10	0.45960	0.47370
30	2.95	3.00	0.45518	

(ii) By completing the table, and incorporating any other information you need:

(a) Calculate the 30-year zero coupon Gilt yield and swap rate.

(b) Calculate the five-year forward Gilt yield and the five-year forward swap rate for the period from 25 to 30 years.

[6]

(iii) Suggest reasons for any surprising features of your results in (ii)(a) and (ii)(b).

[3]

[Total 14]

6 A bank has been selling a "home equity release" product for several years. The product is aimed at homeowners without a mortgage, usually married couples, who want to receive an enhanced pension in return for giving up some of the value of their property upon death.

The terms of the product are as follows:

- The bank lends the homeowners a fixed sum (typically a proportion of the property value), which they are free to spend as they wish.

- The homeowners pay no interest directly on the loan, but interest accumulates at a fixed rate *f%* per annum over the rest of their lives.

- They live rent free in the property until the last survivor dies, when the property is sold and the loan repaid from the proceeds:

  - If the proceeds exceed the loan, then the difference passes to the homeowners' estate.

  - If the proceeds are less than the outstanding loan, the bank waives the shortfall (referred to as a no negative equity guarantee or NNEG).

The bank is considering how to value the NNEGs on its balance sheet and has concluded that it should use a valuation formula based on the Garman-Kohlagen model:

$$P = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1)$$

where $d_1 = \dfrac{\ln\left(S_0/K\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$ and T is the time of death of the last survivor, assumed to be fixed.

(i) Identify precisely the option that the bank is writing, and state what K, S_0, r, q and σ represent in the formula. [4]

(ii) Discuss how the values for S_0, q and σ could be determined, and outline any assumptions that would need to be made. [5]

(iii) Describe briefly any difficulties the bank might experience in trying to hedge its NNEG exposure. [3]

[Total 12]

7 Consider a non-dividend paying asset X of current value S and volatility σ, in a market in which the risk-free rate is constant for all maturities. Consider also a European Call option Y based on X with maturity T and an "at-the-money" strike equal to the risk-neutral forward price of X.

(i) (a) Write down the Black-Scholes formula for the price of Y.

(b) Demonstrate, by using a Taylor expansion of your formula in (a), or otherwise, that the theoretical price of Y is approximately equal to $\dfrac{S\sigma\sqrt{T}}{\sqrt{2\pi}}$.

[<u>Hint</u>: You may wish to use the fact that $N'(0) = \dfrac{1}{\sqrt{2\pi}}$.]

(c) Using Put-Call parity, derive a similar approximation for the equivalent "at-the-money" Put option on X. [6]

A dealer wishes to put together a trade where she buys 1-year "at-the-money" straddles (Calls + Puts in equal quantities with identical strikes) on X, and sells 3-month "at-the-money" straddles on X, so that her position is funding neutral.

(ii) (a) Calculate the approximate hedge ratio for her trade.

(b) Describe under what future conditions the strategy might be successful or, indeed, unsuccessful.

(c) Sketch a rough graph of the value of the position against the price of X in three months' time, assuming no other parameters have changed. [7]

During the period that the trade is in place, the risk-free rate drops sharply across all maturities, although this does not appear to affect the price of asset X.

(iii) Describe any impact the rate change might have on the option strategy. [2]
[Total 15]

8 You work in the quantitative valuation department of a life insurance company. You calculate the value of the guarantees on the firm's equity-linked investment products using Monte Carlo techniques. The guarantees depend only on equity prices and risk-free bond prices, although the payoffs can be path dependent.

Your economic scenario generator (ESG) is based around a Hull-White model for risk-free interest rates and a log-normal model for equities, as follows:

$$dr = [\theta(t) - ar]dt + \sigma dz_1$$
$$dS = rSdt + \nu S\sigma dz_2$$

The two sources of stochastic uncertainty, z_1 and z_2, are assumed to be correlated in some way.

(i) Describe how your model would be calibrated to provide a market consistent valuation of the guarantee. Your answer should include:

- a list of the parameters that need to be set
- the order in which the parameters would be calibrated
- a list of potential calibration instruments
- compromises that may need to be made in the calibration

[8]

(ii) Outline how you would use the ESG output for *r* and *S* to calculate guarantee values. [3]

The life insurance company's auditors are worried about the possible size of sampling errors in the calculation of the guarantee values and would like evidence that sampling errors are unlikely to be material.

(iii) Describe, given the calibration used, how you could calculate confidence intervals for the values of the guarantees for any specified probability level.

[2]

[<u>*Note*</u>*: Tax can be ignored throughout this question.*]

[Total 13]

END OF PAPER