EXAMINATION

24 April 2009 (am)

Subject CT5 — Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Define and calculate ${}_{5|10}q_{[40]+1}$.

Basis: AM92 Select [4]

2 Calculate the following functions:

(i) $A^{1}_{40:\overline{20}|}$ [3]

(ii) $A_{40:\overline{20}|}$ [1]

Basis: $l_x = 110 - x$ (for $x \leq 110$).
Interest 4% per annum. [Total 4]

3 Employee contributions to a pension fund are paid continuously at the rate of 4% of salary per annum after a fixed deduction from salary of £5,000 per annum paid continuously.

Determine an expression using commutation functions for the present value of the future contributions by a member aged x with salary S in the previous 12 months. [4]

4 Explain, in the context of the lapse rates of life insurance policies, what is meant by:

(a) class selection
(b) temporary initial selection
(c) time selection

Give an example in each case. [5]

5 A population is subject to two modes of decrement α and β where $q_x^{\beta} = \frac{1}{3} + \frac{1}{4} q_x^{\alpha}$.

Derive <u>from first principles</u> $(aq)_x^{\beta}$.

State clearly any assumptions you make. [5]

6 The random variable T_{xy} represents the time to failure of the joint-life status $(x\ y)$. (x) is subject to a constant force of mortality of 0.02 and (y) is subject to a constant force of mortality of 0.03. (x) and (y) are independent with respect to mortality.

Calculate the value of $E[T_{xy}]$. [5]

7 A life insurance company issues a special annuity contract to a male life aged 70 exact and a female life aged 60 exact. Annuity payments are due on the first day of the month.

Under the contract an annuity of £50,000 per annum is payable monthly to the female life, provided that she survives at least 5 years longer than the male life. The annuity commences on the monthly policy anniversary next following the fifth anniversary of the death of the male life and is payable for the balance of the female's lifetime.

Calculate the single premium required for the contract.

Basis: Mortality: PMA92C20 for males, PFA92C20 for females
Interest: 4% per annum
Expenses: Nil [5]

8 (i) Describe three distinct methods of averaging salary that might be defined in the scheme rules of a pension fund. [3]

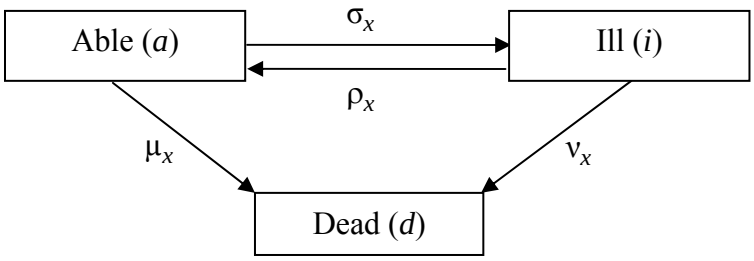
(ii) Define s_x and z_x in the context of a pension fund. [2]

[Total 5]

9 A life insurance company sells a policy with a 10 year term to a healthy life aged 55 exact. The policy provides the following benefits:

- £25,000 payable immediately on death
- £1,000 per annum payable continuously during illness

The company prices the policy using the following multiple state model:



Give a formula for the expected present value of the benefits under the policy. [5]

10 A life insurance company issues a term assurance policy for a term of 10 years to two lives whose ages are x and y, in return for the payment of a single premium. The following benefits are payable under the contract:

- In the event of either of the lives dying within 10 years, a sum assured of £100,000 is payable immediately on the first death if it is the life aged x or £50,000 if the life aged y.

- In the event of the second death within the remainder of the 10 year term, a further sum assured of twice the original claim previously paid is payable immediately on the second death.

Calculate the single premium.

Basis: Mortality: $\mu_x = 0.02$ constant throughout life and $\mu_y = 0.03$ constant throughout life
Interest: $\delta = 4\%$ per annum
Expenses: Nil

[8]

11 A life insurance company issues the following policies:

- 20-year endowment assurance with a sum assured of £75,000 payable at maturity or at the end of the policy year of death if earlier. Level premiums for this contract are paid annually in advance.

- 20-year single premium temporary immediate annuity with an annual benefit payable in advance of £18,000.

On 1 January 2001, the company sold 5,000 endowment assurance policies and 2,500 temporary immediate annuity policies, all to lives aged 45 exact.

(i) Calculate the death strain at risk for each type of policy during 2008.

Basis: Mortality: AM92 Select
Interest: 4% per annum
Expenses: Nil

[4]

During the first seven policy years, there were 65 deaths from the endowment assurance policies and 30 deaths from the temporary immediate annuity policies. During 2008, there were 10 deaths from the endowment assurance policies and 5 deaths from the temporary immediate annuity policies.

(ii) Calculate the total mortality profit or loss to the company during 2008 using the basis in (i) above.

[5]
[Total 9]

12 (i) Explain the terms "unit fund" and "non-unit fund" in the context of a unit-linked life assurance contract. [4]

(ii) Explain why a life insurance company might need to set up reserves in order to zeroise future expected negative cashflows in respect of a unit-linked life assurance contract. [2]

(iii) A life insurance company issues 4-year unit-linked contracts to a male lives aged 50 exact. The following non-unit fund cash flows, $NUCF_t$, (t = 1, 2, 3, 4) are obtained at the end of each year t per contract in force at the start of the year t:

Year t	*1*	*2*	*3*	*4*
$NUCF_t$	375.4	−152.0	−136.2	−118.0

The rate of interest earned on non-unit reserves is 5.5% per annum and mortality follows the AM92 Select table.

Calculate the reserves required at times t = 1, 2 and 3 in order to zeroise future negative cash flows. [4]

[Total 10]

13 A life insurance company issues a 3-year savings contract to unmarried male lives that offers the following benefits:

- On death during the 3 years, a sum of £15,000 payable immediately on death.
- On surrender during the 3 years, a return of premiums paid, payable immediately on surrender.
- On marriage during the 3 years, a return of premiums paid accumulated with compound interest at 4% per annum, payable immediately on marriage.
- On survival to the end of the 3 years, a sum of £5,000.

The contract ceases on payment of any benefit.

Calculate the level premium payable annually in advance for this contract for a life aged 40 exact.

Basis:		
	Independent rate of mortality	AM92 Ultimate
	Independent rate of surrender	10% per annum
	Independent rate of marriage	5% per annum
	Interest	5% per annum
	Expenses	0.5% of each premium

[12]

14 A life insurance company issues a 5-year with profits endowment assurance policy to a life aged 60 exact. The policy has a basic sum assured of £10,000. Simple reversionary bonuses are added at the start of each year, including the first. The sum assured (together with any bonuses attaching) is payable at maturity or at the end of year of death, if earlier. Level premiums are payable annually in advance throughout the term of the policy.

(i) Show that the annual premium is approximately £2,476.

Basis:	Mortality:	AM92 Select
	Interest:	6% per annum
	Initial expenses:	60% of the first premium
	Renewal expenses:	5% of the second and subsequent premiums
	Bonus Rates:	A simple reversionary bonus will declared each year at a rate of 4% per annum

[5]

The office holds net premium reserves using a rate of interest of 4% per annum and AM92 Ultimate mortality.

In order to profit test this policy, the company assumes that it will earn interest at 7% per annum on its funds, mortality follows the AM92 Ultimate table and expenses and bonuses will follow the premium basis.

(ii) Calculate the expected profit margin on this policy using a risk discount rate of 9% per annum.

[14]
[Total 19]

END OF PAPER